

# USING COGNITIVE AND SITUATIVE PERSPECTIVES TO UNDERSTAND TEACHER INTERACTIONS WITH LEARNER ERRORS

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*Cognitive and situative theories have both proved very useful in furthering our understandings of mathematics learning. An important current area of investigation is to synthesize these perspectives in order to provide more robust theories of learning and to bring theory and practice into better relations with each other. This paper contributes to this endeavour in two ways: 1. by using both theories to understand learner errors, and 2. by focusing on teaching as well as learning.*

## COGNITIVE AND SITUATIVE<sup>1</sup> PERSPECTIVES

The differences between cognitive and situative perspectives are best captured by Sfard (1998). She argues that cognitive/constructivist<sup>2</sup> perspectives view knowledge as a commodity and the metaphor for learning this knowledge is one of acquisition. We acquire or gain knowledge, through the construction of ever more powerful schemata, concepts or logical structures (Hatano, 1996; Sfard, 1998). Self-regulation is the primary mechanism for learning in this perspective. Contextual and social influences, including teaching, are either ignored, or are seen as means for enabling the acquisition of individual knowledge (Greeno & MMAP, 1998). Social processes are secondary processes, which constrain and influence the primary process of self-regulation (Piaget, 1964).

Situative perspectives view learning as participation in communities of practice (Lave & Wenger, 1991; Wenger, 1998). To learn mathematics is to become a better participant in a mathematical community and its practices, using the physical and discursive tools and resources that the community provides (Forman & Ansell, 2002; Greeno & MMAP, 1998), and adding to them (Wenger, 1998). Situative perspectives argue that a focus on conceptual structures is not sufficient to account for learning. Rather, interaction with others and resources are both the process and the product of learning and so learning cannot be analysed without analysing interactional systems.

Researchers who suggest syntheses of cognitive and situative approaches argue for different possibilities in such syntheses. Schoenfeld (1999) and Sfard (1998) argue

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<sup>1</sup> I follow Greeno et al. (1998) in using “situative” rather than “situated” to distinguish a perspective on learning from a particular way of learning. A situative perspective argues that all learning is situated.

<sup>2</sup> Sfard points to differences between information processing views of cognition and neo-Piagetian views, which take a more constructivist and meaning-making approach. I work with neo-Piagetian notions of cognition and constructivism and use these interchangeably in this paper.

that each approach has its strengths and weaknesses, and that we should draw on each in ways that enable coherent progress on particular research projects. For example, Sfard argues that while it may not be helpful to account for learners' thinking only in terms of cognitive structures, it is also not helpful to suggest that we do not impose structure on the world, and that this structuring does not somehow become part of us and help us to make better sense of new situations. Greeno et al. (1998) argue that a situative view is in fact an expanded cognitive view and that we need to develop concepts that will enable us to take features of both into account.

## TEACHING MATHEMATICS

Cognitive and situative theories are primarily theories of learning and as such they entail theories of knowledge. It is more difficult to speak about cognitive and situative approaches to *teaching* mathematics because while theories of learning offer implications for pedagogy and general pedagogical principles, they do not directly lead to particular pedagogical approaches. Pedagogical principles do not derive from theories of learning in a one-to-one relationship. The two different theories might suggest very similar approaches, which are distinguished at the level of explanation rather than the level of practice.

One example of this is the common classroom practice of group work. Both cognitive and situative theories suggest that learners talking through their ideas in groups is a useful pedagogical approach. A cognitive perspective suggests that as learners articulate their ideas, they are likely to clarify their thinking, and develop more complex concepts or schemata (Hoyles, 1985; Mercer, 1995). A situative perspective suggests that as learners consider, question and add to each other's thinking, important mathematical ideas and connections can be co-produced. For cognitive perspectives the group is a social influence on the individual; for situative perspectives the group is the important unit, which produces mathematical ideas beyond the individual ideas. Either one, or both, of these purposes for group work might be operating in a classroom at any particular time.

As learners talk through their ideas, either in groups or in whole class situations, they make errors. Teachers' understandings of learner errors and misconceptions are key to reform visions in many countries. In this paper, I begin to develop ways in which we might think about learner errors from both cognitive and situative perspectives. This is an exploratory paper, drawing on an example of classroom interaction where the teacher deals with a number of errors, one of which proves particularly resistant. I argue that this error needs to be seen in both cognitive and situative terms, and in so doing, I begin to expand the notion of misconceptions to take account of situational factors and teaching-learning interactions.

## ERRORS AND MISCONCEPTIONS

Research into learners' misconceptions has been a key strand of constructivist research (Smith, DiSessa, & Roschelle, 1993). This research shows that many errors

are systematic and consistent across time and place, remarkably resistant to instruction, and extremely reasonable when viewed from the perspective of the learner. To account for these errors, researchers posit the existence of misconceptions, which are underlying conceptual structures that explain why a learner might produce a particular error or set of errors. Misconceptions make sense when understood in relation to the current conceptual system of the learner, which is usually a more limited version of a mature conceptual system<sup>3</sup>. Misconceptions alert us to the fact that “building” on current knowledge also means transforming it; current conceptual structures must change in order to become more powerful or more applicable to an increased range of situations. At the same time the new structures have their roots in and include earlier limited conceptions. Learners’ misconceptions, when appropriately coordinated with other ideas, can and do provide points of continuity for the restructuring of current knowledge into new knowledge (Hatano, 1996; Smith et al., 1993).

Misconceptions can also produce correct contributions (Nesher, 1987). The seminal story of Benny (Erlwanger, 1975) is an example of a learner who constructed many of his own rules for mathematical operations. His rules were partially sensible modifications of appropriate mathematical operations. They were derived from his instructional program and his correct understanding of some mathematical principles. They produced many correct answers and Benny was considered to be a good mathematics student by his teacher. However, many of his underlying understandings of mathematics were incorrect and were never picked up by his teacher.

The notion of misconceptions as part of a cognitive framework suggests that an individual’s conceptual structure can account for her productions in the classroom, and that shifts in conceptual structure can account for learning. Situative perspectives argue that a focus on conceptual structures is not sufficient to account for learning and certainly cannot account for teacher-learner interaction in the classroom. Therefore situative perspectives have not focused explicitly on errors or misconceptions. This is of concern for reform visions of teaching, where teachers are asked to focus on learners’ thinking, which often exhibits errors or misconceptions. However, situative perspectives can give us additional ways to understand learners’ errors. Situative perspectives view learning mathematics as increasingly appropriate participation in mathematical practices using mathematical tools (Forman & Ansell, 2002; Greeno & MMAP, 1998). From this perspective, correct contributions are seen as appropriate uses of tools and resources in a setting. Incorrect productions can be seen as partial or inappropriate uses of the tools and resources in the setting, the use of inappropriate tools and resources, or non-engagement in mathematical practices. Situative perspectives argue that what a learner says and does in the classroom makes sense from the perspective of her current ways of knowing and being, her developing

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<sup>3</sup> For this reason, many authors prefer the terms “alternative conceptions” or “naive conceptions” (Smith et al., 1993), preferring to indicate presence rather than absence. I use the term misconceptions to indicate an absence in relation to accepted mathematical knowledge.

identity in relation to mathematics, and to her previous experiences of learning mathematics, both in and out of school. If learners have come to expect particular ways of working in a mathematics classroom, and of what counts as an appropriate contribution in a classroom, they will continue to make use of these expectations. A learner contribution can be an attempt at speaking about an idea in order to grapple with it, to engage someone else in a process of solving a problem or coming to a joint understanding, or to participate in the conversation, either appropriately or inappropriately. It can also be an attempt to resist the classroom conversation or to disrupt it. Researchers can attempt to document these patterns of interaction and show that patterned regularities exist in these kinds of interactions (Greeno & MMAP, 1998). Social and cultural attunements and patterned regularities may be just as widespread, systematic and resistant to instructional intervention as misconceptions are.

### AN EXAMPLE

This example comes from a larger research study in which I look at how teachers interact with learners' contributions. In the study, I videotaped and analysed two weeks of lessons of five Grade 10 and 11 teachers in South Africa, and conducted a set of interviews with each teacher. For the classroom analysis, I developed a coding scheme for learner contributions (including errors) and teacher moves in response to learner contributions. My methods have been discussed in detail elsewhere. In this paper, I draw on one example from one of the Grade 10 classrooms to show how cognitive and situative perspectives can help to understand a learner error.

The learners had worked on a task the previous day in pairs and handed their work to the teacher. The task was:

Consider the following conjecture: " $x^2 + 1$  can never be zero". Prove whether this statement is true or false if  $x \in \mathbb{R}$ .

The teacher, Mr. Peters, had read all the responses and chosen three, which he used to structure his lesson the following day. The first response was from Grace and Rethabile, that  $x^2 + 1$  cannot be 0 because  $x^2$  and 1 are unlike terms and so cannot be added together. Many other learners had made the same argument. Mr. Peters asked them to explain their reasoning and Rethabile argued:

what we wrote here, I was going to say that the  $x^2$  is an unknown value and the 1 is a real number, sir, so making it an unknown number and a real number and both unlike terms, they cannot be, you cannot get a 0, sir, you can only get  $x^2 + 1$

and

Yes, sir. There's nothing else that we can get, sir. but the 0, sir

As other learners contributed to this discussion, some made a different error, saying that  $x^2 + 1 = 2x^2$ , i.e., they completed the expression, which is a well known algebraic error (Tirosh, Even, & Robinson, 1998). Mr. Peters dealt relatively easily with this error, creating a class discussion and helping many of the students to see their

mistake. Grace and Rethabile's error was of a different kind and proved more resistant. They had not completed the expression incorrectly, and they had the correct answer, that  $x^2 + 1$  cannot be 0, but for the wrong reasons. Their justification was incorrect and, as the second contribution above shows, their reasoning was confused.

Mr. Peters' interpretation of the girls' error<sup>4</sup> was that they saw  $x^2 + 1$  as an immutable unit which could not be simplified, rather than as a variable expression that could take different values depending on the values of  $x$ . He therefore asked the following question:

So it will only give you  $x^2 + 1$ , it won't give you another value. Will it give us the value of 1, will it give us the value of 2?

By asking whether  $x^2 + 1$  could take a range of values and suggesting some possibilities other than zero, Mr. Peters was trying to help the learners to see  $x^2 + 1$  as a variable expression. Rethabile drew on the first part of Mr. Peters' question to argue:

It will give us only 1, sir, because  $x$  is equal to 1, sir

Mr. Peters followed up this response by asking:

How do you know  $x$  is equal to 1?

which led to Grace making the following, rather confusing contribution:

Sir, not always sir, because, this time we dealing with a 1, sir, that's why we saying  $x^2$  equals to 1, sir, because, that's how I see my  $x$  equals to 1, sir, because, a value of 1, only for this thing, sir

Subsequent contributions by Rethabile and other learners suggested that they thought  $x = 1$  because, as they justified it: "there is a 1 in front of the  $x$ ". Again, this is a common error that many experienced teachers would recognise. Mr. Peters noticed this error and worked on it with the class, as he did with many other common errors. In his interviews Mr. Peters showed a deep understanding of the mathematical thinking and misconceptions that might underlie common errors. However, the crucial error made by Grace and Rethabile proved both more difficult to work with, and more difficult for both Mr. Peters and myself as the researcher to understand. There are many points of contradictory arguments and confused reasoning, for example, if they saw  $x$  as 1, why did they not argue that  $x^2 + 1$  was 2?

Mr. Peters spend the remainder of the lesson having the class discuss two other solutions, the first where learners substituted different values to show that  $x^2 + 1$  could produce a range of positive values, and the second where learners argued that  $x^2$  was always positive or 0, so  $x^2 + 1$  would always be positive. Mr Peters spent a lot of time on each of these solutions, emphasising both the testing of the conjecture by substitution and the justification of it by logical argument. In this way, he made available other learners' reasoning as resources for Grace and Rethabile to help them

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<sup>4</sup> Mr. Peters discussed this in an interview with me.

think about the parameters of the task. Even after this substantial discussion, Grace was not convinced, and asked the question:

What about, what's the final, if it's not the zero what is it, sir, if it's not the zero, sir, what's the answer?

Her question suggests that she was still seeing the expression as one that needed to be completed, rather than one that could take on a range of values. Even though others in the class had reached a conclusion that agreed with hers (that  $x^2 + 1$  could not be zero), she did not understand the basis for their conclusion, and needed to know what  $x^2 + 1$  could be, given that it was not 0. She did not seem to accept that it could take a range of positive values. At the end of the lesson, I asked Grace whether  $x^2 + 1$  could equal 10 and suggested that she think about it at home and write a response for me for the next day. She wrote two solutions: first that  $x^2 + 1$  could not equal 10 or any other number because  $x^2 + 1 = x^2 + 1$ ; and second that  $x^2 + 1$  could equal 10, if  $x = 3$ .

## **DISCUSSION**

How can we best understand this interaction from cognitive and situative perspectives? From a cognitive perspective, the two girls and many other learners in the class were struggling with the idea of  $x^2 + 1$  as a variable expression that could take multiple values. Mr. Peters understood this and worked with the learners' ideas, building the lesson around them and using other learners' contributions to do this. He focused on the mathematical reasoning that was required to do the task. However this approach did not help Grace, and possibly other learners, to understand what was faulty in their argument and to restructure their thinking to accept a mathematically correct argument for their conclusion. From a situative perspective, Mr. Peters had provided the learners with a task, which would enable their engagement with the mathematical practices of reasoning and justification. He set up pair work to enable learners' communication and justification processes. In the whole-class discussions he required learners to justify their answers, he probed and pressed their thinking and spoke to them about how they should justify. He also spent much time on other learners' appropriate mathematical reasoning to provide resources for Grace and Rethabile to draw on. Yet Grace and Rethabile still struggled to participate appropriately in the classroom. When asked to justify their thinking, both by the teacher and the researcher, they showed further errors in their thinking. These errors show a number of ways in which the girls were not comfortable in participating in mathematical practices. They did not appreciate the justificatory nature of the task. Having spent many years simplifying expressions, they wanted to continue to do so. They were uncomfortable in reasoning mathematically in the ways in which the task required. They complied with the teacher's requests for justification by trying to say something, even if it was contradictory to their previous position. Grace's written response to the researcher's question shows that she had serious difficulties in reasoning mathematically and could comfortably hold two contradictory positions at the same time. It might also show that she wrote whatever she could think of, hoping

that some of it would satisfy me. This might also have been the case in class discussions; that the learners drew on whatever they could think of to be able to comply with the teacher's requests to participate. This relates to the learners' identities as participants in school discourse, rather than mathematical practices. The ways in which they accessed the resources that Mr. Peters provided did not help them to shift their ways of reasoning mathematically nor to participate appropriately in a mathematics discussion. We might even say that through the interaction they co-produced further errors, inappropriate mathematical reasoning and little engagement with important mathematical practices.

In the larger study, I show that Mr. Peters dealt with errors that were relatively familiar to him as an experienced teacher and that he had mathematical and cognitive explanations for them (see also Tirosh et al, 1998). In addition, he understood that his learners were struggling to come to terms with a different way of engaging in mathematics, that of mathematical reasoning and justification, and talked to them about how to do this. He knew that their prior experiences of school mathematics made it difficult for them to engage in the practices that he was trying to teach. As experienced and successful as he was in his teaching, he was still faced with systematic, patterned errors that came out of both the learners' conceptual structures and their ways of participating in mathematics classrooms. How might he go forward with his quest to teach more genuine mathematics to his learners? Taking a situative perspective, some literature suggests teaching the norms of inquiry classrooms (McClain & Cobb, 2001) or the learning practices required to engage in mathematics in this way (Boaler, 2002). These both take account of the patterns of schooling that need to be changed. From a more cognitive perspective, Sasman et al. (1998) have documented how learners easily hold contradictory mathematical positions, or change their positions from one day to the next. In this paper, I have argued that we have to bring these explanations together. We have to understand both the cognitive misconceptions that learners are working with and their difficulties with mathematical reasoning, as well as their issues of participation in class, including identity issues in defending their positions for the teacher and other learners and the ways in which they understand and use the mathematical tasks and resources presented to them.

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