MEDIATION AND INTERPRETATION: EXPLORING THE INTERPERSONAL AND THE INTRAPERSONAL IN PRIMARY MATHEMATICS LESSONS

Mike Askew

King's College, University of London

This paper describes a theoretical model for examining teaching and learning in primary mathematics classrooms. The model in intended to be both analytical – to provide insights into classroom incidents – and heuristic – to inform planning and practice. This paper reports on the elements of the model, which are based on Vygotskian theory and encompass both the interpersonal and the intrapersonal. An example is provided illustrating how the model can be used to examine the meaning making processes of individual pupils.

1 INTRODUCTION

The focus of this paper arises from interest in how teachers and pupils co-construct mathematical meaning, through the dialectic between the processes of participating in mathematics lessons, and the processes of individual cognition (Cobb & Bauersfeld, 1995). This interest is both theoretical and empirical. Over the course of a five-year longitudinal research programme (the Leverhulme Numeracy Research Programme) a team of researchers has sought ways of developing a theoretical model to help to explain the differential acquisition of mathematics that was observed (see for example, Askew, Brown, Denvir, & Rhodes, 2000).

2 THEORETICAL BACKGROUND

The model is based in a Vygotskian theoretical framework that assumes learning to precede intellectual development through mediated transactions (Cole, 1996; Wertsch, 1991). Matusov (1998) distinguishes between a view of development as coming about through participation or through internalisation, arguing that a preference for one or other of these theoretical positions comes from different world-views. In discussing Matusov's work, Daniels (2001) distinguishes between 'skills and functions in the 'internalisation thesis' and 'meaning' in the 'participation antithesis' (p. 40).

However, rather than preferring one position – participation or internalisation – we consider it is necessary to work with both (whilst sharing other writers' concerns with the use of the metaphor of 'internalisation'). While pupils' participation in mathematics lessons may shape the mathematical understandings that they acquire, it cannot determine then uniquely: one only has to witness the range of meanings demonstrated by pupils who have participated in the 'same' lesson. Hence in the development of a model, we seek to understand both the interpersonal and the intrapersonal and the relationship between the two. Other models that influenced our thinking, in particular that of Saxe (1991) incorporated both these aspects, but we sought to develop a model that would also enable us to examine the inter- and intra-personal on their own, as well as in conjunction. Thus there are two parts to the model: an observable set of parameters examining mediating means and a more interpretive set of parameters exploring personal meanings.

3 THE THEORETICAL MODEL: MEDIATING MEANS

At the observable level we found the following four parameters to be the most helpful, in terms of framing both our observations and the subsequent analysis: *tasks*, *artefacts*, *talk* and *actions*.

3.1 Tasks

The mathematics lessons that we observed were all based around a *task* or *tasks* that the teacher initiated for the pupils to work on, and in this, we consider them to be typical of mathematics lessons in England. The teacher herself may have determined the actual nature and content of the *tasks* or she may have directed the pupils to work from a text-book or work-sheet. Whatever the origin, we take *tasks* to be a key mediating means for working with pupils on mathematical meaning. At the observable level, we take *tasks* to be publically set up and initiated, linked, as we argue below, to individuals' sense making; to individuals' *activities*.

3.2 Artefacts

The everyday usage of *artefact* is simply to refer to a material object. While classrooms are clearly full of material objects, our definition of *artefacts* goes beyond 'brutally physical objects' (Bakhurst, 1995). *Artefacts* have a dual nature, having not only a 'material' dimension but also an 'ideal' (conceptual) dimension:

> (T)he artefact bears a certain significance which it possesses, not by virtue of its physical nature, but because it has been produced for a certain use and incorporated into a system of human ends and purposes. The object thus confronts us as an embodiment of meaning, placed and sustained in it by 'aimed-oriented' human activity (Bakhurst, 1995)

In this sense mathematics classrooms are the home of many artefacts: hundred squares, number lines, base ten blocks, etc. Children's fingers or other body parts would also be considered as artefacts when used for counting or calculating. We also consider more transitory objects to be artefacts: symbols, diagrams and so forth. Whilst *talk* could be considered under this definition to be an *artefact*, for reasons set out below, we choose to treat it separately.

Artefacts do not come into being with the 'ideal' in an immediately apprehendable form: the ideality must be mediated, usually through *talk* and *actions*. While a hundred square could be a material presence for a two-year-old, it would not have the 'ideal' dimension that it might for a ten-year-old. We are interested in how pupils come to 'read into' artefacts the 'ideality' inscribed in the material?

3.3 Talk

As Cole (1996) points out, a 'material' and 'ideal' view of artefacts means that

> the properties of artifacts (sic) apply with equal force whether one is considering language or the more usually noted forms of artifacts such as tables and knives which constitute material culture (p. 117).

Thus we might argue for considering teachers' talk within the category of artefact. However we chose to separate out 'talk' from 'artefacts' for several reasons. Firstly, mathematical classrooms are unusual in the extent to which there are a number of pedagogical artefacts (that are not talk) that exist and take meaning only within the context of classrooms. Many classroom mathematical artefacts are primarily pedagogical and usually only found within classrooms. The artefactual 'paraphernalia' of mathematics classrooms seems worthy of attention its own right.

Secondly, *talk* is probably the principal *artefact* through which teachers and pupils co-construct meanings. Without *talk* classroom participants would have difficulty imbuing pedagogical artefacts or lesson tasks with meanings. *Talk* is not only an *artefact* in its own right: it is also a mediating means for developing shared meanings for *tasks* and other *artefacts*. Thirdly, *talk* is unique amongst *artefacts* in its self-referential nature. Verbal explanations are used to clarify other verbal explanations in a way that physical *artefacts* are not.

Finally, unlike *talk*, the majority of *artefacts* used or produced in mathematics lessons are usually introduced by the teacher. In lessons characterised by a high level of discussion involving pupil-pupil *talk* as well as teacher-pupil *talk* then pupils are involved in the production of mediating means for their peers.

3.4 Actions

Many of the *artefacts* of mathematics classrooms go beyond providing visual images of mathematics and are designed to be acted upon, either by the teacher or pupil or both. Even when not designed to handled (for example a wall mounted 100 square), *actions* are invoked in working with *artefacts* (for example, imagining movement around the squares on a 100 square). Also, in helping pupils appreciate the meaning of mathematical operations that might be associated with symbols, *action* based metaphors are often invoked, verbally and/or through physical models. For example, subtraction as 'take-away' or division as 'share'. In such instances, actions are important mediating means to help link *talk* and *artefacts*.

Although discussion of each of these parameters is treated separately above, they are interdependent. For example the setting up of a classroom *task* may involve use of *talk*, *artefacts* and *actions*.

4 THE THEORETICAL MODEL: PERSONAL MEANINGS

So far we have focused our attention on observable aspects of mathematics lessons. But as Lemon and Taylor (1998) remind us, examining the material world only provides a partial story:

> We never perceive only raw matter, nor do we perceive only mental phenomena. We always experience the action between the two. (p. 230)

Looking at *tasks*, *talk*, *artefacts* and *actions* as mediating means can give some insights into the sorts of experiences in which pupils have the opportunity to participate. However, we also need to consider the sense that participants make of such experiences. While we cannot directly observe such sense making, we can take teachers' and pupils' particular responses to and uses of mediating means as indicators of how they are interpreting their experiences. So linked to our four parameters of mediating means, we have worked with three interpretive parameters of: *activity*, *tools* and *images*.

4.1 Activity

Tasks are publicly set up, *activities* are *tasks* as privately interpreted. In setting up classroom *tasks*, teachers will have their individual interpretations of the *activities* that the *tasks* are intended to provoke (although such understanding may be tacit and the distinction between a *task* and an *activity* not explicitly addressed). Pupils will always interpret classroom *tasks* in the light of their previous experiences and current understandings. However carefully a teacher sets up a *task*, one cannot assume that the individual pupils' interpretations of that *task*, the *activities* that they engage in, are either similar to each other's, or fit with the *activity* expectations of the teacher. Hence the use of the term *activity* to distinguish any one individual's interpretation from the 'public' presentation of a *task*.

4.2 Tools

Cole (1996) argues for the treatment of 'a tool as a subcategory of the more general conception of an artifact' (p. 117). However, rather than consider *tools* as subcategories of *artefacts*, we define them as the personal meanings attached to artefacts.

We begin this definition by drawing on the distinction between interpersonal meanings and intrapersonal meanings and linking these with *artefacts* and *tools* respectively. As indicated we consider *artefacts* as embodying ideas, interpersonal meaning, alongside having some material existence. A hundred square on the classroom wall is intended to be 'aimed-oriented' as embodying particular aspects of the number system. The interpersonal ‘meaning’ embodied in an *artefact* is, in a sense, objective, transcending the sense making of any particular individual.

On the interpretative plane, we want to argue for a conception of *tools* as more personal arising from the interpersonal meaning unique to any individual working with an *artefact*. Just as the relationship between *tasks* and *activities* needs to be examined, so too the relationship between *artefacts* and *tools*.

4.3 Images

We define *images* as broader than visual images to include verbal or kinaesthetic images. We do not consider that there is any simple one-to-one correspondence between external mediating means and the *images* that might be provoked. For example, an external visual *artefact* may produce an internal kinaesthetic or verbal *image* just as it might produce a internal visual *image*. We are not interested in the mechanisms whereby *images* are provoked, nor in understanding the mechanisims by which external mediating means become internal *images*. We are interested in the impact of individuals' *images* on interpersonal mathematical meaning making.

Once again, although described separately, these are not independent of each other. The use of particular *tools* may depend on access to certain *images* and together they may affect the nature of *activity*. The following analysis further illustrates the interplay between the parameters.

5 ANALYTICAL APPLICATION OF THE MODEL

As an example of the use of the framework as an analytical tool, we present data from one lesson observed over the course of five-years of lesson observations. (For a more detailed account of the research from which this example is drawn see Brown, 2002). Given the level of detail that the analysis yields, restrictions of space prevent the reporting on the response of more than one pupil; Mayur. However, the full analysis demonstrates that although set up by the teacher to be working on the ‘same’ *task*, the pupils' individual responses meant that they engaged in distinct *activities* and consequently, we suggest, were likely to have established different meanings.

The group observed, of which Mayur was a member, had not been singled out by the researcher for particular attention, there just happened to a spare chair at the table that they were working at during the observation. After the lesson, the teacher indicated that the children at this table were in the “middle” group of maths attainment. When the researcher (MA) joined them they were each working through calculations from identical worksheets: finding unitary fractions of whole numbers, for example, 1/4 of 36 or 1/5 of 40 (all with an exact whole number answer). There were 12 such calculations for the children to answer.

At the end of the lesson, the children's completed worksheets suggested that the other children in the group had carried out all the calculations correctly whereas Mayur, while getting some correct, had made several errors in his calculations. But as our observations of Mayur's methods of working show, the differences in his answers were not simply the result of correct or incorrect calculations but arose from different interpretations of the *task*: the *activities* that he engaged in changed as he worked through the *task*. (And the full analysis shows that, although arriving at correct answers, neither could the other children be considered to have been engaged in the same *activity*)

Most of the pupils on the table were observed to have chosen one method of calculation and then used that for all the calculations. In contrast, Mayur did not consistently use only one method. He did the first four calculations using a tallying method, for example, finding '1/4 of 36' by marking tallies in rows of 4, each row under the previous one and counting on in 4's until he reached 36 (doing his working on scrap paper that ended up in the waste-bin). Counting the number of rows of tallies gave him his answer. Thus Mayur's initial *actions* involved partitioning the total into requisite groups through the use of the *artefact* of groups of tallies.

However, for the fifth calculation –'1/3 of 21'– Mayur initiated a change of *artefacts* and *actions*. Rather than recording tallies in groups of three as he had done previously, he verbally counted on in threes, keeping track of this by holding up one finger for each multiple of three pronounced. Once he reached 21 he counted the number of raised fingers (seven). Similarly for '1/5 of 30' he counted on in fives, raising a finger for each multiple of five, thus ending up with six raised fingers.

Rather than count in ones (as other children in the group were observed doing) Mayur's initial *artefacts* and *actions* – lining up of the tallies under each other – meant that in a sense his *artefact* controlled his *actions*: once a group of tallies was complete there was an umabiguous signal to start the next set of tallies. This facilitated his use of skip counting in the pattern of multiples. In doing so, he attended to counting up in groups: to a repeated addition method of solution. His use of the pattern of multiples as a *tool* then allowed the development of what were, for him, more efficient methods through different *artefacts* and *actions*, but which in turn appeared to affect his *images* and *activity*.

In coming to represent the divisor as a unitary group – a single finger representing a group, rather than tally all the elements – Mayur employed a different *artefact* (and *tool*), a 'compacted' representation. Instead of relying on paper and pencil to model the full quotitioning of the number to be divided, he only had to count the number of fingers he held up. But with this change of *artefact* there was a resultant shift in his attention and *activity* as his subsequent work demonstrated.

On question seven –‘1/3 of 30’– Mayur announced (to no one in particular) that he was going to ‘cheat’. Looking back at his answer to ‘1/3 of 21’ (7) he immediately held up 7 fingers, counted on in threes from 21 to 30, putting out three more fingers and writing down ‘10’. The next calculation was to find ‘1/10 of 20’, but rather than use tallies or fingers, Mayur immediately wrote down ‘10’.

MA: Why is the answer to that ten?

Mayur: You have to find which table the number is in. Twenty is in the tens table, so the answer is ten.

Similarly he wrote down ‘10’ as the answer to ‘1/8 of 40’ but changed this to ‘4’.

Mayur: I got it wrong. It's not which table it's in, but where in the table.

He wrote down ‘5’ as the answer to ‘1/2 of 50’.

MA: Why is that five?

Mayur: It’s in the ten times table and it’s the fifth one’.

Finally, checking back over his work, Mayur changed his answer to ‘1/3 of 30’ from ‘10’ to ‘3’ and ‘1/10 of 20’ from ‘10’ to ‘2’!

Mayur’s attention to the *action* of holding out the total number of fingers was, initially, correctly linked to the divisor. However, in counting up to the dividend he focused (literally and metaphorically) more on the number of fingers than on the running total. A shift from ‘count on in 4’s to 36’ to ‘9 makes up 36’ (with the 9 being linked to groups of 4 becoming more tacit). So rather than starting with the divisor his attention shifted to the dividend as being the most significant item of information. In attempting to be even more efficient, he began to attend first to the dividend and his *activity* become one of ‘spotting’ the obvious table that that particular dividend would be in. Given, say, a multiple of 10, then the calculation, for him, must be something to do with the ten times table.

Note that initially Mayur went as far as re-interpreting the *task* as the *activity* of ‘which table is this number occurring in’: if the dividend was 30 then the answer must be 10 (30 is obviously in the ten times table). Subsequently he ‘self-corrected’ himself to ‘where in the table’ the dividend was, but still a ‘table’ of his own determination (the most obvious one) rather than determined by the divisor: if the dividend was 30 then the answer must be 3 (it’s the third multiple in the tens table).

6 DISCUSSION

With the *action* of drawing up tallies Mayur, was still producing *artefacts* that represented all the information in the calculation: the size of each group, his progress towards the total number to be divided and when the total was reached. But in the move to holding up fingers, the only external *artefact* then available was the total number of groups. All the other information Mayur had to hold in his head. Along with focusing on the number of fingers he had held up, his attention shifted away from building up to a total (by skip counting) and instead to focus on the position of the divisor in the pattern of multiples. So while his *tools* may have been more efficient, they influenced his *activity*.

We are not suggesting that alternative *activities* are ones that the children may consciously develop, but that different *activities* are potentially present through the choice of diiferent *artefacts* (for example, columns, fingers) as mediating means and consequently the *tools* children use to carry out the task. The relationship between tools and activity is dialectical – each is informed and informed by the other. It is not simply a case of children understanding *tasks* and then selecting appropriate *artefacts* to use, that each of these together influence the *activities* and *tools* and hence the mathematical meanings. In further work we are examining the role of *talk* in these processes.

7 REFERENCES

Askew, M., Brown, M., Denvir, H., & Rhodes, V. (2000). Describing primary mathematics lessons observed in the Leverhulme Numeracy Research Programme: A qualitative framework. In T. Nakahara & M. Koyama (Eds.), *Proceedings of the 24th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 2-17, 12-24). Hiroshima: University of Hiroshima.

Bakhurst, D. (1995). Lesson from Ilyenkov. *The Communication Review, 1*(2), 155-178.

Brown, M. (2002). Researching primary numeracy. In A. Cockburn & E. Nardi (Eds.), *Proceedings of the 26th Conference of the International Group for the Psychology of Mathematics Education,* (Vol. 1, pp. 1-015 to 011-030). Norwich: University of East Anglia.

Cobb, P., & Bauersfeld, H. (Eds.). (1995). *The emergence of mathematical meaning: interaction in classroom cultures*. Hillsdale, NJ: Lawrence Erlbaum Associates.

Cole, M. (1996). *Cultural Psychology: A Once and Future Discipline*. Cambridge, Mass and London, England: The Belknap Press of Harvard University Press.

Daniels, H. (2001). *Vygotsky and Pedagogy*. London and New York: Routledge/Falmer.

Lemon, N., & Taylor, H. (1998). Caring in casualty: The phenomenology of nursing care. In N. Hayes (Ed.), *Doing qualitative analysis in psychology* (pp. 227-243). Hove, East Sussex: Psychology Press Ltd.

Matusov, E. (1998). When solo activity is not privileged: participation and internalization models of development. *Human Development, 41*, 326-349.

Saxe, G. B. (1991). *Culture and cognitive development: studies in mathematical understanding*. Hillsdale, NJ: Lawrence Erlbaum Associates.

Wertsch, J. V. (1991). *Voices of the mind: A sociocultural approach to mediated action*. Hemel Hempstead: Harvester Wheatsheaf.