THE INTRODUCTION OF CALCULUS IN 12TH GRADE: THE ROLE OF ARTEFACTS

Michela Maschietto[1]

Dipartimento di Matematica Pura ed Applicata

Via Campi 213/b – I 41100 Modena

The paper concerns the analysis of the role of artefacts and instruments in approaching calculus by graphic-symbolic calculator at high school level. We focus on an element of the introduction of calculus: the global/local game. We discus the hypothesis that the zoom-controls of calculator support the production of gestures and metaphors that foster the shift from a global to a local point of view. The analysis of protocols confirms that the exploration of several functions through the zooming process was supported by gestures and language. They appeared during the zooming process and in pupils' answers to the tests when the calculator was no more available and when the task concerned mathematical objects.

INTRODUCTION

This paper refers to our research about the introduction of calculus at secondary school level in a graphic-symbolic calculator environment (Maschietto, 2002). In particular, we focused on this introduction in terms of a transition between mathematical fields and we tackled it in terms of a reconstruction of relationships among mathematical objects. At the beginning of the teaching of calculus at high school level, pupils meet functions as algebraic and geometrical objects mainly under two points of view, the global point of view and the pointwise point of view. A global point of view on functions involves two aspects. The first aspect consists of considering functions as entities defined by a formula and/or a graphic representation. The second aspect concerns global properties of functions. A pointwise point of view consists of considering a function by the values taken on one or several chosen points, belonging to its domain. The beginning of the study of calculus is characterised by the addition of the local point of view. This forces to pay attention, for example, to a specific value and to the points near to it or, in the graphic setting, to what happens around a chosen point. Then within the objects characterising the transition and emerging from researches in mathematics education, we chose to study the articulation between a global point of view and a local point of view about functions, which we called the 'global/local game'. For analysing this game, we developed a didactical engineering (Brousseau, 1997), where we considered some cognitive research (above all Longo, 1998; Lakoff and Núñez, 2000) that studies the

1 Research funded by the MIUR and the Università di Modena e Reggio Emilia (COFIN03 n. 2003011072)

conceptualisation in mathematics (Longo is particularly interested in cognitive analysis of the foundations of mathematics, while Lakoff and Núñez are mostly interested in the analysis in terms of embodied cognition and metaphors). The situations are proposed in a graphic-symbolic calculator environment (TI-89, TI-92).

In this paper, we pay attention to the role of artefacts and instruments in approaching the global/local game under a Vygotskijan perspective.

THEORETICAL FRAMEWORK

With respect to artefacts, Vygotskij distinguishes between the function of mediation of *technical tools* and that of *psychological tools* (or *signs* or *tools of semiotic mediation*), discusses on their relation and offers a list of examples:

> language, various systems for counting, mnemonic techniques, algebraic symbol systems, works of art, writing, schemes, diagrams, maps, and mechanical drawings, all sorts of conventional signs and so on (Vygotskij 1974).

In the process of internalisation, technical tools (e.g. a concrete instrument to be handled in problem solving, graphic-symbolic calculator) become psychological tools (e.g. signs), starting in this way the cultural process:

> The use of signs leads humans to a specific structure of behaviour that breaks away from biological development and creates new forms of a culturally-based psychological process. (Vygotsky, 1987).

Elsewhere, the same author emphasizes the need to combine three different contributions in the process of internalisation:

> Children solve practical problems helping themselves with language, eyes and hands. This unit of perception, language and action, which in the end produces interiorization of the visual field, constitutes the central theme for all types of analyses regarding the origin of exclusively human forms of behaviour (Vygotskij, 1987).

This internal visual field is a part of the student's internal context where to carry on mental experiments, also supporting the production of mathematical reasoning. This emphasis on the body (eyes, hands and action) is consistent with the quoted position of Lakoff & Núñez (2000) according to which mathematical ideas are, to a large extent, grounded in sensory-motor experience.

An important contribution to the analysis of the process of internalisation, when artefacts are into play, is given by Vérillon and Rabardel (1995). In their studies, drawing on cognitive ergonomics they suggest the possibility that the use of an artefact causes in the subject the activation of schemes of use that transform it into an instrument. In particular, Artigue and al. (Artigue, 2002) have exploited these studies in order to develop an instrumental approach to both mathematical software and graphic-symbolic calculators in mathematics education.

A further elaboration of this idea is given, among others, by Bartolini Bussi, Mariotti & Ferri (in press), who integrate the cognitive analysis with the didactical analysis of the teacher's role. In a teaching experiment on artefacts taken from the history of perspective drawing (i. e. one of the roots of modern projective geometry), they validate two research hypotheses that concern: (1) the relationship between the intrinsic polysemy of any artefact and the expected - desired polyphony of classroom activity, through the essential role played by the teacher; (2) the relationship between the physical features of some artefacts and the bodily activity involved. They guess that similar hypotheses may be validated in other fields of experiences, related to new technologies (e.g. microworlds). In this paper, we aim at reformulating and discussing the second hypothesis with respect to graphic-symbolic calculator environment.

HYPOTHESIS AND METHODOLOGY

The second hypothesis above highlights a link between experienced events (and dynamic features of situations for pupils) and processes of conceptualisation in mathematics. Drawing on these researches, we focus on the link about the global/local game and we precise this hypothesis of embodiment as it follows.

The Hypothesis of Embodiment. In suitable activity with GS calculators, the Zoom-controls (ZoomIn and ZoomBox ZoomOut, ZoomStd) support the production of gestures and metaphors that foster the shift from a global to a local point of view, and produce a specific language which is maintained also later when the calculator is no more available (e.g. in paper and pencil environment) and when the task concerns mathematics objects.

We mean that it is possible to introduce some kind of manipulation (that is dynamics) in the treatment of graphic representations of functions through the zoom-controls. Then, the transformations of the graphic representation of a function through the use of zoom-controls and the experience of perceptive phenomena of local linearity (that can be formulate like that "when a curve is enlarged around a point of differentiability, it becomes locally linear") which these transformations may give <u>rise to the formulation</u> of a specific language, the production of gestures and the construction of metaphors from pupils. These elements might be exploited in processes of construction of mathematics objects such as tangent line and linear approximation of a curve at a chosen point (local linearity leads to mathematical analytic concepts such as the concept of limit and derivative).

We developed a "didactical engineering" to introduce the global/local game through the notion of local linearity. It was structured according to the following path: from identifying the graphic phenomenon of local linearity ("micro-straightness") during the first session to its mathematics formulation during the second and the third session. The first session is based on the use of the artefact: through the zoom-controls, pupils are lead to explorations of graphic representations of functions. In the second session, calculator (an instrument according to Rabardel and al.-1995- because of the activation of schemes of use) is used in order to discuss about mathematical character of micro-straightness (This part is not analysed in this paper).

Each session was organised in two parts: group activities first and then collective work orchestrated by the teacher. The collective work was not strictly conceived as a mathematical discussion according to Bartolini Bussi (1996), nevertheless it was managed in order to favour the sharing of results of the explorations among pupils and teacher and to let gestures and metaphors to emerge (according to Boero & al., 2001). The didactical engineering (3 sessions) was implemented in three Italian classrooms (fourth year of a 'Liceo Scientifico' with 17-18 year old students) in May 2000 (exp_A) and May 2001 (exp_B and exp_C). We analyse some excerpts of protocols in the following paragraph.

THE ANALYSIS OF PROTOCOLS

We present three excerpts: the first excerpt shows some gestures that were associated to a strategy of exploration and connected to zoom-controls in the group activities of the first session, the second excerpt concerns the collective work of the first session, the third excerpt concerns some answer to tests we gave to pupils at the end of each experiment.

The first session – The zooming process

In group activities, pupils had to explore six functions around chosen points, some of these functions are differentiable everywhere and others have singular points. At the beginning the exploration is guided, then the students are asked to use the zoom control more freely. In the text of the worksheet, we impose the shift from the calculator to paper and pencil in order to represent some screen.

In the transcript below (taken from exp_A), we have described the gestures used by pupils. We have underlined the words uttered at the same time of gestures and we have described them below. However, it is necessary to consider that the duration of gesture was not limited to the pronunciation of the associated word.

Students: PM, RE, GA. Researcher: C

58 PM: We always consider the same <u>point</u> (G58a), at the beginning we see how the <u>graph changes</u> (G58b) making <u>ZoomIn</u> (G58c), then from the standard window we verify how the curve changes making <u>ZoomOut</u> (G58d) this time [he said to C] we have a <u>bigger curve</u> (G58e) as a graph.

<u>G58a</u>. The point is showed by two fingers touching each other in the air (thumb and forefinger). PM indicated the point on the screen of his calculator and kept this position with the other fingers close to each others. <u>G58b</u>. The palm of the right hand was considered as the plane of the graphic representation. <u>G58c</u>. The joint fingers (right hand) shifting towards the palm (left hand), which was facing downward, indicated that the ZoomIn control was being used here. This gesture was made twice. <u>G58d</u>. His fingers started as joint from the palm of the hand and they separated in a movement upward (his hands was opening). <u>G58e</u>. The curve is traced in the air, with the same trend of the curve displayed by the calculator at this moment.

59 C: Do you think that “studying around a point” means…

60 PM: But, is it wrong?… because if we see the standard graph, zoom standard (G60a), as the element of reference, we take a point (G60b) and then we make ZoomIn (G60c) and then we make ZoomOut (G60d), we can better understand how it [*the curve*] develops in a detailed way and then however in…

G60a. His fingertips (thumb and forefingers of the two hands) made now a little rectangle. G60b. The point was indicated by two fingers close to each other, but not touching each other. G60c. This is accompanied by a repeated movement downward. G60d. The return to zoom standard to make a ZoomOut is accompanied again by the construction of the little rectangle.

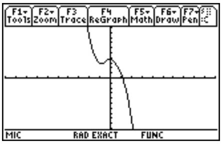
The ZoomIn control is used in order to see some of the characteristics of the curve in a detailed way and is associated to a movement downward meaning an “entrance into the curve”. The ZoomOut control, that is used to obtain a bigger curve and to better study its characteristics, is associated to a movement upward meaning an “exit from the curve”, that corresponds to a moving away from the curve. PM also creates a space in front of him for controlling these processes (the standard window of the calculator becomes a little rectangle that is constructed by his fingers (G60a) under his eyes). This excerpt shows how PM enriches his use of artefact with gestures that seem to generalise those which are connected to zoom-controls. They seem also to act as a mediator between the global point of view and the local one. PM’s gestures lead to interpret the particular as downward and the general as upward.

In general, our analysis showed that pupils have different behaviours according to their role (pupil using calculator and pupil drawing curves in paper and pencil environment) in the group activities. For instance, RE’s gestures are different from PM’s ones: since she had to draw the curve appearing on the screen, she tries to appropriate the trend of curve following it by her finger. In the same way, the language, that is associated to explorations and to the characterisation of the local linearity, changes: it is rather dynamic for pupils making the explorations, it is rather static for pupils drawing graphic representations in paper and pencil environment.

The analysis of this excerpt suggests that the zoom-controls introduce a kind of dynamics in manipulating graphic representations. Then it confirms our hypothesis.

The first session – At the beginning of the process of mathematisation

The collective work focuses on the linear invariant, obtained by explorations (for instance, figure 1): teacher solicits pupils to attribute a linguistic expression to the graphic sign displayed by the calculator at the end of several explorations. The analysis of the three experiments shows that the attribution of this term is very delicate and not evident at all. Communication is possible because pupils find a shared context in the common experience of the explorations of graphic representations of function.



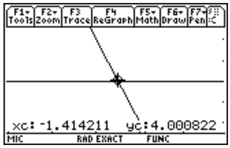


Figure 1: standard window [-10,10]x[-10,10] and micro-straightness

In the collective discussion of exp_A, pupils agreed upon the expression “zoomata lineare”[2], which highlights the idea of the process of enlargement and the use of instruments. It pays less attention to the local property. In the collective discussion of exp_C, the phase of definition had been very rich. At the beginning, pupils proposed some terms (such as “segmentizzazione” or “segmentizzata”[2] where the root of the word “segment” can be recognized. These terms are different from the previous one (exp_A), because they stress the result of the enlargements rather than the process.

In the second session, the defined expression is used by teacher as a tool of semiotic mediation, because it is given as a sign to be interpreted. The task (“Determine the equation of the line which is displayed by the calculator”) leads to remake an exploration and starts the process of mathematisation.

Test – Some use of graphic sign and linguistic expressions

The following excerpts show how pupils recall both linguistic expressions and signs in different way in their answers to the questions of the test. They also show some personal adaptation of shared terms.

The first excerpt (cf. figure 2) presents CF’s answer to the question of giving an example of a micro-straightness curve (“zoomata lineare”) at a point. In the text of test, the use of linguistic expression evokes both the zooming process and its result. Two graphic signs point out the process: (1) a standard graphic representation of a curve with a rectangle and the point P inside, (2) two arrows between the two free-hand drawings. The rectangle is an evidence of localisation and it refers to the use of calculator (perhaps ZoomBox). Because of the absence of a formula to insert into the calculator, this is a mental experiment. The presence of two arrows pays attention to the shift to a local point of view and to the linear invariant.

Instead of exp_A, the evaluation of exp_B is based on the comparison between two tests about tangent line to a curve at a point: a test which is given before the beginning of didactical engineering and a similar test which is given at the end. In figure 3, we point out the linguistic expression “to make a zoomata lineare” in order to justify the existence of the tangent line at the chosen point. This indicates that the pupil associates a precise mathematical meaning to it. As in the previous example, he can not use calculator for the zooming process, which is therefore only evoked.

---

[2] Italian expression; a possible translation “linear zooming”.

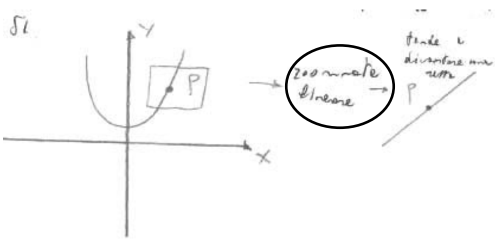


Figure 2: pupil CF (exp_A)

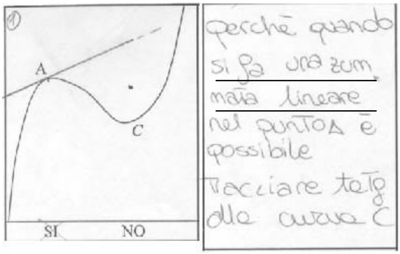
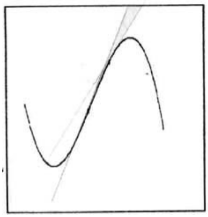


Figure 3: pupil BE (exp_B)

In this example (cf. figure 4), SM has changed the linguistic expression "micro-linear" into a reflexive verb ("si microlinearizza" in Italian words), which indicates a potential transformation of the drawn curve into a line around the chosen points. This answer suggests that zoom-controls have been internalised and they have became a tool of verification. Also in this case, the calculator is not available.



Si, in tutti e due i punti, perche´ nei dintorni di entrambi la curva si microlinearizza.

Yes, in all the two points [*the tangent line can be drawn*], because the curve becomes micro-linear on the neighbourhoods of both [*points*].

Figure 4: pupil SM (exp_C)

CONCLUDING REMARKS

In our research, the introduction of calculus is conceived in terms of global/local game and it focuses on the local linearity. In this paper, we have discussed the Hypothesis of Embodiment, basing our analysis of artefacts on a Vygotskijan theoretical framework. We have studied the mediation of zoom-controls of graphic-symbolic calculator in approaching local linearity. In particular, we have studied how the use of zoom-controls supports the production of gestures and metaphors that foster the shift from a global to a local point of view and that produce a specific language, which is maintained also later when the calculator is no more available. The analysis of protocols confirms our hypothesis. In fact, the exploration of several functions through the zooming process was supported by gestures and language. They appeared not only during the zooming process, but also in pupils' answers to test when the calculator was no more available and when the task concerned a mathematical object such as tangent line at a curve. They were revealed in this particular context, but they had their roots in deeper relationships with space and movement. Gestures seemed to play a particular role in the collective discussions when the process of mathematisation began and pupils did not yet have analytical instruments and associated technique to answer to the questions.

In this paper, we have started to discuss a reformulation of the Hypothesis of Embodiment and its validation in this technological environment. For space constraints, we have not yet discussed the first hypothesis stated by Bartolini Bussi, Mariotti & Ferri, i. e. the Hypothesis of Polysemy. Our future research, on the one hand, will carry on the analysis suggested by the Hypothesis of Embodiment, and, on the other hand will proceed to the reformulation and the discussion of the Hypothesis of Polysemy, that is also expected to be suitable for the environment of graphic-symbolic calculators.

References

Artigue, M. (2002). Learning mathematics in a CAS environment: the genesis of a reflection about instrumentation and the dialectics between technical and conceptual work. *International Journal of Computers for Mathematical Learning*, 7, 245-274.

Bartolini Bussi, M.G. (1996). Mathematical discussion and perspective drawing in primary school. *Educational Studies in Mathematics*, 31 (1-2), 11-41.

Bartolini Bussi, M.G., Mariotti, M.A. and Ferri, F. (in press). Semiotic mediation in the primary school: Dürer's glass. In M. Hoffmann, J. Lenhard & F. Seeger (eds), *Activity and Sign – Grounding Mathematics Education*, *Festschift for Michael Otte*, Kluwer, Dordrect.

Boero, P., Bazzini, L. and Garuti, R. (2001). Metaphors in teaching and learning mathematics: a case study concerning inequalities. *Proceedings of the 25th PME Conference*, Utrecht University, the Netherlands, vol.2, 185-192.

Brousseau, G. (1997). *Theory of Didactical Situations in Mathematics*. Kluwer Academic Publishers.

Lakoff, G. and Núñez, R. (2000). *Where Mathematics Comes From: How The Embodied Mind Brings Mathematics Into Being*. Basic Books, New York.

Longo, G. (1998). The mathematical continuum, from intuition to logic. In J. Petitot et al. (eds), *Naturalizing Phenomenology: issues in contemporary Phenomenology and Cognitive Sciences*. Stanford U.P.

Maschietto, M. (2002). *L'enseignement de l'analyse au lycee: les debuts du jeu global / local dans l'environnement de calculatrices*. PhD Dissertation, Université Paris 7 & Università di Torino, IREM Paris 7.

Vérillon P. and Rabardel P. (1995). Cognition and artifacts: A contribution to the study of thought in relation to instrument activity. *European Journal of Psychology in Education*, 10 (1), 77-101.

Vygotskij, L.S. (1974). *Storia dello sviluppo delle funzioni psichiche superiori e altri scritti*. Giunti.

Vygotskij, L.S. (1987). *Il processo cognitivo*. Bollati Boringhieri.