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ON HAMILTON p_2–EQUATIONS IN SECOND-ORDER FIELD THEORY

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ABSTRACT. In the present paper recent results on regularizations of first order variational problems are generalized to Lagrangians affine in the second derivatives. New regularity conditions are found and Legendre transformations are studied.

1. INTRODUCTION AND NOTATION

In this paper we consider an extension of the classical Hamilton–Cartan variational theory on fibered manifolds.

It is known that in field theory to a variational problem represented by a Lagrangian one can associate different Hamilton equations corresponding to different Lepagean equivalents of the Lagrangian (DEDECKER [1], KRUPKA [7]). Accordingly, these Hamilton equations depend upon a Lagrangian (resp. its Poincaré - Cartan form), and an auxiliary differential form corresponding to the at least 2-contact part of the Lepagean equivalent of the Lagrangian. This admits a new approach to the problem of regularity (DEDECKER [1], KRUPKOVÁ [11], [12], KRUPKOVÁ and SMETANOVÁ [13], [14]). Contrary to the classical calculus of variations where regularity is a property of a single Lagrangian, in the generalized approach regularity conditions (different from [3], [4], [8], [15]) depend upon a Lagrangian *and* some "free" functions which can be considered as *parameters*. Within this setting, a proper choice of a Lepagean equivalent can lead to a *"regularization" of a Lagrangian*. Using this regularization procedure one can regularize some interesting traditionally singular physical fields, the Dirac field, and the electromagnetic field (cf. DEDECKER [1], KRUPKOVÁ and SMETANOVÁ [13], [14]).

Throughout this paper, $\pi : Y \to X$ is s fibered manifold, and $\dim X = n$, $\dim Y = m + n$. The r-jet prolongation of π is a fibered manifold denoted by

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$\pi_r : J^rY \to X$ and $\pi_{r,s} : J^rY \to J^sY$, $0 \le s \le r$, we denote the natural jet projections. A fibered chart on Y (resp. an associated fibered chart on J^rY) is denoted by (V, ψ), $\psi = (x^i, y^\sigma)$ (resp. (V_r, ψ_r), $\psi_r = (x^i, y^\sigma, y^\sigma_i, \dots, y^\sigma_{i_1 \dots i_r})$). In what follows, we consider $r = 1$ or $r = 2$.

Recall that every q-form η on J^rY admits a unique (canonical) decomposition into a sum of q-forms on $J^{r+1}Y$ as follows:

$$\pi^*_{r+1,r}\eta = h(\eta) + \sum_{k=1}^{q} p_k(\eta),$$

where $h(\eta)$ is a horizontal form, called the *horizontal part of* η, and $p_k(\eta)$, $1 \le k \le q$, is a k-contact form, called the *k-contact part of* η (see e.g. [5], [6] for review).

We use the following notations:

$$\omega_0 = dx^1 \wedge dx^2 \wedge \dots \wedge dx^n, \quad \omega_i = i_{\partial/\partial x^i}\omega_0, \quad \omega_{ij} = i_{\partial/\partial x^j}\omega_i,$$

and

$$\omega^\sigma = dy^\sigma - y^\sigma_j dx^j, \quad \omega^\sigma_i = dy^\sigma_i - y^\sigma_{ij} dx^j.$$

By an *r-th order Lagrangian* we shall mean a horizontal n-form λ on J^rY. A form ρ is called a *Lepagean equivalent* of a Lagrangian λ if (up to a projection) $h(\rho) = \lambda$, and $p_1(d\rho)$ is a $\pi_{r+1,0}$-horizontal form [5]. For an r-th order Lagrangian we have all its Lepagean equivalents of order $(2r-1)$ characterized by the following formula

$$\rho = \Theta + \nu, \tag{1.1}$$

where Θ is a global Poincaré–Cartan form associated to λ, and ν is an arbitrary n-form of order of contactness ≥ 2, i.e., such that $h(\nu) = p_1(\nu) = 0$ (cf. Krupka [5], [6]). Recall that for a Lagrangian of order 1, $\Theta = \theta_\lambda$ where θ_λ is the classical Poincaré–Cartan form of λ,

$$\theta_\lambda = L\omega_0 + \frac{\partial L}{\partial y^\sigma_i}\, \omega^\sigma \wedge \omega_i.$$

If $r = 2$, Θ is no more unique, however, there is an *invariant* decomposition

$$\Theta = \theta_\lambda + d\phi, \tag{1.2}$$

where

$$\theta_\lambda = L\omega_0 + \left(\frac{\partial L}{\partial y^\sigma_j} - d_k \frac{\partial L}{\partial y^\sigma_{jk}} \right) \omega^\sigma \wedge \omega_j + \frac{\partial L}{\partial y^\sigma_{ij}} \omega^\sigma_i \wedge \omega_j$$

and ϕ does not depend upon λ (Krupka [6]).

With the help of Lepagean equivalents of a Lagrangian one obtains the following intrinsic formulation of the *Euler–Lagrange* and *Hamilton equations*.

Theorem (Krupka [5])**.** *Let λ be a Lagrangian on J^rY, ρ its Lepagean equivalent. A section γ of π is an extremal of λ if and only if*

$$J^{2r-1}\gamma^* i_{J^{2r-1}\xi} d\rho = 0 \tag{1.3}$$

for every π-vertical vector field ξ on Y.

A section δ of the fibered manifold π_{2r-1} is called a *Hamilton extremal* of ρ (Krupka [7]) if

$$\delta^* i_\xi d\rho = 0, \tag{1.4}$$

for every π_{2r-1}-vertical vector field ξ on $J^{2r-1}Y$.

(1.3) are called the *Euler–Lagrange equations* and (1.4) the *Hamilton equations* of ρ, respectively. Notice that while the Euler–Lagrange equations are uniquely determined by the Lagrangian, Hamilton equations depend upon a choice of ν. Consequently, one gets many different Hamilton theories associated to a given variational problem.

In accordance with [13], by *Hamilton p_2-equations* we shall mean Hamilton equations of a Lepagean equivalent ρ of λ where ν is a 2-contact n-form (i. e., $h(\nu) = p_i(\nu) = 0$, $i \geq 1$, $i \neq 2$).

The aim of this paper is to consider Hamilton p_2-equations for a class of second order Lagrangians. Namely, we study Lagragians affine in second derivatives y^σ_{kl}, such that their Lepagean equivalent is of the form $\rho = \theta_\lambda + \nu$, where $\nu = p_2(\beta)$, for an n-form β defined on J^1Y.

Recall that a section δ of the fibered manifold π_r is said to be *holonomic* if $\delta = J^r\gamma$ for a section γ of π. Clearly, if γ is an extremal then $J^{2r-1}\gamma$ is a Hamilton extremal; conversely, however, a Hamilton extremal need not be holonomic, and thus a jet prolongation of some extremal. This suggests a definition of regularity proposed by Krupka and Štěpánková [9] in consequence with a study of second order Lagrangians with projectable Poincaré–Cartan forms: Throughout this paper a Lepagean form is called *regular* if every its Hamilton extremal is holonomic. Taking a Lepagean equivalent of λ in the form $\rho = \theta_\lambda + p_2(\beta)$, where β is defined on J^1Y, we can see that regularity conditions involve λ and β, and one can ask about *a proper choice* β, such that ρ is regular. We study this question in Section 2. Section 3 is then devoted to the question on the existence of certain *Legendre coordinates* for regularizable Lagrangians. In Section 4 we deal with Lagrangians, affine with second derivatives, admitting a Lepagean equivalent projectable onto J^1Y. Our results are a direct generalization of techniques and results from [13], [14] and provide, as a special case, the results of [9] and [2].

2. Regularization of variational problems for second-order Lagrangians affine in second derivatives

We shall consider Lagrangians affine in the second derivatives and its Lepagean forms (1.1), (1.2) where $\phi = 0$, ν is 2-contact, and

$$\nu = p_2(\beta),$$

where β is defined on J^1Y and such that $p_i(\beta) = 0$ for all $i \geq 3$.

In a fiber chart, a Lagrangian λ affine in the second derivatives is expressed by

$$\lambda = L\omega_0, \quad L = \widetilde{L} + \widetilde{L}^{ij}_\sigma y^\sigma_{ij}, \tag{2.1}$$

where functions $\widetilde{L}$, $\widetilde{L}^{ij}_{\sigma}$ do not depend on the variables y^{κ}_{kl} and the functions $\widetilde{L}^{ij}_{\sigma}$ satisfy the condition $\widetilde{L}^{ij}_{\sigma} = \widetilde{L}^{ji}_{\sigma}$. In view of the above considerations we obtain

$$\begin{aligned}\rho = &\left(\widetilde{L} + \widetilde{L}^{kl}_{\nu} y^{\nu}_{kl}\right)\omega_0 + \left(\frac{\partial \widetilde{L}}{\partial y^{\sigma}_{j}} + \frac{\partial \widetilde{L}^{kl}_{\nu}}{\partial y^{\sigma}_{j}} y^{\nu}_{kl} - d_k \widetilde{L}^{jk}_{\sigma}\right)\omega^{\sigma}\wedge\omega_j \\ &+ \widetilde{L}^{il}_{\sigma}\omega^{\sigma}_{i}\wedge\omega_j + f^{ij}_{\sigma\nu}\omega^{\sigma}\wedge\omega^{\nu}\wedge\omega_{ij} + g^{kij}_{\sigma\nu}\omega^{\sigma}\wedge\omega^{\nu}_{k}\wedge\omega_{ij} \\ &+ h^{klij}_{\sigma\nu}\omega^{\sigma}_{k}\wedge\omega^{\nu}_{l}\wedge\omega_{ij},\end{aligned} \tag{2.2}$$

where the functions $f^{ij}_{\sigma\nu}$, $g^{kij}_{\sigma\nu}$, $h^{klij}_{\sigma\nu}$ do not depend on the y^{κ}_{pq}'s and satisfy the conditions

$$\begin{aligned} &f^{ij}_{\sigma\nu} = -f^{ij}_{\nu\sigma}, \quad f^{ij}_{\sigma\nu} = -f^{ji}_{\sigma\nu}, \quad f^{ij}_{\sigma\nu} = f^{ji}_{\nu\sigma}; \\ &g^{kij}_{\sigma\nu} = -g^{kji}_{\sigma\nu}; \\ &h^{klij}_{\sigma\nu} = -h^{lkij}_{\nu\sigma}, \quad h^{klij}_{\sigma\nu} = -h^{klji}_{\sigma\nu}.\end{aligned} \tag{2.3}$$

In general case, the Poincaré–Cartan forms of a second order Lagrangians is defined on J^3Y, but for Lagrangians of the forms (2.1) θ_λ is projectable onto J^2Y. Our choice of the 2-contact part ν of ρ conserves the Lepagean form (2.2), (2.3) defined on J^2Y.

In the following theorems necessary conditions for regularity are found, which according to the definition of regularity in this paper, guarantee that extremals and Hamilton extremals of $\lambda = h\rho$ are in *bijective* correspondence.

Theorem A. *Let* $\dim X \geq 3$. *Let* λ *be a second-order Lagrangian affine in the variables* y^{σ}_{ij}, *the formula* (2.1) *be its expression in a fiber chart* (V, ψ), $\psi = (x^i, y^\sigma)$ *on* Y. *Let* ρ *be a Lepagean equivalent of* λ *of the form* (2.2), (2.3).

Assume that the matrix

$$\left(A^{klj}_{\nu\sigma} \mid B^{klpq}_{\nu\kappa}\right), \tag{2.5}$$

with mn^2 *rows (resp.* $mn + mn(n+1)/2$ *columns) labelled by* (ν, k, l) *(resp.* (σ, j, κ, p, q)*, where* $1 \leq p \leq q \leq n$)*, where*

$$A^{klj}_{\nu\sigma} = \left(\frac{\partial \widetilde{L}^{kl}_{\nu}}{\partial y^{\sigma}_{j}} - \frac{1}{2}\left(\frac{\partial \widetilde{L}^{jk}_{\sigma}}{\partial y^{\nu}_{l}} + \frac{\partial \widetilde{L}^{jl}_{\sigma}}{\partial y^{\nu}_{k}}\right) - g^{kjl}_{\sigma\nu} - g^{ljk}_{\sigma\nu}\right),$$

and

$$B^{klpq}_{\nu\kappa} = \left(h^{kpql}_{\nu\kappa} + h^{lpqk}_{\nu\kappa}\right),$$

has rank $mn(n+3)/2$.

Then ρ *is regular on* ${\pi_2}^{-1}(V)$*, i.e., every Hamilton extremal* $\delta : \pi(V) \supset U \to J^2Y$ *of* ρ *is of the form* $\delta = J^2\gamma$*, where* γ *is an extremal of* λ.

Proof. Expressing the Hamilton p_2-equations (1.4) in fiber coordinates we get along δ the following system of first-order equations for section δ:

mn^2 equations

$$\left(\frac{\partial \widetilde{L}^{kl}_{\nu}}{\partial y^{\sigma}_{j}} - \frac{1}{2}\left(\frac{\partial \widetilde{L}^{jk}_{\sigma}}{\partial y^{\nu}_{l}} + \frac{\partial \widetilde{L}^{jl}_{\sigma}}{\partial y^{\nu}_{k}}\right) - g^{kjl}_{\sigma\nu} - g^{ljk}_{\sigma\nu}\right)\left(\frac{\partial y^{\sigma}}{\partial x^{j}} - y^{\sigma}_{j}\right) + 2\left(h^{kijl}_{\nu\sigma} + h^{lijk}_{\nu\sigma}\right)\left(\frac{\partial y^{\sigma}_{i}}{\partial x^{j}} - y^{\sigma}_{ij}\right) = 0, \tag{2.6}$$

mn equations

$$\begin{aligned}&\left(\frac{\partial^2 \widetilde{L}}{\partial y^{\sigma}_{j}\partial y^{\nu}_{k}} + \frac{\partial^2 \widetilde{L}^{pq}_{\kappa}}{\partial y^{\sigma}_{j}\partial y^{\nu}_{k}} y^{\kappa}_{pq} - \frac{\partial}{\partial y^{\nu}_{k}} d_p \widetilde{L}^{jp}_{\sigma} - \frac{\partial \widetilde{L}^{kj}_{\nu}}{\partial y^{\sigma}} - 4 f^{jk}_{\sigma\nu} + 2 d_i g^{kij}_{\sigma\nu}\right) \\
&\times\left(\frac{\partial y^{\sigma}}{\partial x^{j}} - y^{\sigma}_{j}\right) + \left(\frac{\partial \widetilde{L}^{ij}_{\sigma}}{\partial y^{\nu}_{k}} - \frac{\partial \widetilde{L}^{kj}_{\nu}}{\partial y^{\sigma}_{i}} + 2 g^{kij}_{\sigma\nu} - 2 g^{ikj}_{\nu\sigma} - 4 d_l h^{kilj}_{\nu\sigma}\right) \\
&\times\left(\frac{\partial y^{\sigma}_{i}}{\partial x^{j}} - y^{\sigma}_{ij}\right) + 2\left(h^{ikjl}_{\sigma\nu} + h^{lkji}_{\sigma\nu}\right)\left(\frac{\partial y^{\sigma}_{il}}{\partial x^{j}} - y^{\sigma}_{ilj}\right) \\
&+\left(2\frac{\partial f^{ij}_{\sigma\kappa}}{\partial y^{\nu}_{k}} + \frac{\partial g^{kij}_{\kappa\nu}}{\partial y^{\sigma}} - \frac{\partial g^{kij}_{\sigma\nu}}{\partial y^{\kappa}}\right)\left(\frac{\partial y^{\sigma}}{\partial x^{j}} - y^{\sigma}_{j}\right)\left(\frac{\partial y^{\kappa}}{\partial x^{i}} - y^{\kappa}_{i}\right) \\
&+ 2\left(2\frac{\partial h^{lkij}_{\kappa\nu}}{\partial y^{\sigma}} + \frac{\partial g^{lij}_{\sigma\kappa}}{\partial y^{\nu}_{k}} - \frac{\partial g^{kij}_{\sigma\nu}}{\partial y^{\kappa}_{l}}\right)\left(\frac{\partial y^{\sigma}}{\partial x^{i}} - y^{\sigma}_{i}\right)\left(\frac{\partial y^{\kappa}_{l}}{\partial x^{j}} - y^{\kappa}_{lj}\right) \\
&+ 2\left(\frac{\partial h^{lkij}_{\sigma\nu}}{\partial y^{\kappa}_{p}} + \frac{\partial h^{kpij}_{\nu\kappa}}{\partial y^{\sigma}_{l}} + \frac{\partial h^{plij}_{\kappa\sigma}}{\partial y^{\nu}_{k}}\right)\left(\frac{\partial y^{\kappa}_{p}}{\partial x^{i}} - y^{\kappa}_{pi}\right)\left(\frac{\partial y^{\sigma}_{l}}{\partial x^{j}} - y^{\sigma}_{lj}\right) = 0,\end{aligned} \tag{2.7}$$

and m equations

$$\begin{aligned}&\left(\frac{\partial \widetilde{L}}{\partial y^{\nu}} + \frac{\partial \widetilde{L}^{pq}_{\kappa}}{\partial y^{\nu}} y^{\kappa}_{pq} - d_j\left(\frac{\partial \widetilde{L}}{\partial y^{\nu}_{j}} + \frac{\partial \widetilde{L}^{pq}_{\kappa}}{\partial y^{\nu}_{j}} y^{\kappa}_{pq}\right) + d_j d_k \widetilde{L}^{jk}_{\nu}\right) \\
&+\left(\frac{\partial^2 \widetilde{L}}{\partial y^{\sigma}_{j}\partial y^{\nu}} + \frac{\partial^2 \widetilde{L}^{pq}_{\kappa}}{\partial y^{\sigma}_{j}\partial y^{\nu}} y^{\kappa}_{pq} - \frac{\partial^2 \widetilde{L}}{\partial y^{\sigma}\partial y^{\nu}_{j}} - \frac{\partial^8 \widetilde{L}^{pq}_{\kappa}}{\partial y^{\sigma}\partial y^{\nu}_{j}} y^{\kappa}_{pq} - \frac{\partial}{\partial y^{\nu}} d_k \widetilde{L}^{jk}_{\sigma}\right. \\
&\left.+\frac{\partial}{\partial y^{\sigma}} d_k \widetilde{L}^{jk}_{\nu} + 2 d_i f^{ij}_{\sigma\nu}\right)\left(\frac{\partial y^{\sigma}}{\partial x^{j}} - y^{\sigma}_{j}\right) \\
&+\left(\frac{\partial \widetilde{L}^{kj}_{\sigma}}{\partial y^{\nu}} - \frac{\partial^2 \widetilde{L}}{\partial y^{\sigma}_{j}\partial y^{\nu}_{k}} - \frac{\partial^2 \widetilde{L}^{pq}_{\kappa}}{\partial y^{\sigma}_{j}\partial y^{\nu}_{k}} y^{\kappa}_{pq} + \frac{\partial}{\partial y^{\sigma}_{k}} d_p \widetilde{L}^{jp}_{\nu} + 4 f^{jk}_{\nu\sigma} - 2 d_i g^{kij}_{\nu\sigma}\right) \\
&\times\left(\frac{\partial y^{\sigma}_{k}}{\partial x^{j}} - y^{\sigma}_{kj}\right) \\
&+\left(\frac{\partial \widetilde{L}^{kl}_{\sigma}}{\partial y^{\nu}_{j}} - \frac{1}{2}\left(\frac{\partial \widetilde{L}^{jk}_{\nu}}{\partial y^{\sigma}_{l}} + \frac{\partial \widetilde{L}^{jl}_{\nu}}{\partial y^{\sigma}_{k}}\right) - g^{kjl}_{\nu\sigma} - g^{ljk}_{\nu\sigma}\right)\left(\frac{\partial y^{\sigma}_{kl}}{\partial x^{j}} - y^{\sigma}_{klj}\right)\end{aligned} \tag{2.8}$$

$$+\,2\left(\frac{\partial f^{ij}_{\sigma\nu}}{\partial y^{\kappa}}+\frac{\partial f^{ij}_{\kappa\sigma}}{\partial y^{\nu}}+\frac{\partial f^{ij}_{\nu\kappa}}{\partial y^{\sigma}}\right)\left(\frac{\partial y^{\kappa}}{\partial x^{i}}-y^{\kappa}_{i}\right)\left(\frac{\partial y^{\sigma}}{\partial x^{j}}-y^{\sigma}_{j}\right)$$
$$+\,2\left(2\frac{\partial f^{ij}_{\sigma\nu}}{\partial y^{\kappa}_{k}}+\frac{\partial g^{kij}_{\nu\kappa}}{\partial y^{\sigma}}-\frac{\partial g^{kij}_{\sigma\kappa}}{\partial y^{\nu}}\right)\left(\frac{\partial y^{\kappa}_{k}}{\partial x^{i}}-y^{\kappa}_{ki}\right)\left(\frac{\partial y^{\sigma}}{\partial x^{j}}-y^{\sigma}_{j}\right)$$
$$+\left(2\frac{\partial h^{lkij}_{\kappa\sigma}}{\partial y^{\nu}}+\frac{\partial g^{lij}_{\nu\kappa}}{\partial y^{\sigma}_{k}}-\frac{\partial g^{kij}_{\nu\sigma}}{\partial y^{\kappa}_{l}}\right)\left(\frac{\partial y^{\kappa}_{l}}{\partial x^{i}}-x^{\kappa}_{li}\right)\left(\frac{\partial y^{\sigma}_{k}}{\partial x^{j}}-y^{\sigma}_{kj}\right)=0.$$

The system (2.6) can be viewed as a system of mn^2 (algebraic) linear homogeneous equations for

$$mn+mn\left(\frac{n+1}{2}\right)=mn\left(\frac{n+3}{2}\right)$$

unknowns

$$\left(\frac{\partial y^{\sigma}}{\partial x^{i}}-y^{\sigma}_{i}\right),$$

and

$$\left(\frac{\partial y^{\sigma}_{j}}{\partial x^{i}}-y^{\sigma}_{ij}\right),\quad j\le i.$$

According to the (algebraic) Frobenius theorem, this system has a unique (zero) solution if and only if the rank of the matrix of system, i. e., $\left(A^{klj}_{\nu\sigma}\mid B^{klpq}_{\nu\kappa}\right)$ is equal to the number of unknowns, i. e., $mn\left((n+3)/2\right)$. Let $\dim X=n\ge 3$, then

$$mn^2=mn\left(\frac{n}{2}+\frac{n}{2}\right)\ge mn\left(\frac{n+3}{2}\right),$$

as desired. Since rank of matrix (2.5) is maximal, by assumption, we obtain

$$\frac{\partial y^{\sigma}}{\partial x^{i}}\circ\delta=y^{\sigma}_{i}\circ\delta,\quad \frac{\partial y^{\sigma}_{j}}{\partial x^{i}}\circ\delta=y^{\sigma}_{ij}\circ\delta,\ j\le i,$$

proving that $\delta=J^2\gamma$. Substituting this into (2.8) we get

$$\left(\frac{\partial\widetilde{L}}{\partial y^{\nu}}+\frac{\partial\widetilde{L}^{pq}_{\kappa}}{\partial y^{\nu}}y^{\kappa}_{pq}-d_j\left(\frac{\partial\widetilde{L}}{\partial y^{\nu}_{j}}+\frac{\partial\widetilde{L}^{pq}_{\kappa}}{\partial y^{\nu}_{j}}y^{\kappa}_{pq}\right)+d_jd_k\widetilde{L}^{jk}_{\nu}\right)\circ J^3\gamma$$
$$=\left(\frac{\partial L}{\partial y^{\nu}}-d_j\frac{\partial L}{\partial y^{\nu}_{j}}+d_jd_k\frac{\partial L}{\partial y^{\nu}_{jk}}\right)\circ J^3\gamma\;=\;0,$$

i. e., γ is an extremal of λ. □

Theorem B. *Let λ be a second-order Lagrangian affine in the variables y^{σ}_{ij}, the formula (2.1) be its expression in a fiber chart (V,ψ), $\psi=(x^i,y^{\sigma})$ on Y. Let ρ be a Lepagean equivalent of λ of the form (2.2), (2.3). Suppose that ρ satisfies the conditions*

(2.9) $$h^{klij}_{\sigma\nu}\;=\;0.$$

Assume that the matrix

$$A^{klj}_{\nu\sigma} = \left(\frac{\partial \widetilde{L}^{kl}_{\nu}}{\partial y^{\sigma}_{j}} - \frac{1}{2}\left(\frac{\partial \widetilde{L}^{jk}_{\sigma}}{\partial y^{\nu}_{l}} + \frac{\partial \widetilde{L}^{jl}_{\sigma}}{\partial y^{\nu}_{k}} \right) - g^{kjl}_{\sigma\nu} - g^{ljk}_{\sigma\nu} \right) \tag{2.10}$$

with mn^2 rows (resp. mn columns) labelled by (ν, k, l) (resp. (σ, j)), has the maximal rank (i.e. $\operatorname{rank} A^{klj}_{\nu\sigma} = mn$*).*

Then every Hamilton extremal $\delta : \pi(V) \supset U \to J^2Y$ of ρ is of the form $\pi_{2,1} \circ \delta = J^1\gamma$, where γ is an extremal of λ.

Proof. Substituting (2.9) into Hamilton p_2-equations (2.6), and using the condition $\operatorname{rank} A^{klj}_{\nu\sigma} = mn$ we obtain

$$\frac{\partial y^{\sigma} \circ \delta}{\partial x^{j}} = y^{\sigma}_{j} \circ \delta.$$

The previous condition means $\pi_{2,1} \circ \delta = J^1\gamma$. However, the last equations (2.8) now mean that γ is an extremal of λ. □

3. Legendre transformation on J^2Y for second order Lagrangians affine in second derivatives

Writing the Lepagean equivalent (2.2), (2.3) in the form of a noninvariant decomposition in the canonical basis $(dx^i, dy^\sigma, dy^\sigma_i, dy^\sigma_{ij})$ of 1-forms we get

$$\rho = -H\omega_0 + p^i_\sigma dy^\sigma \wedge \omega_i + p^{ij}_\sigma dy^\sigma_i \wedge \omega_j + f^{ij}_{\sigma\nu}\, dy^\sigma \wedge dy^\nu \wedge \omega_{ij} + g^{kij}_{\sigma\nu}\, dy^\sigma \wedge dy^\nu_k \wedge \omega_{ij} + h^{klij}_{\sigma\nu}\, dy^\sigma_k \wedge dy^\nu_l \wedge \omega_{ij},$$

where

$$\begin{aligned} H &= -L + \left(\frac{\partial L}{\partial y^\sigma_i} - d_j \widetilde{L}^{ij}_\sigma \right) y^\sigma_i + \widetilde{L}^{ij}_\sigma y^\sigma_{ij} + 2 f^{ij}_{\sigma\nu} y^\sigma_i y^\nu_j - \left(g^{kij}_{\sigma\nu} + g^{jik}_{\sigma\nu} \right) y^\sigma_i y^\nu_{jk} \\ &\quad - \frac{1}{2}\left(h^{klij}_{\sigma\nu} + h^{ilkj}_{\sigma\nu} + h^{kjil}_{\sigma\nu} + h^{ijkl}_{\sigma\nu} \right) y^\sigma_{ik} y^\nu_{jl}, \\ p^i_\sigma &= \frac{\partial L}{\partial y^\sigma_i} - d_j \widetilde{L}^{ij}_\sigma - 4 f^{ij}_{\sigma\nu} y^\nu_j - \left(g^{kij}_{\sigma\nu} + g^{jik}_{\sigma\nu} \right) y^\nu_{jk}, \\ p^{ij}_\sigma &= \widetilde{L}^{ij}_\sigma + \left(g^{ikj}_{\nu\sigma} + g^{jki}_{\nu\sigma} \right) y^\nu_k - 2\left(h^{kilj}_{\nu\sigma} + h^{likj}_{\nu\sigma} \right) y^\nu_{kl}. \end{aligned} \tag{3.1}$$

If $p^{ij}_\sigma = p^{ji}_\sigma$ (i. e., $h^{kilj}_{\nu\sigma} + h^{likj}_{\nu\sigma} = h^{kjli}_{\nu\sigma} + h^{ljki}_{\nu\sigma}$) and

$$\det \begin{pmatrix} \dfrac{\partial p^i_\sigma}{\partial y^\nu_k} & \dfrac{\partial p^i_\sigma}{\partial y^\nu_{kl}} \\[2ex] \dfrac{\partial p^{ij}_\sigma}{\partial y^\nu_k} & \dfrac{\partial p^{ij}_\sigma}{\partial y^\nu_{kl}} \end{pmatrix} \neq 0,$$

then

$$\psi_2 = (x^i, y^\sigma, y^\sigma_i, y^\sigma_{ij}) \to (x^i, y^\sigma, p^i_\sigma, p^{ij}_\sigma) = \chi \tag{3.2}$$

is a coordinate transformation over an open set $U \subset V_2$. We call it *Legendre transformation*, and the χ (3.2) the *Legendre coordinates*. Accordingly, $H, p^i_\sigma, p^{ij}_\sigma$ are called a *Hamiltonian* and *momenta*, respectively. Since the functions $f^{ij}_{\sigma\nu}$, $g^{kij}_{\sigma\nu}$, $h^{likj}_{\nu\sigma}$ (2.3) may depend upon the momenta p^i_σ (not upon p^{ij}_σ), the Hamilton p_2-equations (1.4) in these "Legendre coordinates" take a rather complicated form:

$$\begin{aligned}\frac{\partial H}{\partial y^\sigma} = &-\frac{\partial p^i_\sigma}{\partial x^i} + 4\frac{\partial f^{ij}_{\sigma\nu}}{\partial x^j}\frac{\partial y^\nu}{\partial x^i} + 2\left(\frac{\partial f^{ij}_{\kappa\nu}}{\partial y^\sigma} + \frac{\partial f^{ij}_{\kappa\sigma}}{\partial y^\nu} + \frac{\partial f^{ij}_{\nu\kappa}}{\partial y^\sigma}\right)\frac{\partial y^\kappa}{\partial x^i}\frac{\partial y^\nu}{\partial x^j}\\ &- 4\frac{\partial f^{ij}_{\sigma\nu}}{\partial p^k_\kappa}\frac{\partial p^k_\kappa}{\partial x^i}\frac{\partial y^\nu}{\partial x^j} + \frac{\partial g^{kij}_{\sigma\nu}}{\partial x^j}\frac{\partial y^\nu_k}{\partial x^i} + 2\left(\frac{\partial g^{kij}_{\kappa\nu}}{\partial y^\sigma} - \frac{\partial g^{kij}_{\sigma\nu}}{\partial y^\kappa}\right)\frac{\partial y^\kappa}{\partial x^i}\frac{\partial y^\nu_k}{\partial x^j}\\ &- 2\frac{\partial g^{kij}_{\sigma\nu}}{\partial p^l_\kappa}\frac{\partial p^l_\kappa}{\partial x^i}\frac{\partial y^\nu_k}{\partial x^j} + 2\frac{\partial h^{klij}_{\kappa\nu}}{\partial y^\sigma}\frac{\partial y^\kappa_k}{\partial x^i}\frac{\partial y^\nu_l}{\partial x^j},\\ \frac{\partial H}{\partial p^i_\sigma} = &\frac{\partial y^\sigma}{\partial x^i} + 2\frac{\partial f^{jk}_{\kappa\nu}}{\partial p^i_\sigma}\frac{\partial y^\kappa}{\partial x^j}\frac{\partial y^\nu}{\partial x^k} + 2\frac{\partial g^{kjl}_{\kappa\nu}}{\partial p^i_\sigma}\frac{\partial y^\kappa}{\partial x^j}\frac{\partial y^\nu_k}{\partial x^l} + 2\frac{\partial h^{kljm}_{\kappa\nu}}{\partial p^i_\sigma}\frac{\partial y^\kappa_k}{\partial x^j}\frac{\partial y^\nu_l}{\partial x^m},\\ \frac{\partial H}{\partial p^{ij}_\sigma} = &\frac{1}{2}\left(\frac{\partial y^\sigma_i}{\partial x^j} + \frac{\partial y^\sigma_j}{\partial x^i}\right).\end{aligned}$$

However, if $d\eta = 0$, where

$$\eta = f^{ij}_{\sigma\nu}\, dy^\sigma \wedge dy^\nu \wedge \omega_{ij} + g^{kij}_{\sigma\nu}\, dy^\sigma \wedge dy^\nu_k \wedge \omega_{ij} + h^{klij}_{\sigma\nu}\, dy^\sigma_k \wedge dy^\nu_l \wedge \omega_{ij},$$

then

$$\frac{\partial H}{\partial y^\sigma} = -\frac{\partial p^i_\sigma}{\partial x^i}, \quad \frac{\partial H}{\partial p^i_\sigma} = \frac{\partial y^\sigma}{\partial x^i}, \quad \frac{\partial H}{\partial p^{ij}_\sigma} = \frac{1}{2}\left(\frac{\partial y^\sigma_i}{\partial x^j} + \frac{\partial y^\sigma_j}{\partial x^i}\right).$$

In general case the regularity of the Lepagean form (2.3), (2.3) and regularity of Legendre transformation (3.2) do not coincides. By the following Theorem C the existence of Legendre transformation (3.2) guarantees that Theorem B holds.

Theorem C. *Let λ be a second-order Lagrangian affine in the variables y^σ_{ij}, the formula* (2.1) *be its expression in a fiber chart (V, ψ), $\psi = (x^i, y^\sigma)$ on Y. Let ρ be a Lepagean equivalent of λ of the form* (2.2), (2.3). *Suppose that ρ satisfies the conditions $h^{klij}_{\sigma\nu} = 0$. Suppose that ρ admits the Legendre transformation*

$$\psi_2 = (x^i,\, y^\sigma,\, y^\sigma_i,\, y^\sigma_{ij}) \to (x^i,\, y^\sigma,\, p^i_\sigma,\, p^{ij}_\sigma) = \chi$$

defined by (3.1), (3.2).

Then $\pi_{2,1} \circ \delta = J^1\gamma$, where γ is an extremal of λ.

Proof. Since, the functions $h^{klij}_{\sigma\nu}$ vanish, the Jacobi matrix of the Legendre transformation takes the form

$$\begin{pmatrix} \frac{\partial p^i_\sigma}{\partial y^\nu_k} & \frac{\partial p^i_\sigma}{\partial y^\nu_{kl}} \\ \frac{\partial p^{ij}_\sigma}{\partial y^\nu_k} & 0 \end{pmatrix}$$

The above matrix is regular if and only if the matrices $\left(\frac{\partial p^i_\sigma}{\partial y^\nu_{kl}}\right)$, and $\left(\frac{\partial p^{ij}_\sigma}{\partial y^\nu_k}\right)$ have the maximal rank. Explicit computations lead to

$$\frac{\partial p^i_\sigma}{\partial y^\nu_{kl}} = \frac{\partial \widetilde{L}^{kl}_\nu}{\partial y^\sigma_i} - \frac{1}{2}\left(\frac{\partial \widetilde{L}^{ik}_\sigma}{\partial y^\nu_l} + \frac{\partial \widetilde{L}^{il}_\sigma}{\partial y^\nu_k}\right) - g^{kil}_{\sigma\nu} - g^{lik}_{\sigma\nu},$$

i.e. in the notation (2.10), $\left(\frac{\partial p^i_\sigma}{\partial y^\nu_{kl}}\right) = \left(A^{klj}_{\nu\sigma}\right)^T$.

Accordingly, from Theorem B we obtain $\pi_{2,1}\circ\delta = J^1\gamma$, where γ is an extremal of λ. □

For a deeper discussion on Legendre transformations and their geometric meaning we refer to [11], [12].

4. Projectability onto J^1Y

Theorem D. *Let λ be a second-order Lagrangian affine in the variables y^σ_{ij}, i. e., in fibered coordinates expressed by* (2.1). *Let ρ be a Lepagean equivalent of λ of the form* (2.2), (2.3). *The following conditions are equivalent:*

I. *ρ is projectable onto J^1Y.*

II. *ρ satisfies the conditions*

$$h^{kilj}_{\nu\sigma} + h^{likj}_{\nu\sigma} = 0,$$

(4.1)
$$g^{kjl}_{\sigma\nu} + g^{ljk}_{\sigma\nu} = \frac{\partial \widetilde{L}^{kl}_\nu}{\partial y^\sigma_j} - \frac{1}{2}\left(\frac{\partial \widetilde{L}^{jk}_\sigma}{\partial y^\nu_l} + \frac{\partial \widetilde{L}^{jl}_\sigma}{\partial y^\nu_k}\right).$$

Proof. Taking into account

$$\rho = -H\omega_0 + p^i_\sigma dy^\sigma\wedge\omega_i + p^{ij}_\sigma dy^\sigma_i\wedge\omega_j$$
$$+ f^{ij}_{\sigma\nu}\, dy^\sigma\wedge dy^\nu\wedge\omega_{ij} + g^{kij}_{\sigma\nu}\, dy^\sigma\wedge dy^\nu_k\wedge\omega_{ij} + h^{klij}_{\sigma\nu}\, dy^\sigma_k\wedge dy^\nu_l\wedge\omega_{ij},$$

it is sufficient to find conditions of the independence H, p^i_σ, and p^{ij}_σ on y^σ_{ij}'s. Explicit computations lead to

$$\frac{\partial p^i_\sigma}{\partial y^\nu_{kl}} = \frac{\partial \widetilde{L}^{kl}_\nu}{\partial y^\sigma_i} - \frac{1}{2}\left(\frac{\partial \widetilde{L}^{ik}_\sigma}{\partial y^\nu_l} + \frac{\partial \widetilde{L}^{il}_\sigma}{\partial y^\nu_k}\right) - g^{kil}_{\sigma\nu} - g^{lik}_{\sigma\nu} = 0,$$

$$\frac{\partial p^{ij}_\sigma}{\partial y^\nu_{kl}} = -2\left(h^{kilj}_{\nu\sigma} + h^{likj}_{\nu\sigma}\right) = 0,$$

$$\frac{\partial H}{\partial y^\nu_{kl}} = \left(\frac{\partial \widetilde{L}^{kl}_\nu}{\partial y^\sigma_i} - \frac{1}{2}\left(\frac{\partial \widetilde{L}^{ik}_\sigma}{\partial y^\nu_l} + \frac{\partial \widetilde{L}^{il}_\sigma}{\partial y^\nu_k}\right) - g^{kil}_{\sigma\nu} - g^{lik}_{\sigma\nu}\right) y^\sigma_i$$
$$- \left(h^{kjli}_{\nu\sigma} + h^{kilj}_{\nu\sigma} + h^{ljki}_{\nu\sigma} + h^{likj}_{\nu\sigma}\right) y^\sigma_{ij}.$$

□

Corollary. *Every second-order Lagrangian affine in the variables y^{κ}_{ij} has a Lepagean equivalent projectable onto J^1Y.*

Remark. If the functions $f^{ij}_{\sigma\nu}$, $g^{kij}_{\sigma\nu}$, $h^{klij}_{\sigma\nu}$ (2.2), (2.3) vanish, i. e., $\rho = \theta_\lambda$ the projectability conditions (4.1) take the form (cf. [9])

$$\frac{\partial \widetilde{L}^{kl}_{\nu}}{\partial y^{\sigma}_{j}} - \frac{1}{2}\left(\frac{\partial \widetilde{L}^{jk}_{\sigma}}{\partial y^{\nu}_{l}} + \frac{\partial \widetilde{L}^{jl}_{\sigma}}{\partial y^{\nu}_{k}}\right) = 0.$$

Theorem E. *Let λ be a second-order Lagrangian affine in the variables y^{σ}_{ij}, the formula (2.1) be its expression in a fiber chart (V,ψ), $\psi = (x^i, y^\sigma)$ on Y. Let ρ be a Lepagean equivalent of λ of the form* (2.2), (2.3) *and suppose that it is projectable onto J^1Y. If ρ satisfies the conditions*

$$h^{klij}_{\sigma\nu} = 0,$$

$$\frac{\partial f^{ij}_{\sigma\nu}}{\partial y^{\kappa}_{k}} = \frac{1}{2}\left(\frac{\partial g^{kij}_{\kappa\sigma}}{\partial y^{\nu}} - \frac{\partial g^{kij}_{\kappa\nu}}{\partial y^{\sigma}}\right)$$

$$g^{ikj}_{\nu\sigma} - g^{kij}_{\sigma\nu} = \frac{1}{2}\left(\frac{\partial \widetilde{L}^{ij}_{\sigma}}{\partial y^{\nu}_{k}} - \frac{\partial \widetilde{L}^{kj}_{\nu}}{\partial y^{\sigma}_{i}}\right) \tag{4.2}$$

$$\frac{\partial g^{lij}_{\sigma\kappa}}{\partial y^{\nu}_{k}} - \frac{\partial g^{kij}_{\sigma\nu}}{\partial y^{\kappa}_{l}} = 0.$$

and the matrix

$$C^{kj}_{\nu\sigma} = \left(\frac{\partial^2 \widetilde{L}}{\partial y^{\sigma}_{j}\partial y^{\nu}_{k}} + \frac{\partial^2 \widetilde{L}^{pq}_{\kappa}}{\partial y^{\sigma}_{j}\partial y^{\nu}_{k}} y^{\kappa}_{pq} - \frac{\partial}{\partial y^{\nu}_{k}} d_p \widetilde{L}^{jp}_{\sigma} - \frac{\partial \widetilde{L}^{kj}_{\nu}}{\partial y^{\sigma}} - 4 f^{jk}_{\sigma\nu} + 2 d_i g^{kij}_{\sigma\nu}\right),$$

with rows (resp. columns) labelled by (ν,k) (resp. (σ,j)), is regular, then ρ is regular, i.e., every Hamilton extremal $\delta : \pi(V) \supset U \to J^1Y$ of ρ is of the form $\delta = J^1\gamma$, where γ is an extremal of λ.

Proof. Expressing the Hamilton p_2-equations (1.4) of a Lepagean equivalent ρ projectable onto J^1Y in fiber coordinates and using (4.2) we get along δ the following system of first-order equations:

mn equations

$$\left(\frac{\partial^2 \widetilde{L}}{\partial y^{\sigma}_{j}\partial y^{\nu}_{k}} + \frac{\partial^2 \widetilde{L}^{pq}_{\kappa}}{\partial y^{\sigma}_{j}\partial y^{\nu}_{k}} y^{\kappa}_{pq} - \frac{\partial}{\partial y^{\nu}_{k}} d_p \widetilde{L}^{jp}_{\sigma} - \frac{\partial \widetilde{L}^{kj}_{\nu}}{\partial y^{\sigma}} - 4 f^{jk}_{\sigma\nu} + 2 d_i g^{kij}_{\sigma\nu}\right) \times \left(\frac{\partial y^{\sigma}}{\partial x^j} - y^{\sigma}_{j}\right) = 0, \tag{4.3}$$

m equations

$$\left(\frac{\partial \widetilde{L}}{\partial y^{\nu}} + \frac{\partial \widetilde{L}^{pq}_{\kappa}}{\partial y^{\nu}} y^{\kappa}_{pq} - d_j\left(\frac{\partial \widetilde{L}}{\partial y^{\nu}_{j}} + \frac{\partial \widetilde{L}^{pq}_{\kappa}}{\partial y^{\nu}_{j}} y^{\kappa}_{pq}\right) + d_j d_k \widetilde{L}^{jk}_{\nu}\right) \tag{4.4}$$

$$+\left(\frac{\partial^2\widetilde{L}}{\partial y^\sigma_j\partial y^\nu}+\frac{\partial^2\widetilde{L}^{pq}_\kappa}{\partial y^\sigma_j\partial y^\nu}y^\kappa_{pq}-\frac{\partial^2\widetilde{L}}{\partial y^\sigma\partial y^\nu_j}-\frac{\partial^2\widetilde{L}^{pq}_\kappa}{\partial y^\sigma\partial y^\nu_j}y^\kappa_{pq}-\frac{\partial}{\partial y^\nu}d_k\widetilde{L}^{jk}_\sigma\right.$$
$$\left.+\frac{\partial}{\partial y^\sigma}d_k\widetilde{L}^{jk}_\nu+2d_i f^{ij}_{\sigma\nu}\right)\left(\frac{\partial y^\sigma}{\partial x^j}-y^\sigma_j\right)$$
$$+\left(\frac{\partial\widetilde{L}^{kj}_\sigma}{\partial y^\nu}-\frac{\partial^2\widetilde{L}}{\partial y^\sigma_j\partial y^\nu_k}-\frac{\partial^2\widetilde{L}^{pq}_\kappa}{\partial y^\sigma_j\partial y^\nu_k}y^\kappa_{pq}+\frac{\partial}{\partial y^\sigma_k}d_p\widetilde{L}^{jp}_\nu+4f^{jk}_{\nu\sigma}-2d_i g^{kij}_{\nu\sigma}\right)$$
$$\times\left(\frac{\partial y^\sigma_k}{\partial x^j}-y^\sigma_{kj}\right)$$
$$+2\left(\frac{\partial f^{ij}_{\sigma\nu}}{\partial y^\kappa}+\frac{\partial f^{ij}_{\kappa\sigma}}{\partial y^\nu}+\frac{\partial f^{ij}_{\nu\kappa}}{\partial y^\sigma}\right)\left(\frac{\partial y^\kappa}{\partial x^i}-y^\kappa_i\right)\left(\frac{\partial y^\sigma}{\partial x^j}-y^\sigma_j\right)=0.$$

The matrix $C^{kj}_{\nu\sigma}$ is regular. Hence, from equations (4.3) we obtain the formula

$$\frac{\partial y^\sigma\circ\delta}{\partial x^j}=y^\sigma_j\circ\delta. \tag{4.5}$$

Substituting this into (4.4) we get

$$\left(\frac{\partial\widetilde{L}}{\partial y^\nu}+\frac{\partial\widetilde{L}^{pq}_\kappa}{\partial y^\nu}y^\kappa_{pq}-d_j\left(\frac{\partial\widetilde{L}}{\partial y^\nu_j}+\frac{\partial\widetilde{L}^{pq}_\kappa}{\partial y^\nu_j}y^\kappa_{pq}\right)+d_jd_k\widetilde{L}^{jk}_\nu\right)\circ J^3\gamma$$
$$=\left(\frac{\partial L}{\partial y^\nu}-d_j\frac{\partial L}{\partial y^\nu_j}+d_jd_k\frac{\partial L}{\partial y^\nu_{jk}}\right)\circ J^3\gamma\ =\ 0,$$

proving our assertion. □

Remark. **a)** Let λ be a second-order Lagrangian (2.1), suppose that the functions $\widetilde{L}^{ij}_\sigma$ satisfy the conditions

$$\frac{\partial\widetilde{L}^{ki}_\sigma}{\partial y^\nu_j}=\frac{\partial\widetilde{L}^{ki}_\nu}{\partial y^\sigma_j}$$

This means that $\widetilde{L}^{ij}_\sigma$ take the form

$$\widetilde{L}^{ij}_\sigma=\frac{1}{2}\left(\frac{\partial f^j}{\partial y^\sigma_i}+\frac{\partial f^i}{\partial y^\sigma_j}\right)$$

and the Lagrangian equivalent with a first-order Lagrangian.

We can choose the functions $g^{kij}_{\sigma\nu}$ in a regular Lepagean equivalent (in the sense of Theorem E) in the following form

$$g^{kij}_{\sigma\nu}=\frac{1}{2}\left(\frac{\partial\widetilde{L}^{kj}_\nu}{\partial y^\sigma_i}-\frac{\partial\widetilde{L}^{ki}_\nu}{\partial y^\sigma_j}\right)+t^{kij}_{\sigma\nu},$$

where the functions $t^{kij}_{\sigma\nu}$ do not depend on the variables y^{ν}_{kl} and satisfy the conditions

$$t^{kij}_{\sigma\nu} = -t^{kji}_{\sigma\nu},\; t^{kij}_{\sigma\nu} = -t^{jik}_{\sigma\nu},\; t^{kij}_{\sigma\nu} = -t^{ikj}_{\nu\sigma}.$$

b) Let λ be a second-order Lagrangian (2.1) and suppose that the functions $\widetilde{L}^{ij}_{\sigma}$ satisfy the conditions

$$\frac{\partial \widetilde{L}^{kl}_{\nu}}{\partial y^{\sigma}_{j}} - \frac{1}{2}\left(\frac{\partial \widetilde{L}^{jk}_{\sigma}}{\partial y^{\nu}_{l}} + \frac{\partial \widetilde{L}^{jl}_{\sigma}}{\partial y^{\nu}_{k}}\right) = 0.$$

Then we can choose the functions $g^{kij}_{\sigma\nu}$ as follows:

$$g^{kij}_{\sigma\nu} = \frac{1}{4}\left(\frac{\partial \widetilde{L}^{kj}_{\nu}}{\partial y^{\sigma}_{i}} - \frac{\partial \widetilde{L}^{ij}_{\sigma}}{\partial y^{\nu}_{k}}\right) + t^{ijk}_{\sigma\nu},$$

where the $t^{kij}_{\sigma\nu}$'s are as above.

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