The use of computer algebra in middle secondary mathematics. Part 2*

Experimental and Active Learning with DERIVE

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Abstract: The development of mathematics and mathematics education always was influenced by the tools. Now we are at the beginning of a new phase of this development caused by the possibilities of Computer Algebra Systems (CAS). This article deals with the most important results of the Austrian project: More pupil-oriented, experimental learning, new didactical concepts and changes in the exam situation. The examples I have chosen are from the 7th, the 8th and the 9th grades.

Kurzreferat: *Experimentelles und aktives Lernen mit DERIVE.* Die Entwicklung der Mathematik und des Mathematikunterrichts ist immer durch Werkzeuge bestimmt worden. Wir stehen nun am Beginn einer neuen Phase dieser Entwicklung, die durch die Möglichkeiten der Computeralgebrasysteme (CAS) bedingt ist. In diesem Beitrag wird über die wichtigsten Ergebnisse des österreichischen Projekts “Mehr schülerorientiertes, experimentelles Lernen, neue didaktische Konzepte und Veränderungen der Prüfungssituation” berichtet. Die ausgewählten Beispiele beziehen sich auf die Klassen 7–9.

ZDM-Classification: H20, R20, U70

1. The Austrian project

At the beginning of the 90s, as far as I know, Austria was the first country in the world to buy a general licence for a computer algebra system – namely DERIVE – for all its grammar schools. The next move, then, was not only to equip teachers with the necessary disks and manuals, but, most of all, to quickly offer them didactic guidelines and tools.

On this level, the research project “Computer Algebra Systems in the Classroom” has been going on since February 1993 (Heugl, 1996). The thesis and the examples of my article are results of this project.

For evaluating the results, it is necessary to know that there are 3 different sorts of classes involved in the project:

Type I:
: Students can make use of Computer Algebra Systems (CAS) in any given situation – in class, at home, and during exams. Consequently, the computer can be used both as a tool for doing calculations and as a didactic tool and tutor.
Equipment varies:
In some groups, all students have a PC at home and go to the computer lab when working at school. This means that very often two students have to share one PC. This disadvantage, however, can have favourable aspects, too: Pair work often leads to good results in the classroom situation, because students will often discuss mathematical problems more frequently than when working alone. Homework is partly done on disks; tests are written on the computer.

Type II:
: CAS are regularly used in the classroom (at least once a week). Students work in the computer lab. In those classes where they cannot use the computer at home and in the exam situation, CAS are mainly a didactic tool.

Type III:
: Control groups who work on similar tasks as the other classes using traditional methods, without having CAS at their disposal.

The partition of the classes by the 3 types: All together about 700 pupils in 39 classes took part in the project. The partition by grades: 18 Type I, 16 Type II, 5 Type III.

Our first concept was to start the project in the 9th grade. Elementary algebra and the first introduction to the idea of functions should be taught without the use of computers. But after a short time we started to explore the influence of the White Box/Black Box Principle in the teaching and learning process. One result was that is was interesting also to use the CAS in the white-box-phase of learning, especially in elementary algebra. Therefore we started the project in the 7th grade.

2. The influence of CAS in teaching and learning mathematics

General issues of a first practical test:

One goal of the Austrian project was to examine the influence of CAS on the didactical concepts. In a study organized by R. Nocker, the results of classroom observa-

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tions of 20 lessons with and 37 lessons without the use of computers were compared (Nocker, 1996). Lessons from the 9th to the 11th grade had been observed. A special pattern was used to cover the important aspects of classroom methodology in teaching mathematics. The main results of the study confirm the experience of the project-teachers also in the 7th and 8th grade:

- The change from a teacher-oriented to a more pupil-centered learning process,
- a change from working with the whole class to pair work and single work,
- a change from receiving information to active production of knowledge,
- pupils explore mathematics using experimental concepts.

2.1 New didactical principles

The White Box/Black Box Principle

This principle was first explicitly formulated by Bruno Buchberger of the RISC Institute at the University of Linz, Austria (Buchberger, 1992). When teaching mathematics according to this recursive model, the learning process proceeds in two phases:

- The White Box Phase – the phase of cognitive learning: Students should be led to a mathematical concept, an algorithm, a mathematical theory. Necessary calculation skills should be trained “by hand”. The CAS is used as a Black Box for algorihms the students had explored in earlier White Boxes or as a didactic tool to support the searching of the new White Box.
- The Black Box Phase – the phase of skilful application: Algorithms which had been studied in the White Box Phase could be used as Black Boxes in the problem solving process.

Examples:

(1) Look at the examples of chapter 3.2: White Box: Learning calculating rules for term transformations. Black Box: Using the CAS as a black box when solving equations etc.

(2) White Box: Studying the Gauß Algorithm for solving systems of linear equations. – Black Box: Using the CAS to solve systems of linear equations in application problems.

(3) White Box: Studying Newton’s method, development of a module by the students. Black Box: Using this module as Black Box in application problems.
Student’s activities in the White Box: Formulating the problem – developing the idea behind this method – calculating some problems – discussing the convergence, proving etc. – creating a module (e.g. a DERIVE utility file) which could be used in the Black Box Phase.

The Black Box/White Box Principle

- Now the first phase is a Black Box Phase – heuristic, experimental phase:
  Using the CAS as a black box enables students to discover mathematical theories, concepts or algorithms, and to find strategies for solving problems experimentally, by step by step investigation of black box operations of the CAS (“we are also able to do what the CAS can do”).
- The White Box Phase – “exactifying” phase:
  Making safe the supposition of the black box phase by proving, testing, calculating etc.

Examples:

(1) Black Box: Using the **Expand** and the **Factor**-command as a black box. White Box: Exploring the black box by experimenting with the support of the CAS (Motto: “We should also be able to understand and do what the computer did”)

(2) Black Box: Using the **Taylor** command of DERIVE as a Black Box. Discovering the attributes of the Taylor polynomials by experimental learning. Comparing the Taylor polynomials of several functions (e.g.: $\sin(x)$, $\cos(x)$, e^x). Formulating a supposition. (Barzel, 1991).
White Box: Proving the Taylor formula, discussing the convergence problem, applying the Taylor formula (Barzel, 1991).

The Module Principle

The modular way of thinking and working which is typical of informatics science and informatics applications could support and also change mathematical thinking and the strategies of learning mathematics especially if the learning process is supported by the use of computers. B. Buchberger created the idea of a new science “Mathformatic”, which means in the future mathematics could not give up the informatics and the informatics could not exist without the mathematics.

The programming features of CAS allow to create modules which can be used on the one hand as didactic tools in the white box phase and on the other hand as black boxes during the problem solving process.

We distinguish 3 sorts of modules:

- *Modules created by the students* in the white box phase which are used in the black box phase via function call. Typical applications: Complex calculations, visualisation, simulation, modules used as didactical instruments (examples: a “Newton module”, a module for plotting Riemann rectangles, a module for simulating growth processes, etc.). In the problem solving process, it may be useful sometimes to combine several modules into a more complex module which solves the whole problem completely (e.g. a trigonometric module to dissolve any triangles the students need).
- *Modules created by the teacher* to support the student’s learning process which need not necessarily become white boxes, e.g. a module which is plotting histograms to help the students to get a better understanding of binomial and normal distributions.
- *Modules which are made available by the CAS*, e.g. modules for solving differential equations.

In the 7th and the 8th grade the Module Principle does not play as important a role as at upper secondary level II. Exceptions: A module for creating a table of values or the Iterates-function of DERIVE for using recursive models.

2.2 The influence of CAS on the path of students “into” mathematics

B. Buchberger describes this path as a spiral, which we call the "Creativity Spiral". At the beginning we always find problems (Buchberger, 1993). The next stage is formulating conjectures concerning new algorithms, new concepts or concerning strategies for proving. In the next stage theories are formulated, typical activities are exactifying and proving. In the following stage the new theories or the new algorithms are applied for solving the problems. Solving new problems often requires new concepts, new algorithms etc.

Another point of view divides the learning process into *3 phases:*

The heuristic phase

Typical activities are: finding conjectures, trying to devise problem solving strategies, student oriented experimental ways of learning, methods of trial and error, testing models, testing and interpreting results.

The exactifying phase

Typical activities: corroborating assumptions, deducing algorithms, proving theories etc. People often think and students often experience that these activities are the only ones when doing mathematics.

The application phase

Typical activities: applying algorithms, solving problems, modelling, using CAS as a black box for operations which students have developed themselves in phases I and II, testing and interpreting.

In traditional mathematics education, the experimental or heuristic phase often does not exist. One reliable result of the Austrian project is the growing importance of the heuristic phase in learning mathematics.

3. Experimental learning in the heuristic phase

A possible definition of heuristic is: "the art of finding a way" or "the teaching of the ways to scientific recognition".

Pupils in schools and especially students at universities often experience mathematics as something finished and final. What they are not shown is the way to mathematical recognition. Freudenthal once said: "If mathematicians showed their students the way to their mathematical recognition they would feel naked". With the aid of CAS this heuristic phase should and could get a greater importance in the future. Not results, recipes or unequivocal ways given by the teacher to the pupils should be the main goals, pupils should have the opportunity to try several ways, to decide on their individual way or strategy.

Mathematical methods are often connected with deductiv, logical conclusions. But typical aspects for the way to mathematical recognition – we say the heuristic phase – are:

- plausible, inductive conclusions,
- experimental phases,
- the concept of trial and error,
- not only the result but the strategy for getting results is the subject of the thinking process.

In didactical literature you can find several heuristic strategies e.g. generalising, specialising, working forward or backward, etc. (Fischer/Malle, 1985, S. 205–220). The use of CAS makes new strategies available and others could be used more easily.

3.1 Some computer-aided strategies

(1) Experimenting
Looking for assumptions by systematical trial.

(2) Testing with the CAS
While numerical tests were also possible with numeric calculators, the CAS also allow to use algebraic and graphic tests. It is possible to examine the equivalence of several results. Pupils can check whether the mathematical result is suitable for the practical problem.

(3) Examining special cases
This typical strategy of mathematics and also of natural science is also supported by the CAS.

(4) Testing the effect of parameters in the solution
This strategy often needs complex calculations which are now done by the CAS.

(5) Changing the prototype of the mathematical object
In traditional mathematics learning often only one prototype is available e.g. either the algebraic or the graphic prototype. CAS make parallel work with several prototypes possible.

(6) Developing strategies for finding and removing mistakes
What is important, is the activity of the pupils. Structured trial and error concepts should be acquired by the pupils themselves.

(7) Visualising
A special quality of mathematics is the graphic representation of abstract facts. CAS allow us to get graphs faster and more exactly.

(8) Zooming
The possibility to enlarge or to make graphs smaller is an additional possibilty of visualisation.

(10) Simulating
The computer changes the definition of the idea of solution. Until now a problem was solved when you could find certain numbers or an algebraic prototype of the solution like a term or an equation. Now a recursive prototype of a function in connection with the simulation by the CAS is also accepted as a solution.

3.2 Examples for experimental and active learning in the heuristic phase

In the following examples DERIVE is used in the White Box Phase of the elementary algebra. Pupils had to explore rules and formulas for term-transformations by experimenting with the CAS or they used the CAS for training and applying rules they found before. The examples are results of experiments by W. Klinger and G. Razenberger at the Gymnasium in Stockerau with pupils of the 7th grade.

Example 1: Training and applying of rules they learned before:

#1 :	$(u + v^2) = u^2 + 2uv + v^2$	User
#2 :	$(u - v^2) = u^2 - 2uv + v^2$	User
#3 :	$u^2 - v^2 = (u + v)(u - v)$	User

Exercise 1: Find out values for a, b and c:

#4 :	$4x^2 + a + 25 = (b + c)^2$	User

Pupils used the command **Manage Substitute**

#5 : $4x^2 + 20x + 25 = (2x + 5)^2$ Sub(#4)

The conjecture is tested by using the **Factor**-command

#6 : $(2x + 5)^2 = (2x + 5)^2$ Fctr(#5′)
#7 : $x = @1$ Solve(#6)

Exercise 2: Not all conjectures are correct:

#13 : $4x^2 + 2xy + a = (b + c)^2$ User

Pupils suppose: $a := y$, $b := 2x$, $c := y$.

#14 : $4x^2 + 2xy + y^2 = (2x + y)^2$ Sub(#13)
#15 : $4x^2 + 2xy + y^2 = (2x + y)^2$ Fctr(#14)
#16 : $4x^2 + 2xy + y^2 = 4x^2 + 4xy + y^2$ Expd(#15′)

DERIVE helps to find the correct solution

#17 : $2xy = 2(2x)c$ User
#18 : $c = \frac{y}{2}$ Solve(#17)

Testing is a very important activity

#19 : $4x^2 + 2xy + \frac{y^2}{4} = [2x + \frac{y}{2}]^2$ Sub(#13)
#20 : $4x^2 + 2xy + \frac{y^2}{4} = 4x^2 + 2xy + \frac{y^2}{4}$ Expd(#19)

Example 2: Experimental discovering of equivalence transformations

By shuttling between several windows of the screen (Fig. 1) pupils can compare the result of several equivalence transformations and decide on their individual strategy. Some pupils prefer to situate the variable on the left side, others don't like fractions etc. What is very important that pupils can also find out non suitable transformations which would lead to a wrong result if they were working by hand. An example can be seen in window 4.

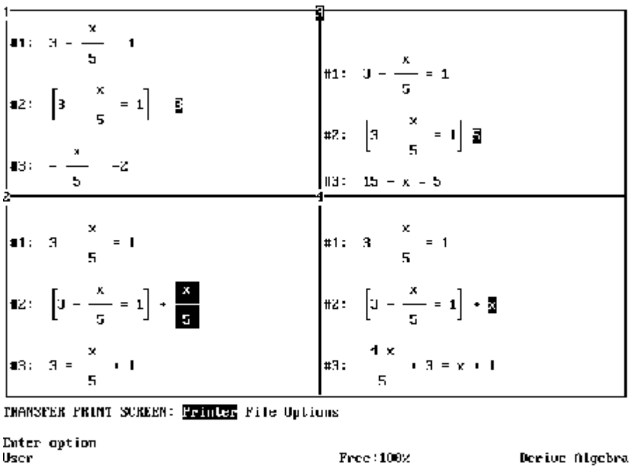

Fig. 1

Example 3:Optimization without differential calculus

Given is a rectangular with $l = 10\,\text{dm}$ and $b = 8\,\text{dm}$. At the angles congruent squares were cut of to get a cuboid with a maximum volume. Demanded is the length of the square.

You could find this exercise in nearly every mathematics school book dealing with calculus. Pupils often solve this problem automatically by looking for the zeros of the first derivation. Some examine the maximum with the second derivation. Because of calculating they forget that the main goal is optimization.

CAS allow us to deal with this examples in the 9th grade without calculus by exploring the graph of the function. Using the *Trace* modus of DERIVE pupils can move along the graph, looking for a senseful definition area, looking for maximums and minimums (Fig. 2). By zooming they could get a better exactness.

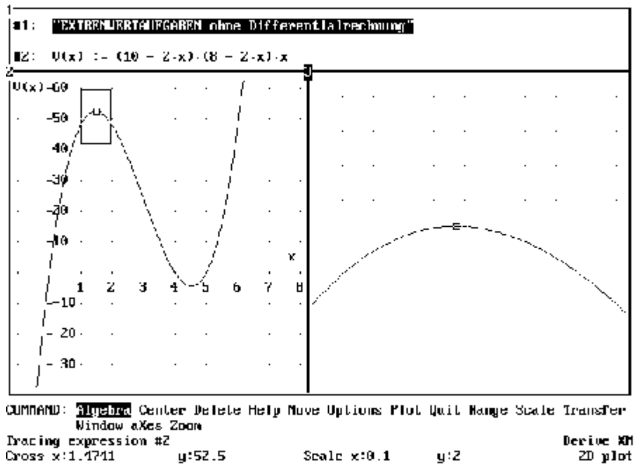

Fig. 2

Pupils experience the problems of application oriented mathematics by discussing questions like: "What exactness is meaningful?" or "The maximum is between 1,4258 and 1,5193. Should we take the average value?" etc.

Experimenting and interpreting are the main activities and not operating.

3.3 The interface operating – interpreting in elementary algebra

One problem of the computer-aided learning process is that the pupils have to examine and to interpret results they did not produce themselves. On the other hand the CAS allows the pupils to learn and to use test-strategies which support the decision about the correctness and the use of the solution.

Example 4: To pay interest of a capital K with the percentage p (Heugl/Klinger/Lechner, 1996)

Determine a formular for the new capital.

Pupils found several formulas e.g.:

#1 : $K \cdot [1 + \frac{p}{100}]$ User
#2 : $K + \frac{K \cdot p}{100}$ User
#3 : $K + K \cdot \frac{p}{100}$ User
#4 : $K \cdot \frac{1+p}{100}$ User

Some of the results were wrong

#5 : $K \cdot (1 + p)$ User
#6 : $K + \frac{p}{100}$ User
#7 : $K \cdot \frac{1+p}{100}$ User

Now it was necessary to find strategies to prove the correctnes and the equivalence of the terms.

Strategy 1: Applying the DERIVE commands **Simplify**, **Factor**, **Expand**. Important is that the *Annotate* mode allows the teacher to reconstruct the pupils' way.
Expand transfers the equation #1 into the equation #8 which shows the equivalence of #1 and #2.

```
#8 :    K·p/100 + K        Expd(#1)
```

With **Factor** pupils could prove the equivalence of #2 and #4.

```
#9 :    K·(p+100)/100      Fctr(#2)
```

Simplify does not change #1 because for DERIVE the factorised forms seems to be more "beautiful". But this command allows the user to show that #3, #2 and #1 are equivalent.

```
#10 :   K · [p/100 + 1]    Simp(#1)
#11 :   K · [p/100 + 1]    Simp(#3)
#12 :   K · [p/100 + 1]    Simp(#2)
```

The same strategy allows the pupils to realise that the formulas of #5, #6 and #7 are wrong.

```
#12 :   K · p + K          Expd(#5)
#13 :   K·p/100 + K/100    Expd(#7)
```

Strategy 2: Using the difference of terms
The result of the difference in line #14 is 0. That means that the terms #1 and #2 are equivalent.

```
#14 :  K · [1 + p/100] − [K + K·p/100] = 0   User = Simp(User)
```

The difference of #4 and #7 is unequal 0. The terms are not equivalent.

```
#15 :  K · (100+p)/100 − K + (1+p)/100 = 99·K/100   User = Simp(User)
```

Strategy 3: Using equations
To examine the equivalence of the terms T_1 (line #1) and T_4 (line #4) we built the equation $T_1 = T_4$.

```
#16     K · [1 + p/100] = K·(p+100/100     User
```

Now there are several possibilities to work the equation:
a) Applying the **soLve**-command: If the terms are equivalent every element of the given domain is solution. DERIVE shows this result with the symbol "@n"

```
#17 :   K = @1     Solve(#16)
#18 :   p = @2     Solve(#16)
```

If the terms are not equivalent, the result will be concrete values of the variables K or p.

```
#19 :   K + K·p/100 = K + p/100    User
#20 :   K = 1                      Solve(#19)
#21 :   p = 2                      Solve(#19)
```

b) Working on the equation or of one part of the equations with the commands **Expand**, **Factor** or *Simplify*:

```
#22 :   K·p/100 + K = K·p/100 + K      Expd#16
```

Factorising of the equation of two non equivalent terms (#23):

```
#23 :   K·(p+100)/100 = (100·K+p)/100     Fctr#19
```

c) Equivalence transformations with the help of CAS like multiplying by 100 etc. Examples will be shown in strategy 5.
d) Visualising: Terms with one variable would allow to draw the graphs of the left and the right side of the equation and to examine the equivalence by watching the graphs.

Strategy 4: Building lines of equations
The pupils found this strategy themselves. For example if we want to compare the terms T_1, T_2, T_4 and T_7 we built the "equations line" $T_1 = T_2 = T_4 = T_7$:

```
#24 :  K · [1 + p/100] = K + K·p/100 = K · (100+p)/100 = K · (1+p)/100
```

Now we can apply the DERIVE commands **Expand**, **Factor** or **Simplify** to the whole line of equations. Simplify shows the equivalence of T_1 und T_2 and also that T_4 and T_7 are not equivalent:

```
#25 :  K · [p/100 + 1] = K · [p/100 + 1] = K·(p+100)/100 = K·(p+1)/100
```

It is better to use **Expand**:

```
#26 :  K·p/100 + K = K·p/100 + K = K·p/100 + K = K·p/100 + K/100
```

DERIVE also allows us to apply equivalence transformations in the line of equations

```
#27 :  [K · [1 + p/100] = K + K·p/100 = K · (100+p)/100 = K · (1+p)/100] · 100
#28 :  K · (p + 100) = K · (p + 100) = K · (p + 100) = K · (p + 1)
```

Strategy 5: Looking for a factor
To compare the terms T_1 and T_4 we are looking for a factor FAKT with the attribute: $T_1 \cdot \text{FAKT} = T_4$. If the terms are equivalent the result is 1.

```
#29 :   [K + K·p/100] · FAKT = K · (100+p)/100     User
#30 :   FAKT = 1                                   Solve(#29)
#31 :   [K + K · p/100] · FAKT = K · (1+p)/100     User
#32 :   FAKT = (p+1)/(p+100)                       Solve(#31)
```

4. Experimenting in the exam situation and practising by using the CAS

The aspect "practising" has an interesting status in the didactical discussion. On the one hand every teacher knows that practising is necessary for the learning process, but on the other hand this aspect seems to be very unattractive for didactical research. In my opinion the automation of certain skills is also necessary in the age of computer assisted learning. In addition to operating skills new skills have to be strengthened by practising e.g. skills like text-translating, skills for testing with the help of CAS or skills concerning the handling of the software, like the use of the DERIVE-function Iterates.

Some changes in the practising phase:

– When the pupils have to do exercises for calculation skills by hand not the teacher is examining the results, the pupils are doing it themselves by using the CAS.

They are also trying to find and to remove mistakes by experimenting with the CAS.

– Instead of many complicated term transformations more practising of structure recognition.
– Instead of solving many equations "by hand" experimental learning by using CAS to find the suitable equivalence transformation.
– Practising as a component of the problem solving process instead of isolated "drill phases".
– More practising of modelling and interpreting instead of operating.
– New aims for practising such as training visualising skills or testing.

It is a fact that the *exam situation* has a great influence on the motivation of the learners and on the learning strategies and also on the subjects of learning. Our study shows that pupils also use experimental strategies in the exam situation. But it is necessary that the teacher must provide enough time in the exam situation for experimental phases and for finding mistakes supported by the CAS. There was no written exam which was only carried out by working with the computer. A combination of working by "pencil and paper" and using the CAS was the usual way of working.

One problem was that in our computer-labs we mostly have 14 PCs. On the other hand normally there are more than 14 pupils in one class. In the exam situation there were used two strategies: The pupils were divided in two groups or only one or two examples had to be solved by using the computer.

Example 5: One example of a written exam in the 8th grade:

Subjects which could be taught at the earliest in the 10th grade before using CAS, because pupils needed the formula of geometric series and calculating rules of logarithms now we can teach in the 7th or the 8th grade.

1) A bank offers 4,1% interest rate for a saving account.
a) From the mentioned interest in Austria, you have to pay a tax of 25%. Give the net interest rate you get for this saving account.
b) Mrs. "Happiness" has won 1 Million shillings by the national lottery. She invests this amount in this bank and withdraws annually 45.000 ATS (up to now). What will be her saving account after 20 years?
c) After how many years will it not be possible to withdraw the total annual amount?

Solve this problem with DERIVE (Options/Notation/Decimal)

Name of the file: 4C1A. Answers in the exam book.

A girl named Martina produced the following results:

1a) She used the correct model and calculated with the numeric calculator:
$4{,}1 \cdot 0{,}75 = 3{,}075$
Not correct was her answer: "The net interest rate is 0,03075%"

1b) and 1c) were solved with DERIVE as a tool for modelling and operating. Beginning in the 7th grade pupils had learned to use recursive models for solving such problems.

Using the Iterates-function they came to following results:

```
#1: Notation := Decimal  User
#2: ITERATES((x - 45000)·1.03075, x, 1000000, 20)
                                              User
    Simp(#2)
#3: [1000000, 984366.2, 968251.7, 951641.7,
    934520.9, 916873.7, 898683.8, 879934.6,
    860608.8, 840688.8, 820156.3, 798992.3,
    777177.6, 754692, 731515.1, 707625.4,
    683001.1, 657619.7, 631457.7, 604491.3,
    576695.7]
```

The answer of 1b) was written in the exam book: "After 20 years the saving account is 576.695,7 ATS".

```
#4: ITERATES((x - 45000)·1.03075, x, 1000000, 40)
                                              User
    Simp(#4)
#5: [1000000, 984366.2, 968251.7, 951641.7,
    934520.9, 916873.7, 898683.8, 879934.6,
    860608.8, 840688.8, 820156.3, 798992.3,
    777177.6, 754692, 731515.1, 707625.4,
    683001.1, 657619.7, 631457.7, 604491.3,
    576695.7, 548045.3, 518514, 488074.5,
    456699.1, 424358.8, 391024.1, 356664.3,
    321248, 284742.6, 247114.7, 208329.8,
    168352.1, 127145.2, 84671.2, 40891.1,
    -4235.21, -50749.1, -98693.4, -148112,
    -199050.2]
```

To come to the answer of 1c) Martina had to count the number of the positive values of the list of line #5. Her answer was: "After 36 years Mrs. 'Happiness' cannot withdraw the total annual account".

5. A short resume and an outlook to further projects

Changes caused by the use of CAS

from	→	to
Doing	→	planning, interpreting
Reproductive learning	→	active, experimental learning
Teacher-oriented learning	→	pupil-centered learning
Using strategies	→	developing strategies
Knowledge about calculations	→	knowledge about strategies
Complex calculation skills	→	less complex calculation skills "by hand"
Exercises	→	problems (modelling, operating, interpreting)
Calculation-oriented math education	→	applying-oriented math education.

Now we are starting a new project concerning the influence of the algebraic calculator TI-92 in teaching and learning mathematics. One result of our first project was: So long as we need the computers in the computer labs we cannot expect a greater acceptance of the use of CAS. We will need algebraic calculators in every schoolbag. And now earlier as we expected we have such a pocket calculator. Its possibillities will again influence the didactical concepts, the contents and the exam situation. Therefore we think that such projects are necessary.

More than 50 classes with more than 1000 pupils will take part in this project. We expect first results in 1998.

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Vorschau auf Analysethemen der nächsten Hefte

Für die Analysen der Jahrgänge 30 (1998) und 31 (1999) sind folgende Themen geplant:
- Analysis an Hochschulen
- Mathematik in der Ingenieurausbildung
- Fächerübergreifender Unterricht
- Mathematik und Frieden.

Vorschläge für Beiträge zu o.g. Themen erbitten wir an die Schriftleitung.

Outlook on Future Topics

The following subjects are intended for the analysis sections of Vol. 30 (1998) and Vol. 31 (1999):
- Calculus at universities
- Mathematics and engineering education
- Cross curricular activities
- Mathematics and peace.

Suggestions for contributions to these subjects should be addressed to the editor.