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ON KÄHLERIAN COHERENT STATES*

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Abstract. A reformulation of Rawnsley's Kählerian coherent states (in the framework of geometric quantization) is used in order to investigate the interplay between their local and global properties (projective embeddings) and the relationship with Klauder quantization (via path integrals and the introduction of a metric on the classical phase space). A Klauder type formula is established for the projection operator onto the quantum Hilbert space (the kernel of a Bochner Laplacian) in terms of a phase space path integral. As a further application, a Riemann surface diastatic identity is derived, yielding, via Green function theory, a short proof of the Abel-Jacobi theorem (and conversely), together with some coherent state induced theta function identities.

1. Introduction

Coherent states, originally introduced by Schrödinger [20], and generalized in various directions (see e. g. the recent surveys [2] for a thorough introduction to the subject), provide an extremely useful tool for dealing with many aspects of quantum mechanics and play a relevant role in most quantization prescriptions. In this note we further elaborate the notion of coherent state in the Kählerian setting in geometric quantization, due to J. Rawnsley (see e. g. [19, 7, 22]), in view of establishing a relationship with Klauder's quantization. Specifically, Klauder's approach to quantization (via heat equation regularized path integrals over phase space, after introducing a metric thereon, [10, 11] yields the quantum Hilbert space as the (degenerate, in general) Landau ground state space of a "quantum" Hamiltonian (a generalized Laplacian) (cf. also [9, 13] and references therein, see [23] as well) and this is, in turn, close in spirit to the

*This paper is dedicated to the memory of my friend and collaborator Giorgio Valli (1960–1999).