Cross-Curriculum Applications of Mathematics

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Abstract: In Australia applications of mathematics in other curriculum contexts are located within the jurisdiction of mathematics courses. That is mathematics reaches out, but the applications are decided by mathematics personnel rather than designed in collaboration with subject experts from other areas. This may be achieved either by school-based decision making, or by means of a common application or investigation set at a state level. Examples from both approaches are provided in the paper. Tertiary mathematicians support the introduction of applications, but university training of applied mathematicians has not as yet linked closely with the increased emphasis introduced at secondary level.

Kurzreferat: *Fächerübergreifende Anwendungen der Mathematik.* Anwendungen der Mathematik in anderen Disziplinen liegen in Australien in der Zuständigkeit der Mathematik. Das heißt, die Mathematik wird erweitert, aber die Anwendungen werden eher vom mathematischen Lehrkörper beschlossen als daß sie in Zusammenarbeit mit Experten anderer Fachgebiete entworfen würden. Dies könnte entweder durch Entscheidungen auf Schulebene oder mittels allgemeiner Anwendungen oder Untersuchungen auf Staatsebene erreicht werden. Beispiele für beide Fälle werden gegeben. Mathematiker im Tertiärbereich unterstützen die Einführung von Anwendungen, aber die Hochschulausbildung angewandter Mathematiker ist noch nicht eng genug verknüpft mit der wachsenden Bedeutung, wie sie sich im Sekundarbereich zeigt.

ZDM-Classification: M10

1. Introduction

The Australian education system is state-based with each state implementing its own subject curricula. While there are common topics and themes across the curricula they tend to remain self-contained and this applies also on a within-state basis. Several states such as Victoria and Queensland have specific requirements in relation to applications and modelling at senior secondary level. However these are confined within the boundaries of the mathematics subjects so that there is limited opportunity for cross-curricular developments as such, for while applications and models are developed by teachers and students they tend to emanate from within the mathematics classroom. That is, they address a selection of real-world problems but not in collaboration with a particular curricular area outside mathematics. Physicists will use mathematics for example in physics, and a mathematical problem may be set in an ecological context - but not usually devised in collaboration with a biologist or a geographer. This means that while mathematics students may learn to address real world applications such applications do not involve formal cross-curricular co-operation.

The application of mathematics to other curriculum areas to the extent described above is facilitated by mandated assessment criteria which require that investigations and modelling be conducted in real-world contexts as well as within the confines of mathematics itself. In Queensland, choice of example and assessment lies within the province of school-based curriculum decisions, and district and state panels are entrusted to exercise a monitoring and quality control function. In Victoria a common set of tasks is provided from which students select an extended example. Several weeks are allowed for the students to work on the task and a common date is set for submission of the final report. In this case the nature of the task effectively directs the students to engage with information that is within the scope of other curriculum areas. However as noted above this approach is instigated and driven from within mathematics rather than as a collaborative cross-curricular venture.

2. Cross-disciplinary applications

Three examples follow which illustrate the nature of this approach to applications.

Lighter Baseball Bats (Money et al. 1992)
How does the weight of a baseball bat affect the speed with which the ball, if you manage to hit it, soars back in the direction from which it came? The laws of physics, in particular the principle of conservation of momentum, govern the relationship between the weights and speeds of the bat and the ball. In consequence it might be thought that the heavier the bat the better, but this overlooks the fact that players, young ones in particular, find it much harder to swing a heavy bat than a light one. Adult professional players use much heavier bats than 'Little League', but they eventually have to decide on a compromise between a slight increase in power and a wavering in the accuracy of their swing. It helps them make this decision if they understand the mathematical model underlying these relationships. In this investigation you are asked to choose a particular player and develop a model which can be checked against real world data. Obtaining all the data you need may present some measurement difficulties and the following information should at least enable you to get started:

Ball design:

- The weight of a baseball must be between 142 and 149 grams (5 to 5.25 ounces);
- The circumference must be between 23 and 23.5cm;
- The coefficient of restitution,

$$e = \frac{Vball - Vbat \text{ after collision}}{Vball - Vbat \text{ before collision}} = 0.55$$

- The length of a professional league baseball bat may not exceed 107cm;
- The diameter may not exceed 7cm.

Bat speed data:
A skilled 10-year old player was tested with a range of different bats to obtain the following data:

Bat weight (g)	300	350	400	450	500	550
Bat speed (km/h)	93	88	83	79	75	72
Bat weight (g)	600	650	700	750	800	850
Bat speed (km/h)	68	65	62	60	57	55

This problem is typical of those that can be set within the school-based model of curriculum design such as applies in Queensland. Here the cross-curricular content is Sport and Physical Education. Students are encouraged to engage with the content area to the extent of practical involvement in order to gain personal insight into the nature of the problem. Staff from the sporting section may be involved through interest, and indeed share direct advice and intuitive knowledge with the students who then bring this experience back into the solution of the mathematical problem.

The next two problems were set within the Victoria Certificate of Education as common assessment tasks at pre-university level.

The Art Gallery (Board of Studies 1995b)

Question 1:
A room in an art gallery contains a picture which you are interested in viewing. The picture is two metres high and is hanging so that the bottom of the picture is one metre above your eye level. How far from the wall on which the picture is hanging should you stand so that the angle of vision occupied by the picture is a maximum? What is this maximum angle?

Question 2:
On the opposite wall there is another equally interesting picture which is only one metre high and which is also hanging with its base one metre above eye level, directly opposite the first picture. Where should you stand to maximise your angle of vision of this second picture?

Question 3:
How much advantage would a person 20 centimetres taller than you have in viewing these two pictures?

Question 4:
This particular art-gallery room is six metres wide. A gallery guide of the same height as you wishes to place a viewing stand one metre high in a fixed position which provides the best opportunity for viewing both pictures simply by turning around. The guide decides that this could best be done by finding the position where the sum of the two angles of vision is greatest. Show that the maximum value which can be obtained by this approach does not give a suitable position for the viewing stand.

Question 5:
The gallery guide then decides to adopt an alternative approach which makes the difference between the angles of vision of the two pictures, when viewed from the viewing stand, as small as possible. Where should the viewing stand be placed using this approach? Comment on your answer.

In order to obtain a feel for this problem it is necessary either to visit an art gallery or to set up a room with pictures that simulate the context. In either case students are brought to consider both the geometry and the aesthetics involved in displays of paintings. As a matter of interest the best solutions incorporated the use of compound angle formulae, inverse trigonometric functions, maximisation using calculus as well as extensive use of functions and their graphical representation.

Times and Tides (Board of Studies 1995a)

Background:
Townsville is a large town situated on the coast of Queensland, opposite Magnetic Island, the position of which contributes to a complex tidal system. This is in an area where a knowledge of tidal patterns is of great importance for both the local marine and tourist industries. Although these patterns are based on many variables, they are predicted with extreme accuracy by the National Tidal Facility and published in tables (as provided for students).

In this investigation you should use only the data provided. You are, however, encouraged to comment on any assumptions and limitations associated with the data provided.

You may choose to develop a computer program to assist you, or you may use a recognised computer package but remember to include your own analysis of the problem.

The tidal data provided for a complete year included

- Times and heights of high and low tides on a daily basis relative to a fixed level.
- Phases of the moon.

Possible Starting Points

1. Hightides and lowtides: In this starting point you will explore the variability of the tides for Townsville and the relationship between the height of the daily highest tide and the daily lowest tide.

a.

i. Using a random number table, or any other random number generator, randomly select 30 days from the table. Briefly explain your method of selection. Draw a scatterplot of daily highest tide versus daily lowest tide for this sample. Perform a correlation analysis for this sample. What conclusions can you draw from your scatterplot and your analysis?

ii. Using the method of least squares derive an equation which enables you to predict the daily lowest tide when given the daily highest tide. Comment on the reliability of your equation. On your scatterplot, graph the equation to show your line of best fit.

iii. Produce a table which predicts the expected daily lowest tides from daily highest tides between 1.9m and 3.7m at 10cm increments.
Comment on any days of your sample for which the prediction is poor. You may refer to tidal heights on other days to support your observations.

b.

i. Find the mean height of all the daily highest tides in your sample and then the mean height of all the daily lowest tides. Compare the extent to which daily highest and daily lowest tides in your sample vary from these two means.

ii. Perform the same calculations for the daily sums and differences of these tides. Comment on your results.
To what extent do these results support the conclusions obtained from your regression analysis?
Discuss the reasonableness of your conclusions in the light of the data provided.

2. Tides and the Moon:
At least 112 different variables have been identified as important when predicting tidal heights and times. One of the most significant of these is the phase of the Moon. These phases are shown on the tidal tables in the following way using the symbols shown in the charts (not included here).

New Moon	First Quarter
Full Moon	Last Quarter

The time between these phases remains constant as the Moon orbits the Earth. In this starting point you will need to explore the relationship between the tidal heights and the Moon.

a.

i. Choose three months where each month is from a different season of the year. Examine the tidal heights (high and low) on the days of both the full and the new Moons and the days immediately before and after each. (For example, in July, for the full Moon phase, you will need to consider the tides on July 3, 4 and 5.)

ii. Compare the patterns you find with those obtained for the days of the first and last quarters and the days before and after them.
Comment on what you have found.

b.

i. Choose a period between two new moons which does not include any days you have already analysed and which covers at least two months. Considering only the daily highest tide each day, calculate for this period
   - a 10-point moving average and
   - a 29-point moving average.

ii. Compare the two outcomes graphically, clearly marking in the phases of the Moon.

iii. Comment on what this analysis shows. You may like to consider further data. Discuss the reasonableness of your conclusions in the light of the data provided.

The better approaches to this problem involved the use of sampling procedures, stem and leaf plots, correlational analysis (q-correlation and Pearson), least squares regression, confidence limits, and curve fitting. In addressing the problem successful students found it necessary to read broadly in the areas of tidal variations and lunar phases, which brought geographical knowledge to a strong interplay with mathematical techniques.

Through these and other life-related problems students over the last five years in two Australian states have been required to apply their mathematics to a range of problems in a variety of areas, rather than systematically link their mathematics in specific cross-curricular initiatives. This can be seen as both advantageous in demonstrating the breadth of applications to which mathematics can be put, and disadvantageous in that opportunities for developing systematic applications in any other given curriculum area are limited.

3. Applied mathematics research

Given the type of introduction to interdisciplinary applications of mathematics provided at the senior levels of high school, it is useful to note the type of activity that characterises applied research among academics. A few selected examples are provided for this purpose from projects funded by the Australian Research Council (1995 & 1996), that in total span a complete range of disciplines, including physical sciences, social sciences, marine and environmental science, and medical applications. Many (most?) are using computer technology as a central component in modelling.

Complexity of arithmetic iterations is at the source of improving the security of basic crypto-systems and a project is exploring new constructions for cryptographically strong random numbers. The analysis of tumour growth is (among other areas) one involving competition between two unstable states that give rise to fractal patterns and these phenomena are being explored by developing mathematical models to provide insight into the origin and growth of these complex patterns. In socio-economic systems leaders and followers are often central figures as when the production plans of a small company (follower) are impacted by a market-dominating corporate leader in the same field and a project is investigating leader-follower behaviour in both theoretical and computational terms. The phenomenon of long-range dependence has been noted in a wide range of scientific, economic, and engineering applications and a project is designing efficient methods for statistical analysis of long-range dependence data. Methods and models are being developed for application to industrial projects involving air quality control in a major city and rural basin, and to forecasting electricity demand taking account of weather impact and spatial variability. More traditionally computer modelling is simulating and investigating complex dynamics associated with the impact of turbulent floods on craft such as barges and hydrofoils. In a different context a medically based program has been investigating factors associated with the onset of schizophrenia over a period of eight years. This project has international links with groups at London and Mannheim and includes an interesting finding based on a huge data set that identifies onset as early as 6-8 years and as late as 60-70 years.

4. Conclusion

In conclusion it is appropriate to comment on links between schooling and academic mathematics with respect to applications and modelling. It would be fair to say that the research training of postgraduate students still follows a traditional training model. This involves coursework in advanced mathematics followed by mentoring by one or more practising research mathematicians. Research topics are determined by the interests and skills of the supervisors and their schools. Even at this level cross-disciplinary projects are limited, being located typically in one mathematics department. Funding mechanisms that direct resources to parent departments add further pressure to this tendency. The introduction of applications of mathematics into senior school curricula have the support of the academics, and the property that they remain firmly within the jurisdiction of mathematics curricula is perhaps a reflection of the territorialism that applies also within academia. Mathematical training of the type needed to support research projects of the type indicated above is certainly not a consequence of systematic building upon application skills gained at school for in this respect secondary and tertiary mathematics continue to exist as essentially separate entities rather than a cohesive progression. That is while cross-curricular applications of mathematics associated with pre-university education are seen on the one hand as belonging to the mathematics community and aimed at broadening the skills of students, on the other they are not trusted to the extent that tertiary curricula build consciously upon the abilities so developed. Some distance remains to be travelled before approaches to applications of mathematics involving knowledge from other disciplines can be genuinely described as cross-curricular.

5. References

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