BEING SENSITIVE TO STUDENTS' MATHEMATICAL NEEDS: WHAT DOES IT TAKE?

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This study is about student teachers using questioning to explore the mathematical reasoning of secondary school students aged between 11 and 14 years of age. The student teachers involved were first year students following a four years' initial teaching training course in Malta. The Teaching Triad developed by Barbara Jarowski was used to analyze the students' reports about their questioning. Another aim of the work is to provide the student teachers themselves with a reflective tool for analyzing their own questioning.

THEORETICAL CONSIDERATIONS

The Teaching Triad, developed by Jarowski from an ethnographic study of investigative mathematics teaching, provides a useful method for analyzing teaching. The decisions inherent in teacher discourse are regarded as a complex interplay of decisions of three types; Management of Learning (ML), Student Sensitivity (SS) and Mathematical Challenge (MC). This model was later used to analyze the classroom interactions of other secondary school teachers (Jarowski & Potari, 1998) as well as those of university tutors working with their students (Jarowski, 1999; Jarowski, 2002). In secondary schools, the emerging characteristics of effective teacher interventions involved a harmonious balance of SS and MC interventions. Through SS, the teacher is not only sensitive to the affective responses of the student (SSA) but is also sensitive to the students' cognitive needs (SSC). In providing mathematical challenge, the teacher is prompting students to engage in mathematical thinking and possibly to develop such thinking. This model thus takes into account the social aspect of the teaching/learning situation to include the important role of the teacher in directing such learning. Attention is directed to teachers' planning and to their interventions intended to extend students' mathematical thinking as well as those intended to sustain their interest. As it were, the teacher can be thought of tapping the students' zone of proximal development as described by Vygotsky (1978).

> What then is defined by the zone of proximal development, as determined through problems that children cannot solve independently but only with assistance? The zone of proximal development defines those functions that have not matured but are in the process of maturation, functions that will mature tomorrow but are currently in an embryonic state. These functions could be termed the "buds" or "flowers" of development rather than the fruits of "development". (p. 86)

Treating the teaching/learning interface from the Formative Assessment perspective, Wiliam (1998), argues that support given to learners constitutes formative assessment only if the following five conditions are met:

1. a mechanism exists for evaluating the current level of achievement;
2. a desired level of achievement, beyond the current level (the *reference* level) is identified;
3. there exists some mechanism by which to compare the two levels, establishing the existence of a gap;
4. the learner obtains information about *how* to close the gap;
5. the learner *actually uses this information* in closing the gap. (p.3)

While both belonging to a tradition of socio-constructivist research, the Teaching Triad model and Wiliam's conditions for formative assessment focus on different aspects of the teaching/learning situation. Jarowski's model accounts for an overall view of the teaching/learning situation, emphasizing the need for teachers to plan for learning, to be sensitive to their students' cognitive and affective needs and to offer them mathematical challenge. On the other hand, Wiliam's conditions zoom in on student sensitivity and mathematical challenge and explain how through student sensitivity cognitive decisions (SSC), the teacher helps provoke students' engagement by providing appropriate mathematical challenge. Finally, Wiliam considers that formative assessment also includes that the learner uses such challenge to extend his/her achievement.

THE PRESENT STUDY

The quality of teachers' questioning is undoubtedly one of the crucial factors affecting the quality of pupils' learning. While in a teaching/learning situation, the use of questions can serve a multitude of purposes, the focus of this paper is limited to their use in evaluating pupils' thinking. In this study the use of student teachers' questioning when working in a one-to-one interviewing situation is explored using the Teaching Triad as an analytic tool. This work aims to articulate the strengths and weaknesses of different episodes of questioning intended to evaluate pupils' thinking. The student teachers themselves could also benefit from being involved in this method of analysis, in that they could use it as a tool in reflecting about their questioning.

The data used in this paper comes from an assignment given by the author to first year student teachers following a four-year initial teacher education course. These were asked to give a written test to a secondary school pupil[1] aged between eleven and fourteen years. The student teachers were to use one of the tests on Fractions,

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[1] Although the word "student" may be more appropriate here, in this paper, the secondary school students are being referred to as "pupils". This is done so as to distinguish more clearly from the term "student teacher".

Algebra or Measurement produced by the CSMS (Concepts in Secondary Mathematics and Science) team based at Chelsea College, University of London (Hart, 1981; Brown et al, 1984). After correcting their pupils' work, the student teachers were to interview their pupils on some of the test items with the aim of exploring their pupils' mathematical reasoning and establishing, where possible, two gaps in their pupils' understanding of the topics concerned. The student teachers were further directed to audio-tape their interviews and to write a report about their work. Emphasis was made that they were meant to probe into their pupils' reasoning and that they were not being asked to teach their pupils.

The student teachers concerned were following a B.Ed. (Hons) course specializing in the teaching of mathematics at secondary level. In their first year, the focus of the mathematics course is mathematics content at an undergraduate level. During this academic year, they also have a school observation course where they are assigned tasks of a general level that are not specific to mathematics teaching. By this time, they would not have had any formal teaching experience in mathematics as part of their course. For the forty student teachers concerned, the purpose of this work was an assignment following their first methodology course. This fourteen-hour course was delivered by the present author and involved discussions about (i) the nature of mathematics education, (ii) behaviourist theories of learning, (iii) socio-constructivist views on mathematics learning and (iv) talk in the mathematics classroom.

For the student teachers, the idea behind the assignment was to focus on an individual pupil's reasoning prior to setting further activities in an attempt to further his/her mathematical thinking. The whole cycle of a teaching/learning process can be considered as including all the five of Wiliam's conditions for formative assessment cited in the previous section. On the other hand, the student teachers' work was limited to the first three of these. The test and the interview constitute the mechanism for determining the current level of achievement as well as identifying a desired level of achievement beyond the current level. The student teacher was then to use this information in order to describe more fully, where possible, two gaps in the pupils' knowledge.

In analyzing the excepts provided by the students, use was made of the Teaching Triad. Since the purpose of the students' work was to explore their pupils' reasoning, the analysis is focused on the student sensitivity exhibited by the student teacher in the cognitive and affective domains (SSC and SSA). Three excerpts from the student teachers' interviews are discussed in the next section.

RESULTS

Case I

When the student teachers were given the assignment, it was emphasized that the aim was to sort out their pupils' thinking and for this reason, it was important for them to refrain from prompting the correct answers themselves. Still, some students ended up prompting their pupils in this manner. A number of student teachers were lured into this strategy and after handing their assignments, a few students also claimed that they were tempted into prompting their pupils to get the right answer and found it very difficult to refrain themselves from doing so. One particular example is taken from Caroline's work while she was interviewing Kathy on question 4 of the Algebra test (see Fig. 1). As with all other names used in this paper, Kathy and Caroline are not the real names.

4(ii).	n multiplied by 4 can be written as $4n$. Multiply each of these by 4:		
	8	$n + 5$	$3n$
	$\underline{32n}$	$\underline{4n + 5}$	$\underline{12n}$

Fig 1: A reproduction of Kathy's written response to qn 4(ii)of the Algebra test

From the written script, Caroline noted that Kathy multiplied 8 by 4 and $3n$ by 4 correctly but failed to get the correct answer in multiplying (n + 5) by 4. In fact, Kathy's answer to this multiplication was $4n + 5$.

I: How did you work it out?
K: Uff, I got confused there. See…4 times 8 equals 32…not so?
I: Yes, go on. How did you work the one with $3n$?
K: $3n$ times 4 equals… so 3 times 4 is 12 and there is n as well. So there is 12 times n equals $12n$… yes?
I: Ehe. What about the one in the middle?
K: Well, $n + 5$ times 4 is $4n + 5$. Not so?
I: If you had the first and last one (before multiplying by 4) what do you have?
K: So the 8 and $3n$ plussed?
I: Ehe. What happens?
K: Not $8 + 3n$?
I: So $8 + 3n$ is the same as $3n + 8$, no? So if we do $3n + 8$ times 4, aren't we doing ($3n$ times 4) plus (8 times 4).
K: Ehe, so the answers are added … giving $12n + 32$.
I: Ok, so you are saying that $3n + 8$ times 4 give…?
K: $12n + 32$.
I: Right. So when we had $n + 5$ times 4 – isn't it like you had (n times 4) + (5 times 4)?
K: Yes, I see. I understood! The answer is $4n + 20$
I: Fine. Well done!

This excerpt in fact confirms that one particular gap in Kathy's reasoning is the need for her to appreciate the property of distributativity of multiplication over addition, a gap that was already apparent from the correction of Kathy's script. The interview yielded nothing new about the cognitive meanings Kathy attaches to these algebraic expressions. Instead, Caroline gave Kathy a rule that could be applied in similar problems- a rule that in no way connects to other mathematical ideas. The affective response in the last two lines of the excerpt similarly reflect that both Caroline and Kathy were pleased with this outcome. Implicitly, the student teacher is here confirming what Kathy has already learnt, namely that getting the answer right is what counts. The uncertainty in Kathy's first comment, "Uff, I got confused there", remains unexplored. Why was she confused? Was she simply unsure of the answers or of some of them? Overall, Kathy is not being challenged to engage in any mathematical thinking further than the procedural thinking involved in knowing how to act in a similar situation.

Case II

In a number of student teacher assignments, the verbal prompts given by the student teachers suggest that the pupils' initial method of solution of the set problems were completely disregarded by the student-teacher. A case in point is that of Stefan's assignment who gave the Fractions Test to Maria. Stefan chose to interview Maria on question 24 whose written solution to this question is shown in Fig. 2. The excerpt from the interview regarding this question follows.

24. A relay race is run in stages of $\frac{3}{4}$ km each. Each runner runs one stage. How many runners would be required to run a total distance of $\frac{1}{8}$ km?
$\frac{1}{8}$ $\frac{1}{8}$ $\frac{1}{8}$

Fig. 2. A reproduction of Maria's written response to question 24

I: Maria, look at this question (pointing to question 24 now. Why did you write $^1/_8$, $^1/_8$ and $^1/_8$?
M: Heq… because $^1/_8$, $^1/_8$ and $^1/_8$ and …emm, I don't know!

At this point in the assignment Stefan comments that it was evident that Maria was really confused and thus he decided to help her build her confidence. The interview continues:

I: Maria, How many Lm2 (two maltese liri) are there in Lm10? (Stefan also wrote down the problem for Maria on a piece of paper).
M: Lm2, Lm2, Lm2, Lm2 and Lm2 (using her right hand to count). There are 5 in all.
I: Very good Maria, now explain to me how you can do this using multiplication or division
M: Heq, 10 divided by 2 and the answer is 5.
I: Yes, that's right, Maria! … If now you have Lm $2^1/_2$ and Lm $12^1/_2$ instead of Lm2 and Lm10 (Stefan rubbed out the Lm 2 and Lm10 on the problem)

13 M: The same… Lm12$^1/_2$ division by Lm2$^1/_2$.

14 I: That's right. So if you need to find the fraction made by a quantity of another

15 quantity you just need to apply the normal operations we use for 1,2,3,4,

16 …etc. Therefore in question 24, you just need to do $^3/_4$ division by $^1/_8$.

17 M: Yes! I was going to do it like that, that's why I wrote $^1/_8$, $^1/_8$, $^1/_8$ … but then

18 I got confused.

The first comment is about Stefan's student sensitivity as far as the cognitive is concerned. Maria's written response suggests that she was attempting repeated addition to solve the question (see Fig.2). Similarly, Maria's initial response to the to the whole number division problem (line 6) posed by Stefan follows the same strategy. By shifting away from Maria's method towards the more formal division method, Stefan is closing down on the possibility of exposing why Maria got stuck with the repeated addition method when working on the fractions problem. Instead, as far as Maria's mathematical processes are concerned, the interview does nothing more than to indicating even more strongly, that Maria's initial strategy to question 24 was in fact the use of repeated addition.

On the affective side, on seeing that Maria got confused, Stefan took over and gradually prompted "his" method for a solution. The stress that arose when Maria got confused is very understandable and it is clear that if one would like to explore Maria's thinking more fully, it is necessary to learn how to handle such stress and utilize it in a positive manner. The next excerpt provides a clue on how this can be done.

Case III

3. A piece of ribbon 17cm long has to be cut into 4 equal pieces.

Tick the answer you think is most accurate for the length of each piece.

(a) 4 cm, remainder 1 piece

(b) 4 cm, remainder 1 cm

(c) 4$^1/_4$ cm

(d) X $^4/_{17}$ cm

Fig 3: A reproduction of Pauline's response to qn 3 of the Fractions paper.

In this excerpt, the interviewer, Sandra is asking twelve-year old Pauline about her written response to question 3 of the Fractions paper. (see Fig, 3).

1 I: Now… how did you work out number 3? You can use rough work if

2 you like.

3 P: The question… A piece of ribbon 17cm long has to be cut into 4 pieces. When cut, I think it comes to … (*points to* $^4/_{17}$ cm) four over seventeen.

4 I: But why? Did you try to work it out?

5 P: … Boqq… I forgot how I did it
6 I: Ok, don't worry… try it again.
7 P: So, since I saw these two numbers (*points to 4 and 17 in the question*),
8 … I thought this was the answer (*points to* $^{4}/_{17}$ cm).
9 I: Ok.
10 P: But if you think it out, it is four… you take the four times table, it is sixteen
11 not seventeen. So it is 4 cm remainder 1.
12 I: Now do you think that 1 cm can be broken down into 4 pieces in some way?
13 P: Eh…yes
14 I: How long would each piece be?
15 P: 0.2
16 I: Try to add 0.2 for four times.
17 P: You can cut out 0.2,0.2,0.3,0.3. … *interview continues.*

In this case, the interviewer, Sandra is not prompting Pauline with an alternative method. There is an initial negative response from Pauline (line 5) when she says "…boqq… I forgot how I did it". Here Sandra does not give up on unfolding Pauline's ideas who reveals that she was in a sense guessing at an answer. The mere "O.k." (line 9) in Sandra's response implies an acceptance of Sandra's thinking. This served her to think further and to come up with a more meaningful answer in her next response (lines 10-11). Sandra's next question (line 12) again works with Pauline's earlier construction of the string as 4 pieces of 4cm and 1 cm left over. Later on in the interview (not included here), Sandra discovers another of Pauline's difficulties; she could not work out 0.5 ÷ 2.

DISCUSSION

The high incidence of student teachers' prompting of the 'teacher's method' calls for comment. This is especially significant given the emphasis made that once the assignment called for exploration of the students' reasoning, they were to refrain from prompting a solution to the set questions. There may be various reasons why prompting is so persuasive.

For one, in their own learning experiences, these student teachers would have been heavily exposed to the traditional transmission method or the 'teaching by telling method' (Seegers & Gravemeijer, 1997). This behaviourist approach rests on the belief that learners are passive knowledge receivers and need to be told. Consequently the questioning is not directed towards what the pupils already know but at what is considered that they should know. Another reason for prompting could be that the student teachers themselves were not sufficiently flexible in their mathematical thinking to allow them to recognize different possible methods of struggling with the set questions. This is particularly relevant in this case because of the student teachers' inexperience of teaching.

A third reason emerges from the results of the three interviews discussed in this paper. Unlike the previous cases, there was no evidence of prompting in Case III. This interview stands out in the affective responses of the interviewer. She reacted in a very positive way to the pupil's frustrations and accepted that confusion and getting incorrect answers is part of the learning process. The focus was not that the pupil gets the correct answer at each stage, but rather that the pupil engages in thinking about her work. In short, she was showing profound respect for her pupil's mathematical thinking.

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