MAP CONSTRUCTION AS A CONTEXT FOR STUDYING THE NOTION OF VARIABLE SCALE

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We report research on meanings of scale generated by pairs of 14 year-old students engaged in joint map-construction. Characteristics of the learning environment, such as the communicational orchestration, the use of experientially familiar objects in space as starting points for creating figural representations and the interrelated representational registers of tangible objects, graphical and formal symbolic representations were important. The task to create maps allowing for dynamic scale change encouraged students to focus on the proportional aspects of scale in all three phases of the task, while they progressed from a componential to a holistic view of the map.

FRAMEWORK

This study aims at exploring the meanings about scale generated by 14 year-olds while constructing a map of their school campus with s/w allowing them to create building contours whose scale can be dynamically changed. Research focusing on concepts of spatial cognition required for cartography and map reading identifies scale as one of the basic elements to understand maps perceiving scale as a facet of proportional reasoning (Liben and Downs 1989 Bausmith et al 1998). Literature on proportional reasoning in mathematics education is of course vast and has provided extensive reports on students' difficulties and misconceptions (see for example Tourniaire & Pulos 1985, Kuchemann, 1991). The research highlights students' difficulty with the concept and their tendency to insist on attaching additive rather than proportional relationships to mathematical entities even in contexts with a didactical design to bring out proportionality. Research on proportional reasoning in spatial cognition tasks, however, is mainly oriented towards geometry curricula, investigating students' thinking in the context of geometric axiomatic systems where, in the end, figures represent instants of classes of 'ideal' figural constructs. When these figures are used to represent tangible objects in educational settings, the mathematization of figural relationships such as proportionality is a non-obvious task for students (Mariotti, 2002). Few exceptions, such as students' constructions with the 'N' tasks and the 'house' task by Noss and Hoyles, 1996, have highlighted how they in fact do generate their own theorems embedding proportional thinking but that at the same time how these are grounded in the specific context at hand and thus characterized as 'situated abstractions' by these authors. The idea that proportionality lies in changing the size of the same figure without 'distorting' its characteristics was, however, researched with a medium restricted to static constructions of instances of a figure, not allowing students to get a kinaesthetic sense of the dynamic evolution of

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figure distortion or preservation. The context of cartography, requires the selection of physical objects in space and their graphical representation. There is not much information about how (or even if and under what circumstances) students employ proportional reasoning in order to construct a model of observable objects in real space. Research oriented towards geography education integrates proportional reasoning in map-related tasks where, however, scale is studied solely as a method of establishing correspondence between space and its representations (Leinhardt et al 1998). The focus of these studies is on how students use the calculation formula of scale in map-enlargement or reduction tasks (see for example Bausmith et al 1998). We argue that this view overemphasizes the calculation methods that build a correspondence between space (or the initial model) and its representations placing little emphasis on the functional purpose of scale which is the maintenance of spatial relationships. We designed the cartography activity to bring this issue into the forefront of student activity. The research we report aimed at studying the meanings generated by students negotiating spatial relationships, rather than applying or understanding the scale formula. We wanted to understand weather and how they engaged in proportional reasoning during their attempts to construct the contours of the several buildings of their school site and their respective positions, orientations and relative sizes, so that the figural relationships of these representations would be preserved when dynamically changing the scale. The students worked in a *constructionist* learning environment (Harel and Papert, 1991) including a specially-designed cartography microworld which combines symbolic expression to construct figures with dynamic manipulation of the generated graphical output. A programming language (Logo) functioned as tool for symbolic expression and served a dual purpose: a) provided students with a vocabulary for articulation, reflection, refinement and communication of problem solving strategies (Eisenberg 1995) and with means to focus on how the mechanism underlying scale worked b) provided us with resources to gain insight on how students approached the notion of scale by studying how and what they constructed and edited (Noss et al 1997). Dynamic manipulation of the graphical output offered the students a tangible interface to evaluate and refine the symbolic expression of spatial relationships through the continuous DGS-style evolution of figural constructs.

CONTEXT

A cartography activity was designed and implemented in four classes of 14 year-old students, 70 students in total. The activity was designed to facilitate inter and intra-group collaboration and to trigger whole class discussions as context for negotiating spatial concepts involved in map construction. Pairs of groups engaged in joint construction of a computerized treasure hunt game per class and designed it so as to take place in their campus. The game involved the use of the cartography microworld, consisting of an electronic map of the area where the treasure hunt was going to take place, of a database with spatial information connected to the map and of clues placed in different locations in the area represented on the map (see fig1).

The de-coding of the clues was facilitated by a Venn diagram representation that was connected to the database and the map.

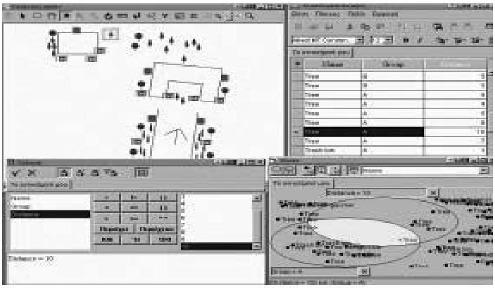

Fig1. An electronic map constructed by a group for the treasure hunt game

The activity involved map construction and use and interweaved navigation in space with work on the representation. Based on Leinhardt's et al (1998) suggestion that students can become more easily acquainted with spatial concepts embodied in maps when they have some knowledge of the place represented on the map, we decided to focus on familiar -for the students- space aiming to support the interplay between representation and referent space. This report is part of a larger study and follows on from a previous study involving seven year olds' spatial orientation (Kynigos and Yiannoutsou, 2002). Here we will focus only on the mathematical facet of the present study that involved construction of the buildings on the map and evolved in two phases. During the first phase students wandered around their school campus and recorded all the information necessary (i.e side lengths of the buildings, distances, type and position of landmarks) for the construction of an accurate map of the area. Accuracy was imposed not only by the task but also by the nature of space. A map of a familiar place is used in a treasure hunt game not to roughly outline space but to offer the necessary information so that a specific symbol on the map (such as a tree or a dust bin) can indicate the exact position of the respective object in space. The space our students represented was bordered by two or three different buildings, and consisted mainly of trees, bushes, dustbins and fire extinguishers. During the second phase, students used a programming language (Logo in this case) to construct a dynamic model of the contours of the buildings based on the measurements they made during the first phase. The idea behind the dynamic model was for the students to express symbolically the spatial relationships so that they could change the scale of their map through direct manipulation with the variation tool (Kynigos, 2002). The variation tool in the microworld can be activated by clicking on a point of the trace of a parametric procedure after it is executed with a specific value for each variable. Dragging the slider which is provided for each variable, causes dynamic change of the figure resulting from the respective 'continuous' change of the value of the variable. An editable step unit allows for change in the effect of continuity. Bundling all these diverse functionalities in one piece of s/w became possible through the use of E-slate as an authoring system (Kynigos, in press). The students of our research had used one year before, the same programming language to construct a size-changing font. This allowed us to focus on the representation of spatial relationships avoiding any "noise" that could be caused by students' unfamiliarity with the representational medium or the mathematical concepts involved. In this report, we thus refer only to the basic feature of the cartography microworld that played an important role in studying students' strategies while expressing spatial relationships symbolically and dynamically manipulating the resulting figural constructs.

METHOD

The study was implemented in four classes of fourteen year-old students and lasted for 19 sessions in each class. We worked in classroom settings aiming to study students' generation of meanings in a framework with rich social interaction facilitating negotiation and floating of ideas. We employed participant observation and collected our data a) from focusing on two groups in each class and b) from the whole class focusing on different groups for a short while in each session. The selection of our data was related to our decision to combine a detailed account of information regarding the work of two groups along with a general picture of the work in the class. Two researchers acted as participant observers focusing on a) verbal exchanges b) gestures, motion in space and c) data captured on the screen. Observers' interventions aimed at prompting students to make their thinking explicit as well as challenging students' actions and explanations. At the end of the activity teachers and the two focus groups from each class were interviewed. Research data consisted of students' work as well as of transcripts of the video-recorded sessions and of the interviews. We implemented discourse analysis to the transcribed data in the framework described by Yackel & Cobb (1996). Unit of analysis was the thematic episode comprising of a series of verbal exchanges around a specific subject. Change of subject indicated new episode.

PERCEIVING OBJECTS AS A SET OF COMPONENTS

In the context of map understanding research with young children, Lieben and Downs (1989) distinguish between a 'componential' and a 'holistic' level. The former involves meaningful constituent parts of a map (e.g. a rectangular area representing a tennis court) being at the forefront of student perception and the latter involves students' ability to make sense of the map by considering the whole of the area represented and the topological relationships between the represented objects. In this sense, all student pairs in the study initially adopted a componential approach, not only by working with the buildings first (we asked them to do this), but by their choice of strategy to measure each segment of a building's perimeter using a specific unit (amongst the units chosen were a foot, a step, a belt and a meter) and to attach a correspondence between this unit and the turtle step unit of their cartography s/w (typically 1-1, 1-5, 1-10). In this report we analyze the two out of the eight pairs of students, who began by writing a Logo procedure using fixed values for segments and turns to begin with. They were reminded by their teacher that since they had split the task of constructing the map, they would have to use each other's building procedures and thus needed to construct them so that they would subsequently be able to coordinate building sizes by being able to change them. The two groups proceeded to substitute each value in their procedure with a variable, not thinking about relating a value with another, as shown in fig. 2 (internal continuous graph of perimeter).

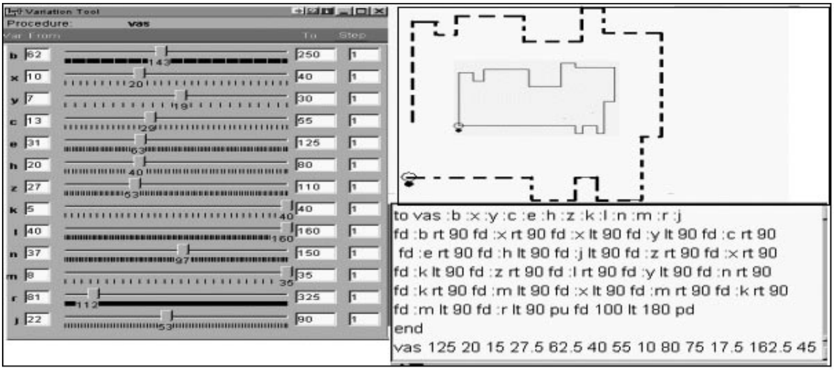

Fig2. Variable scale based on "componential" strategy

When the group tried to enlarge the building using the variation tool sliders they inevitably ended up distorting the building (dotted line contour top right window). It was this problem that brought about dialogue on what change needs to be made to the symbolic code in order for the building to change without distortion, as shown in the following excerpt by one of the groups.

1. S1 We need a lot of variables, because we need a variable here, a variable
2. here [he points on the numbers next to the command forward in the
3. Logo procedure], one here, one here, one here and one here
4. S2 Hold on. Look. Look
5. S1 One, two three four [he counts the numbers in the Logo procedure to
6. decide the number of variables needed]
7. S2 No, this one, and this one will be the same. They have the same
8. value
9. S1 Yeah, right. Ok this is x, this is y, this is z, this is k
10. S2 Hold on, this is, we can
11. S1 Yeah this is the half of it
12. S2 And this is twice as this, this is 3 times this and so on. We can do the
13. whole thing using one variable we don't need all these.

The students' noticing that some sides had the same value seemed to be the departure point for them to attach proportional relationships between line segments (lines 7,11,12). These were expressed by means of giving the procedure one variable and subsequently using a fraction or a product of that variable to express the length of each segment. During this phase, however, the students focused mainly on the line segments constituting the building and had only begun to consider the building as a whole, restricted to the problem of figural distortion.

PERCEIVING OBJECTS AS ENTITIES

Dynamic change of scale was subsequently used in the context of joining buildings to construct the campus map: pairs of groups exchanged their building procedures and created a map consisting of the buildings both groups had constructed. In the extract below we illustrate how during the process of joining maps one pair of groups (S5, S6 and S7, S8) seemed to reason about the map as a representation of interrelated spatial entities.

14. S7 This is not a correct map. Vasilia (*name of one of the buildings*) has the same size with Benakeio (*name of the other building*) [they use the same variable x for both buildings]
15. S5 It doesn't have the same size!
16. S8 Maybe but look, when Benakio is ten, Vasilia should be three. Vasilia
17. is three times smaller than Benakio. Now when Benakio is ten Vasilia is 25. It's
18. the opposite.[They execute each procedure separately first Benakio 10 then
19. Vasileia 25]
20. S7 Hold on, if Vasileia is three times smaller we will divide x by 3

When S7 and S8 attempted to join their map with the map S5 and S6 had constructed they were encountered with two procedures using the same letter (x) to denote variable, but with a different correspondence between x and unit of measurement, as well as different units of measurement. This resulted in the representation of each building being proportionally accurate in itself, but in a distortion between the relative sizes of the two buildings which was obvious to the students right away, since the buildings were part of their everyday reality. The fact that students were representing an experientially familiar space (their campus) seemed to be crucial in the importance they attached to solving the problem of the relative building sizes (lines 15,16,17). Students' efforts seem to focus on coordinating the graphical output not only with the spatial relationships identified in space ("Vasileia is 3 times smaller than Benakeio") but also with the symbolic expression of this relationship ("When Benakeio is 10 Vasileia should be 3"). The next step for the students was the "translation" of the relationship between the sizes of two buildings into a proportional symbolic expression (line 26 "we will devide x by 3"). At this point students returned to the procedure representing Vassileia and divided all inputs to the FD command by 3. They thus changed from viewing the building as a process of joining segments and thinking of the relationships between them to considering the building as an entity and then using that perception to go back to the procedural one.

SITUATING OBJECTS IN TOPOLOGICAL RELATION TO EACH OTHER

Apart from relations between buildings and side lengths, spatial relationships also involved distances between buildings. This became problematic when the students realized that dynamic scale changes of maps containing two buildings distorted the distance between the buildings and thus their topological positioning on the map. What had in fact happened was that it had not initially occurred to them to express these distances proportionally to the building sizes and they either gave fixed values of distances between buildings or between the turtle starting point and the first building. S7 initially thought that variable distances would actually cause map distortion.

21. S7: Now, this distance should not have variables, if we increase the map this building, Vasileia, will go out of the screen
22. S8: We have to put variables in this distance, the whole thing should be shrinking if we try to make it smaller
23. S7:Yes, I know the distance between the two buildings will have variables. This is not the same. I am talking about the distance from the center to Vasileia

It was S8, however, who seemed to adopt a holistic view of the map, realizing that topological relations involved both the objects themselves and their relative positions. His referral to 'the whole thing' as proportional ('shrinking') while addressing a series of proportional relationships is within the framework of situated abstractions as proposed by Noss and Hoyles (1996).

CONCLUSIONS

Some features of the learning environment played an important part in providing the students with opportunities to mathematize (in the sense of Sutherland, 2001) a seemingly geographical science-like task. Of these, the communicational orchestration, the use of experientially familiar objects in space as starting points for creating figural representations and the interrelated representational registers of tangible objects, graphical and formal symbolic representations were important. The task to create maps allowing for dynamic scale change encouraged students to focus on the proportional aspects of scale in all three phases of the task, while they progressed from a componential to a holistic view of the map. Situating mathematization and proportional thinking is such contexts may provide students the ground for generating meanings around proportion which previous research seemed to imply were difficult for students to grasp as concepts (e.g. Tournaire and Pulos, 1985). It may however be interesting to study the nature of these meanings (for instance, why did these students not consider additive relationships and how robust was their choice to attach proportional ones) and to investigate ways in which the tasks, the representational media and the interpersonal interactions may generate mathematization out of phenomenological contexts.

REFERENCES

Bausmith, JM., Leinhardt, G. (1998) "Middle-School Students' Map Construction: Understanding Complex Spatial Displays" In *Journal of Geography* 97, 93 – 107. National Council for Geographic Education

Eisenberg, M. (1995). "Creating software applications for children: Some thoughts about design". In A. diSessa, C. Hoyles and R. Noss with L. Edwards (Eds), *Computers for Exploratory Learning* (175–198). Springer-Verlag.

Harel, I. and Papert, S. (eds) (1991) *Constructionism: Research Reports and Essays,* Ablex Publishing Corporation, Norwood, New Jersey.

Kuchemann, D. (1991) The effect of setting and numerical content on the difficulty of ratio tasks, *Proceedings of the 15th PME Conference,* II, Paris.

Kynigos C. (2002). Generating Cultures for Mathematical Microworld Development in a Multi-Organisational Context. *Journal of Educational Computing Research,* Baywood Publishing Co. Inc. (1 and 2), 183-209.

Kynigos, C. (in press) A "black-and-white box" Approach to User Empowerment with Component Computing. *Interactive Learning Environments.*

Kynigos, C. and Yiannoutsou, N. (2002). Seven Year Olds Negotiating Spatial Concepts and Representations to Construct a Map. *Proceedings of the 26th PME Conference* (3, 177-184), University of East Anglia, Norwitch, UK.

Leinhardt, G., Stainton C. & Bausmith J.M. (1998) "Constructing Maps Collaboratively" In *Journal of Geography 97*, 19 – 30 National Council for Geographic Education

Liben, L.S., Downs, R.M., (1989) «Understanding Maps As Symbols: The development of map concepts in children". In Reese H.W. (Ed) *Advances in Child Development and Behavior* Vol 22 pp 145 – 201. Academic Press INC San Diego California

Mariotti, M. A. (2002) Influence of Technology Advances on Students' Math Learning. In L. English (Ed), *Handbook of International Research in Mathematics Education*, Mahwah, Lawrence Erlbaum.

Noss, R., Healy, L., Hoyles, C. (1997) "The construction of mathematical meanings: Connecting the visual with the symbolic" In *Educational Studies in Mathematics* 33, 203 –233.

Noss, R. & Hoyles, C. (1996) *Windows on Mathematical Meanings*, Kluwer.

Sutherland, R. (2001) Reaction to 'Algorithmic and Meaningful Ways of Joining Together Representatives within the Same Mathematical Activity', M. Heuvel-Panhuizen (Ed.) *Proceedings of the 25th PME Conference, Utrecht University, Netherlands,* 1, 108-110.

Tourniaire, F., Pulos, S., 1985, "Proportional Reasoning: A review of the literature", *Educational Studies in Mathematics*, 16(2), 181-204.

Yackel, E. & Cobb, P. (1996) Sociomathematical norms, argumentation and autonomy in mathematics, *Journal for Research in Mathematics Education,* 27, 4, 458-477.