THE TRANSFORMATION OF MATHEMATICS IN ON-LINE COURSES[1]

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This paper presents some research findings regarding the changes in the mathematics produced by mathematics teachers in on-line distance courses. Predicated on the belief that knowledge is generated by collectives of humans-with-media, and that different technologies modify the nature of the knowledge generated, we have sought to understand how the Internet modifies interactions and knowledge production in the context of distance courses. The research was conducted over a period of several years, during distance courses proffered annually from the mathematics department at UNESP, São Paulo State University, to teachers throughout Brazil, conducted mainly via weekly chat sessions. Findings presented contrast teachers' knowledge production when using the Internet with production of knowledge when using regular dynamic geometry software or plotters.

INTRODUCTION

In this paper, I will report on partial results and new questions that our practice, as a research group, have raised in the process of engaging in virtual interactions with teachers from different parts of Brazil (and in smaller numbers, from other countries in South America). We have developed Internet-based extension courses for mathematics teachers from different levels as one means of addressing social inequalities in Brazil and, at the same time, to research and learn about Internet-based education. Different research questions are being addressed in this project, some of which are related to the nature of the needs that teachers who participate in on-line courses will have, and others to the different opportunities that teachers and researchers may have with the new possibilities offered by the Internet. In this paper, however we will discuss how mathematics can be transformed by the Internet, which we consider to be an interface. There has been a significant amount of research showing that function or geometry software transform the nature of the mathematics that is produced (Noss & Hoyles, 1997). Our own research (Borba, 2004a; Borba & Gracias, 2004) has strongly suggested that different software lead to different possibilities and different mathematics. The most popular case has been the "click and drag" resource of geometry software which enabled many students and teachers to generate conjectures, test them and connect them to "different levels" of demonstration, depending on the level of the students and the teaching objectives.

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However, it appears no questions have been posed regarding the nature of change that on-line interfaces bring to the production of mathematics. In this paper, we will present one model of on-line course that stresses the use of chat, and how such an interface is changing the nature of the mathematics that is being generated in on-line communities such as the one described. Before we do this, however, we will present theoretical views regarding computers and knowledge production, and methodological issues. No literature review on Internet based courses will be presented as very little has been published in most mathematics education journals in English or PME proceedings (see for instance, Pateman, Dougherty & Zilliox, 2003; Høines & Fuglestad, 2004).

THEORETICAL AND METHODOLOGICAL ISSUES

Our research group, GPIMEM[2], has been developing research on the use of different information and communication technologies (ICT) in mathematics education for eleven years. We have developed the theoretical notion of humans-with-media as a means of stressing that knowledge is always constructed by collectives that involve humans and different technologies of intelligence (Levy, 1993), such as orality, paper-and-pencil, and ICT. Different humans, or different technologies, result in different kinds of knowledge production. There is no knowledge produced without humans nor without media.

This notion has provided important insight as we have analyzed how different interfaces, such as graphing calculators or dynamic geometry software, play an important role in knowledge production (BORBA, 2004). In the last five years, we have also started to conduct research on the possibilities provided by Internet. To this end, some members of our group have been researching how collectives formed of humans-with-Internet have constructed knowledge. In particular, we have offered several on-line courses for mathematics teachers as a means of searching for theories and research methodologies that emerge from engagement in different practices (Lincoln & Guba, 1985; Borba & Araújo, 2004). In this sense, we believe that we need to be involved in on-line courses in order to focus on helpful questions and theories.

One transformation that we soon noticed, in terms of research procedures, is that data collection is much more "natural" than in usual face-to-face educational environments. If we are researching in a regular classroom, on a lab environment, we have to deal with issues about how invasive a video-camera may be, or to struggle with students/teachers to write reports on their findings. In on-line distance education courses, filming, voice recording and/or transcribing are "natural" and non-invasive. For instance, using chat as a means of communication generates transcribed data that can be electronically stored (as the reader will see, the use of chat also has other implications in terms of results). Triangulation of data and "member checks" (Lincoln & Guba, 1985) can be easily done through e-mail, as we can always ask: "what did

[2] www.rc.unesp.br/igce/pgem.gpimem.html

you mean when you wrote such-and-such" in a chat session, or in a forum. On the other hand, an immense amount of data is generated by chats, e-mails and forums and, more recently, by video-conferences.

In theoretical terms, some of us have been emphasizing how theoretical perspectives, research questions and research methodologies shape one another (Borba & Araújo, 2004). I believe that the notion that knowledge is always produced by collectives of human-with-media is consistent with the discussion in the last paragraph, in the sense that research procedures and the nature of the interaction change as different media are being used. Research procedures, results and theoretical frameworks shape each other. In the same way, the research question of this paper interacts with these other components: what is the nature of the change provoked by the Internet, a non-human actor, in the production of mathematics? Next we will describe the context of the study.

The On-line practice developed by GPIMEM

Over the last five years, our research group, GPIMEM, has made efforts to connect teachers and researchers who are interested in fostering change in their classrooms. We have offered on-line courses in "Trends in Mathematics Education", or specific topics such as "Teaching and Learning Geometry Using Software". These courses have fostered the development of communities that discuss issues related to the topics presented - teaching and learning of functions and geometry using software, ethnomathematics, modeling, adult education in mathematics, critical mathematics education, and so on.

Courses such as these are of paramount importance in Brazil due to the size of the country and the concentration of knowledge production in the southeastern region, where the states of São Paulo and Rio de Janeiro are located. Internet-based courses are one way of connecting research centers such as São Paulo State University (UNESP) with people in remote locations, where the closest university may be more than several hundred kilometers way.

Each course connects about 20 teachers on-line at regularly scheduled times for a period of about four months. They are designed in such a way that interaction is the key word. The model, which has undergone changes over the last four years, is based on synchronous and asynchronous relationships. We have three-hour chat sessions every week for a four months, and also have bulletin boards and e-mail lists. In the last two versions of the course (2003 and 2004), we used a freeware software environment, Teleduc[3], which requires a server in Linux, but can be accessed by computers that use different platforms. Five courses have been offered since 2000.

These extension courses for teachers have become an environment for research. Our research has shown the transformation of the interaction in these courses, when we

[3] TelEduc é um ambiente de suporte para educação a distância, desenvolvido pelo Nied e Instituto de Computação da Unicamp, sob a coordenação da Profª Drª Heloísa Vieira da Rocha, e disponibilizado no endereço: http://hera.nied.unicamp.br/teleduc.

compare it to our interactions in our regular graduate courses, in which teachers and researchers take part (Gracias, 2003; Borba & Villarreal; *in press*). Based on the assessment made at the end of each course, this model has had a significant impact in terms of bringing members of different communities into the discussion regarding mathematics education, and giving them access to professors from one of the most prestigious mathematics education graduate programs in Brazil, with whom they would otherwise not have an opportunity to interact. The chat has become the principle means of interaction of the course. Forum, an asynchronous tool in on-line environments, has not been used extensively, and the use of e-mail has decreased. A typical course consisted of 11 three-hour synchronous chat sessions. Preparation for a session would be done through asynchronous interactions, mainly e-mail and regular mail. For instance, prior to a session on ethnomathematics, participants would be mailed a book by D'Ambrosio (2001). All the participants were expected to have read it before the session, and two of them (together with myself) would be responsible for raising questions to generate discussion. After the class, a third teacher would generate a summary of the class which would be published in the virtual environment of the course. A different kind of preparation was required when the objective of the class involved doing mathematics; problems regarding the use of function, for instance, would be sent beforehand to the teachers, and they would attempt to solve them before the class. During the chat session, different solutions would be discussed.

The problems were designed to be solved with the use of plotters such as winplot[4]. Since this software is free, teachers could have their own copies installed in their computers. On the other hand, it was not possible to share a figure with the other participants of the course simultaneously. An attached file could be sent to TeLeduc, and everyone could access it minutes later. In this sense, this course was joining together "old" computer interfaces, such as plotters, and "new" ones, such as the Internet. In this paper, we present some of our findings regarding the interaction between teachers and these two types of interfaces.

As a means of explaining this further, I would like to re-emphasize my belief that knowledge is always constructed by collectives of humans-with-media. If the media change, paper and pencil to a plotter, for instance, the manner of teaching the concept of function, for example, will change. For example, a problem that might be particularly provocative and engaging for a collective of students-with-paper-and-pencil could be entirely simple and uninteresting for collectives that include graphing calculators. Might there be analogous changes with intensive use of the Internet? In this paper, we will be presenting excerpts of the interaction of collectives of humans-with-Internet-winplot.

The 20 teachers who took the course each time were, for the most part, high school teachers, but university level teachers, teacher educators and others, such as

[4] http://www.gregosetroianos.mat.br/softwinplot.asp

curriculum developers, have also taken part. It was common in these chat sessions to have simultaneous dialogues, since different teachers would pursue different aspects of a given problem, or would pose a different problem, or talk about something that happened recently in their classroom.

RESULTS

Before we present the main set of data from the 2004 class, we would like to present a short episode that led us to look at the data we have been generating with the on-line courses with different eyes. In the 2003 class, prior to a scheduled chat meeting with all twenty teachers participating in one of the courses, a problem was posed to them regarding Euclidean geometry. Different solutions and questions were raised by all the participants, but one of the student's reflections called our attention. During the discussion, Eliane[5], said: "I confess that, for the first time, I felt the need for a face-to-face meeting right away . . . it lacks eye-to-eye contact". She then followed up, explaining that discussing geometry made her want to see people and to share a common blackboard. In this case, there was no follow-up discussion that to clarify what she meant. While this comment raised some design issues regarding the development of distance education environments, in this paper, we will focus on the conjecture it evoked regarding possible changes in the mathematics practiced in Internet-based environments.

In year that followed, we posed the following problem[6] to the teachers who participated in the course:

> Biology students at UNESP, São Paulo State University, take an introductory course in pre-calculus/calculus. The teacher of this course asks the students to explore, using a graphing calculator, what happens with 'a', 'b' and 'c' in $y=ax^2+bx+c$. Students have to report on their findings. One of them stated: *"When b is greater than zero, the increasing part of the parabola will cross the y-axis . . . When b is less than zero, the decreasing part of the parabola will cross the y-axis."*. What do you think of this statement? Justify your response.

The mathematics involved in the conjecture, and its accuracy according to academic mathematics, is developed in detail in Borba & Villarreal (*in press*). But it is interesting to see how these teachers dealt with it. Some aspects of it were suppressed, since they were seen as irrelevant to the understanding of the dialogue, or because they were part of a different dialogue, as explained in the previous section.

Carlos, a high school teacher, started the debate at 19:49:07 (these numbers indicate the hour, minutes and seconds in when the message reached the on-line course), reporting on what one of his students had said: "When a is negative, or b is positive, the parabola goes more to the right, but when a is negative and b is also negative, the

[5] Eliane Matesco Cristovão, High School teacher, from the 2003 class.
[6] Translation of this problem and of the excerpt from Portuguese into English was done by the author and Anne Kepple.

parabola goes more to the left". He challenged the group to see if the student's sentence could lead them to solve the problem.

Since the debate was not gaining momentum, the professor of the course, the author of this paper, tried to bring the group back to what Carlos had said:

(19:53:15) **Marcelo Borba**: The solution that Carlos' student presented regarding 'a' and 'b'. Does anyone have an algebraic explanation for it?

(19:54:53) **Taís**: It has something to do with the x coordinate of the vertex of the parabola.
(19:55:30) **Carlos:** after a few attempts (constructing many graphs changing the value of 'a', 'b, and 'c') the students concluded that what was proposed by Renata is really true.

The issues at stake are distinct. Carlos tried to do what the professor proposed to the group, but Taís raised a new issue, the vertex idea. As can be observed on the excerpt below, the two issues also have intersections:

(19:57:07) **Taís** : Xv=-b/2a...if 'a' e 'b' have different signs, Xv is positive.

(19:59:16) **Norma**: I constructed many graphs and I checked that it is correct, afterwards I analyzed the coordinates of the parabola vertex Xv= - b / 2a, and developed an analysis of the 'b' sign as a function of 'a' being positive or negative, then I verified the sign of the vertex crossing. . . . with the concavity upwards or downwards, and checked if it was increasing or decreasing. . . .did I make myself clear?

Norma presented her ideas, which according to my analysis, are similar to the one made by Taís, and can be labeled the vertex solution. After further discussion about this, the professor presents another solution based on the derivative of y, y'=2ax+b:

(20:07:03) **Marcelo Borba**: Sandra, . . I just saw it a little differently. I saw it . I calculated y'(0)=b, . . and therefore when 'b' is positive the parabola will be increasing and analogously. . . .

Since a few people said they did not understand this comment, he went back to explain his solution.

(20:10:59) **Marcelo Borba**: . . . as I calculate the value of y', y'>0, then the function is increasing, and therefore I consider y'(0), which is equivalent to the point at which y crosses the y-axis, and y'(0)=b, and therefore 'b' decides the whole thing!!!! Got it?

(20:29:24) **Badin**: The parabola always intercepts the y-axis at the point where the x coordinate is zero. In order for this point to belong to the increasing "half" of the parabola (a>0), it should be left of the x_v, this means x_v should be less than zero. Therefore, -b/2a < 0 is equivalent to -b<0 (remember, a>0). But -b<0 is equivalent to b>0. In other words, if b>0, the point where the graph crosses the y-axis is in the increasing part of the parabola. The demonstration por a<0 is analagous.

At this point, some of the teachers had been discussing the problem and both solutions - the vertex and the derivative – for 40 minutes. The large spaces shown by the clock between the different citations from participants of the course, indicate the size and amount of sections which were not transcribed in this paper, as there were

about four messages per minute. For ten more minutes, additional refinement and shared understanding of the solutions were presented. More examples of people's writing about their understanding in the chat are available in the naturally recorded data. Educational issues regarding the use of winplot, to explore the problem and generate conjectures, were discussed. But what is new about Internet in this case? This is the topic of the next section.

DISCUSSION

Before going further, the reader should be aware that some sentences were omitted to make it easier to follow the interaction, and that the translation suppressed most of the informality and typos that normally occur in this kind of environment. There were other actors involved in the discussion and refinement of the solutions of the problem, but for the purpose of clarity, only a few were included here. When we compare the solution presented by the teachers, the vertex one, to the original situation that took place in a normal classroom situation in 1997, there are similarities and differences. Students used graphing calculators to generate many conjectures for the problem relating coefficients of parabolas of the type $y=ax^2+bx+c$ to different graphs. Similarly, the teachers used winplot (or other software, in some instances) to investigate the problem just described, and later the problem related to Renata's conjecture. In the face-to-face classroom, the professor/author led the discussion, and eventually presented the vertex solution (as he did not know the answer either, at first). The explanation for the conjecture was never written by the students. In an on-line learning environment based on chat, writing is natural, and everyone involved had to express themselves in writing. Although we know that some aspects of writing in a chat situation are different compared to writing with paper and pencil, there is a fair amount of research showing the benefits of writing for learning (see, for instance, Sterret, 1990). The data presented here is insufficient, and the design of the study is inappropriate, to support arguments about "benefits". However, it can be argued that chats transform mathematics education in a similar way that it changed research procedures. Chats, together with human beings, generate a kind of written mathematics that is different from that developed in the face-to-face classroom, where gestures and looks form part of the communication, as well. I believe that collectives of humans-with-Internet-winplot generate a different kind of knowledge, which does not mean that the mathematical results were different. But if process is considered, I believe that we may be on the way to discovering a qualitatively different medium that, like the "click and drag" tool of the dynamic geometry, offers a new way of doing mathematics that has the potential to change the mathematics produced, because writing in non-mathematical language becomes a part of doing mathematics. At this point, it is too early to confirm this, but I believe that this "working hypothesis" (Lincoln & Guba, 1985) regarding the transformation of mathematics by the Internet is one to be pursued in further research.

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