The use of computer algebra in middle secondary mathematics. Part 1[*]

Pupils Learning Algebra with DERIVE
A Didactic Perspective

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Abstract: This article deals with the use of computer algebra systems in mathematics secondary education. First, we present the global framework of a research project carried out in France with pupils of grades 9 to 12, from 1993 through 1995. Then we focus on a specific part of this project concerning two grade 9 classes of different mathematical ability taught by the same teacher. We briefly present the aims and organisation of DERIVE's use in the two classes, before embarking with more details on a specific topic: systems of linear equations. Finally, we present the data provided by the pupils' answers to a questionnaire taken at the end of the academic year; we use them for investigating pupils' assumptions about DERIVE's potential for mathematics learning, and compare these representations with those emerging from the entire population.

Kurzreferat: *Schüler lernen Algebra mit DERIVE. Eine didaktische Perspektive.* Dieser Beitrag behandelt die Nutzung von Computeralgebrasystemen im Mathematikunterricht der Sekundarstufe. Zunächst wird ein globaler Rahmen für ein Forschungsprojekt in Frankreich dargestellt, das 1993 bis 1995 mit Schülern der Klassen 9 bis 12 durchgeführt wurde. Dann wird auf einen speziellen Teil des Projekts eingegangen, bei dem zwei neunte Klassen mit Schülern unterschiedlicher mathematischer Fähigkeiten von demselben Lehrer unterrichtet wurden. Ziele und Organisation der Nutzung von DERIVE in beiden Klassen werden kurz dargestellt, bevor näher auf Details im Rahmen der Behandlung der linearen Gleichungssysteme eingegangen wird. Schließlich werden Ergebnisse einer Schülerbefragung am Ende des Schuljahres präsentiert. Sie werden dazu benutzt, Annahmen von Schülern über den Nutzen von DERIVE für das Mathematiklernen zu untersuchen und sie mit entsprechenden Daten aus der Gesamtbevölkerung zu vergleichen.

ZDM-Classification: H20, R20

1. Introduction

In France, the use of computer algebra systems (CAS) in secondary education began with mu-math, many years ago. In 1990, the French Ministry of Education created a national working group of about 15 teachers whose task was to "explore the potential of CAS - and especially DERIVE - for mathematics teaching and learning, to experiment with and develop engineering products for teachers, and to investigate the impact on mathematics teaching (pedagogical practices, curriculum objectives and contents, assessment)" (Hirlimann, 1996). From 1993 through 1996, we were asked by the Ministry of Education to work in close collaboration with this group and analyse the present state of CAS integration at secondary level.

As didacticians, sensitive to the fact that, in spite of many incentive actions, integration of computer technologies is progressing very slowly in our educational system, we were particularly interested in the following issues:

– What is the potential of CAS for mathematics teaching and learning, and what conditions are required to use CAS's potential to the full?
– To what extent do teachers' perceptions of CAS's use or expectations accord with what is really happening in classroom sessions with CAS? If discrepancies between perceptions, expectations and reality exist, as integration problems incline us to hypothesise, is it possible to identify some didactic characteristics of CAS based teaching and learning practices which might explain them?

In order to answer these questions, we developed a research project whose methodology used both:

– questionnaires (one for teachers and one for pupils) whose aim was to determine the extent to which DERIVE was in use in secondary education, the characteristics of its present use, and its effects on pupils' mathematics representations and practices,
– observations of classroom sessions with DERIVE in the mathematics classes of teacher members of the national group mentioned above.

As a rule we did not interfere with the planning of these sessions, but we did ask the teachers to provide us with an a priori analysis of them, according to a questionnaire which had been negotiated with the group (Artigue et al., 1995).

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As explained in Artigue 1996, these two methodologies were seen as complementary ones:

"The first provides us with an overview of the extent to which DERIVE is currently being used in secondary education. But we are well aware that questionnaires will only provide a posteriori rationalisations from users of this software, and that a great deal of bias may be associated with this type of methodology. The second methodology provides a much more accurate approach, but it has the disadvantage of remaining extremely local. By combining the two, we hope to be able to compensate, at least partially, for the limitations inherent in each of them."

Twenty teachers and nearly five hundred of their pupils filled in the questionnaires, and twelve sessions were observed, from grade 9 to grade 12.

Educational research dealing with the use of CAS mainly concerns high school level, and pre-calculus or calculus* topics. In this article, in contrast, we focus on the lowest grade involved in the research project (grade 9) and on algebra. In France, grade 9 pupils cannot be considered as absolute beginners in algebra, insofar as algebra teaching starts at grade 8. However, many of them have not yet achieved the difficult transition from arithmetic to algebra (Chevallard, 1985, 1989, 1990); they cannot yet think of expressions at a structural level (Kieran, 1992, Sfard, 1995). Generally, teachers are reluctant to use CAS with such pupils who are poorly skilled in algebra. In their opinion, premature contact with such powerful algebraic tools might hinder the necessary learning of algebraic skills. A member of the national group was teaching mathematics in two grade 9 classes and regularly used DERIVE. Like the other teachers of the group, he specified his aims for us and accepted a number of classroom observations. He and his pupils filled in the questionnaires. So it seems especially interesting to investigate what his expectations were with respect to DERIVE's use, what strategies he developed, how pupils reacted, and what results were finally obtained.

In the following, we first present the experimental context, the teacher's expectations and the way he organised DERIVE's use through the academic year. Then we focus on a specific topic related to algebra: linear systems. We give at sketch of the didactic strategy developed by the teacher before presenting and analysing one specific observation which we consider as a prototypical case with respect to the issues at stake. We then proceed to discuss the questionnaires, analyse pupils' answers and compare them to those of the whole population in order to look for possible links between pupils' representations and the specificity of DERIVE's use at this level of schooling.

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* A notable exception is Hunter et al., 1995. But this study is about grade 10 pupils and DERIVE is being used mainly as a tool. It concludes: "A CAS can be benefit for the pupils' learning of abstract elements of algebra as long as the pupils are mathematically ready to use it." Therefore the questions of the benefit of using CAS for pupils "not mathematically ready" remains an open question.

2. Characteristics of DERIVE's use in the two grade 9 classes

2.1 Experimental context, teacher's expectations and organisation

The French grade 9 corresponds to the upper grades of lower secondary. Experimental work with grade 9 pupils took place in a "college" situated near Paris, during the academic year 1993–94. It involved two parallel classes instructed by the same teacher but having rather different mathematical levels. DERIVE and Cabri-Géomètre were the two softwares mainly used during that academic year. They were used both in ordinary classroom sessions with the aid of a datashow and in specific laboratory sessions, where the 30 pupils were working in pairs with PCs.

DERIVE was used in the teaching of numbers, algebra and geometry. As far as numbers are concerned, DERIVE was supposed to favour an evolution of pupils' conceptions, through the possibility of carrying out exact calculations and also manipulate decimal numbers with an arbitrary number of digits. This potential was exploited to question erroneous conceptions reinforced by the widespread use of pocket calculators, and to investigate the distinction between numbers and their different semiotic representations.

In algebra, the use of DERIVE obeyed different aims:

- It was supposed to promote pupils' mathematical work on the syntactic dimension of algebra and allow to engage in this mathematical work, which is often considered by pupils as pure requirement of the didactic contract, in non-didactical situations (Brousseau, 1986 & 1988). Syntactic rules, for instance, can thus appear as requirements of the mathematical activity in some appropriate non-didactic "medium", and distinctions between mathematical rules and properties on the one hand, conventions on the other hand, are made clearer.

- It was supposed to favour pupils' awareness of the role played by formal generalisation in algebra. This was done mainly by means of pattern recognition problems. In such problems, DERIVE was used as a producer of data and phenomena that pupils had to classify, explain and reproduce.

- DERIVE was also supposed to favour investigations of the status of algebraic objects and contribute to the "mathematisation" of algebraic manipulations, by emphasizing their explicit and operational character through instructions or sequences of instructions given to the computer. This aspect was specially addressed when dealing with equations and systems of equations.

In geometry, the primary use of DERIVE was in analytical geometry, which in France is introduced at this level. A specific file was created by the teacher. This file included nine commands allowing pupils to obtain the co-ordinates of a point, of the middle of a segment, of a vector defined by two points, of the image of a point by a central symmetry, and the distance between two points, and also to draw segments, triangles and quadrilaterals. This file was then used for focusing problem solving in analytical geometry on the search for different strategies, methods of proof, and for comparing their respective efficiencies.

The possibilities offered by DERIVE for linking algebraic and graphical work were also taken into account.

The reader more interested in the different facets of the use of DERIVE in this experimental work will find detailed information in Artigue et al., 1995, Rousselet, 1996.

2.2 Systems of equations: the didactic strategy

After this synthetic presentation, in the following, we are going to focus on one specific mathematical domain, i.e. that of systems of linear equations.

Systems of linear equations of size 2×2 are introduced at grade 9, through the modelling of "daily life situations". These are only numerical systems, without parameters, admitting one solution only. Pupils are taught to solve them both algebraically through substitutions or linear combinations and graphically (affine functions and line equations are also part of the syllabus). They have to only work on particular examples, and any general theory is explicitly disregarded.

In Hirlimann (in press), M. Rousselet, who was the teacher of the two classes engaged in the experiment at grade 9, proposes five DERIVE sessions on systems of equations. The two first ones deal with substitution methods, the third with the linear combination method, the fourth concerns graphical solutions, and the last one aims at introducing reflection upon the numerical accuracy of solving methods by comparing the solutions of systems:

$$
\begin{aligned}
19.05x + 9.99y &= 29.98 \\
40.04x + 20.99y &= 38.01 \\
19x + 10y &= 30 \\
40x + 21y &= 38 \\
19.05x + 10y &= 30 \\
40.04x + 21y &= 38
\end{aligned}
$$

graphically associated to lines nearly parallels. The two last systems are presented as two simplifications of the first one, introduced by pupils who wanted to make their mathematical work easier.

Note that these are only the laboratory sessions associated with the solving of systems of equations. They do not represent the whole teaching process which, for example, also includes modelling and treatment of various everyday life, physical or geometrical situations leading to linear systems, explanation and institutionalisation of the different solving methods introduced, and discussions on how one can be sure that a linear system has been solved correctly. One should also note that this proposal goes beyond the ambitions of the syllabus: non regular numerical systems are introduced, the various possible types of systems are distinguished and related to the respective positions of the two corresponding lines.

We would also stress that, even if DERIVE allows a direct solution of such systems, the corresponding instruction is not introduced in any phase of the teaching process. According to the general principles and hypothesis set up above, DERIVE is used in this area in order to:
– help the object "linear equation" gain some mathematical status as an object which can be involved in arithmetical operations, as numerical and algebraic expressions are,
– help pupils give (and maintain) mathematical meaning to the "gestures" of the solving process,
– help pupils link algebraic and graphic solving processes and become aware of the fact that each frame can act as a control frame for the other.

As the ambition of the syllabus remains very limited and, as a consequence, also the time devoted to linear systems, it seems unreasonable at this early stage to introduce an instruction which would negate the mathematical work and reflection the teacher wants to promote here.

2.3 A specific session

We specially observed one session in this area, similar to the third one described above (which was in fact adapted from the observed situation), and we should like to use this observation as a prototype for giving the reader a better understanding of this experimental dimension of the research.

2.3.1 Presentation of the activities proposed to pupils

First, the instruction := is introduced as a means for giving a name to an equation, in the same way it has been previously used with points. The two first tasks proposed aim at exploring how this instruction allows to operate on a particular equation (equa $:= 3x - 4y = 1$) by adding, subtracting a number, multiplying by a number, and to get some familiarity with it. This first phase achieves by a prediction question: what will be obtained by simplifying the expression 2^*equa $- 3$?

The second exercise introduces a new equation ($2x - 5y = 8$) and, starting from this equation, pupils are asked to obtain four given proportional equations by similar operations. The answer is given, as a hint, for the first one.

With the third exercise, we enter the domain of linear systems: a regular system is introduced, and pupils are asked to explore linear combinations of these two equations using DERIVE. The two last ones proposed make x, respectively y " disappear".

Exercise 4 is similar with a new system and only linear combinations of the last type. Pupils are asked to find out what happens.

Exercise 5 closes this exploration phase. For a new system, pupils are asked to find adequate multipliers and solve the system. In exercise 6, they have to do the same with five different regular systems. In exercise 7, they have to go on with two non regular systems and try to interpret the DERIVE messages.

2.3.2 Analysis

Three groups of pupils were observed, and the written products of the fifteen groups were gathered. Observations and texts tend to show that, in that specific environment, the great majority of pupils do not meet particular difficulties at considering an equation as an object which can be involved in arithmetical operations. The main hypothesis at play in the session is thus reinforced. They also show that, in this situation as in many others, DERIVE favours strategies based on trial and formal analogies, and confirm the fact that these are generally productive.

This is especially visible with exercise 2, where pupils had to find adequate multipliers. For the majority of pupils, the first phase did not allow to give sufficient meaning to operations on equations in order to make them able to answer this question very easily. Pupils tended to try multipliers (obviously an economical strategy), hoping that these trials were likely to produce the answer or at least some interesting formal phenomena. One among the observed groups developed another strategy. These pupils had the idea of dividing one equation by the initial one. They were nearly convinced that it could not work but, as they said: "it's worth a try" – so they tried! They strangely obtained: $3 = 3$. Surprised, they laughed, but this equality looked challenging. They did not try to understand it but decided to use it with the first equation and obtained: $6 = 6$. 6 was the hint given for this first equation. So they were now convinced they had found the trick they were looking for. When typing the third equation, they made a sign mistake and thus obtained: $5 = -5$. They were not seriously disturbed. As they expressed, they had got at least one interesting information: there was an evident invariant: 5, so the number they were looking for was 5 or -5. They chose -5 by comparing the coefficients of x in the two equations. Note that, in the written texts, we found various answers 5 to this question.

As this example shows, pupils are sensitive to formal analogies and are likely to exploit them, but these are not necessarily articulated with more analytical levels of functioning. All our observations tend to prove that such an articulation is not spontaneous at all. The teacher has to take charge and motivate pupils by specific mathematical activities. Moreover, the transition from one level of functioning to another one requires specific devolution processes insofar as the two levels do not obey the same didactic contract (Brousseau, 1988).

Moreover, in DERIVE sessions, pupils' attention may be distracted from the mathematical task itself by what we have called "pseudo-transparency phenomena" (Artigue, 1995). This expression labels slight differences in the treatment or the semiotic representations of mathematical objects due to the "computer transposition" of mathematical knowledge and semiotic representations (Balacheff, 1994). In this session as in many others, we have observed such phenomena. Here, they are linked to the fact that linear combinations are not always managed in the same way by the computer when using the commands: "simplify" or "develop". For instance, in exercise 4, many groups did not notice that the linear combinations allowed to obtain equations with only one unknown. They were more sensitive to the fact that, in this particular case, the commands "simplify" and "develop" were leading to the same result!

Finally, in this session as in many others, we can see that it is not easy to foster the kind of mathematical activity the teacher wants to obtain. It is not easy to guarantee an adequate devolution of the tasks. Some characteristics of the DERIVE environment allow pupils to get some detachment from mere execution tasks and develop reflection about what they are doing and why, while some other characteristics allow to economise reflection.

3. Attitudes of grade 9 pupils towards technology and DERIVE

As explained above, the external part of the research project consisted of an evaluation of the integration of CAS in French secondary education with the aid of questionnaires.

We questioned 25 teachers and 459 pupils who had been working with DERIVE. From the teachers' answers, we got a classification of the expected improvements that CAS might bring. By means of a statistical analysis of the pupils' answers, we obtained distinctive attitudes towards the usefulness of using CAS to learn mathematics (Lagrange, 1996).

3.1 Objectives

In this part of the paper, we want to examine what characteristics pupils and teacher had in the two grade 9 classes taken into account here and the possible influence of those characteristics on their assumptions about the help provided by DERIVE for the learning of mathematics.

As a first distinction, we have to notice that, in the French curriculum, grade 8 and 9 pupils learn elementary algebraic techniques mainly in the context of equations while, in further grades, algebra tends to be involved in functional contexts and used as a tool for elementary calculus. A CAS like DERIVE can help both in algebra and calculus. In elementary calculus, pupils can consider DERIVE's capabilities like "push-button" manipulations of a function: a "button" for computing the limit, another for derivations, and so on. In contrast, expanding, factorisation, simplifying is not as simple.

For instance, to factor a polynomial, the user of a CAS has to select a "level" of factorisation which is actually a class of possible outputs and the associated algorithm. In addition, factorisation is systematically complete, while, in the usual practice, factorisation partially is possible and often useful. Therefore, doing algebra with DERIVE is not just reading an output on a computer screen. Pupils must develop specific techniques for using DERIVE's algebraic capabilities. We can thus expect that undertaking the learning of algebra in these conditions can produce attitudes which differ from those of the rest of the population involved in the questionnaires.

Another characteristic is the intensity of the use of DERIVE in these classes. The 25 teachers in the survey were representatives of the population who attempts to integrate DERIVE into French classrooms. Among these 25 teachers, only 10 used DERIVE with their pupils on a regular basis, arranging more than 15 work sessions during the academic year.

We estimate that improving the pupils' mathematical understanding through CAS use is realistic only with sufficient integration of the software. When DERIVE is used only occasionally, pupils cannot get really familiar with its capacities and will not have sufficient insight into the techniques necessary for using those capabilities efficiently. The grade 9 teacher cited in this paper was one of the 10 teachers who performed an actual integration of DERIVE. Therefore, by examining the data obtained in his classes, we can compare the attitudes of the pupils who experi-

enced this integration with those of the entire population of pupils who used DERIVE more or less frequently.

A third characteristic of these lower secondary grades consists of the teacher's aims as stated above: stress on the syntactic dimension of algebra, recognition of general patterns, mathematisation of the algebraic manipulations. Hence, he introduced specific sessions directed towards the understanding of algebraic notions through CAS use. Among the 25 teachers in the survey, roughly one third aimed at getting a better understanding through CAS use in the same way. Another third offered DERIVE just as a tool for calculations in non-specific activities, clearly inducing their pupils to view DERIVE as just a tool for double check. And the last third focused on problem solving, possibly inducing the same view.

In addition, we want to stress that the two 9 grade classes did not have the same level of skill in mathematics. The pupils in class A were reported to be more skilled than those in class B. So we can see how the same teaching strategy can produce different effects in two dissimilar classes.

3.2 Pupils' perception of mathematics and technology in grade 9 classes

We first compare the grade 9 pupils with the entire population, by using data on pupils' use of technology, expertise of DERIVE, and perception of how DERIVE may help in the learning of mathematics. The data were obtained from the answers to the pupils' questionnaire and are summarised in Table 1.

	Entire population	grade 9A	grade 9C
I often use my calculator at school	76%	73%	83%
I often use my calculator at home	78%	63%	33%
I had mathematics sessions with computer sessions in the former year	1 or 2 sessions	10 sessions or more	Between 3 and 10 sessions
Now, I use DERIVE easily	65%	81%	58%
I found it easy to learn how to use DERIVE for solving simple tasks	82%	69%	63%
I found it easy to learn how to use DERIVE adequately	50%	62%	33%
DERIVE helped me to learn the hardest topic I met this year	26%	48%	46%

Table 1

Pupils in the grade 9 classes use their calculators to the same extent at school and less frequently at home

This fact clearly results from the first and second lines of Table 1. It could be expected as, at lower secondary level, pupils are not given much homework to do. The difference between the two classes is however remarkable. In class B, pupils apparently invest less in homework, or are less likely to use the calculator, or both.

Pupils in the grade 9 classes initially had much more knowledge of how to use the computer to learn mathematics (line 3, Table 1)

Like most pupils, pupils in the entire population had a very limited use of computers in the former years. In contrast, the two classes are in a school where teachers are likely to use computers.

The grade 9 classes have different levels of familiarity with DERIVE (line 4, Table 1)

About two out of three pupils feel at ease with DERIVE both in the entire population and in the grade 9 classes. At first sight, one could expect that the grade 9 pupils would be more familiar with DERIVE as they used it significantly more often than other classes. In addition, there is a strong difference between the two classes: the highest achievers in mathematics obviously feel easier with DERIVE.

The grade 9 classes perceive the learning of DERIVE differently (lines 5 and 6, Table 1)

In both classes, pupils think it is harder to learn to use DERIVE for simple tasks than in the entire population, but students in class A do not differentiate clearly between what is required for solving simple tasks on the one hand and an adequate use on the other hand. Moreover, there is a strong difference between the answers given in the two classes with respect to "adequate use". Obviously, pupils in class B do not have the feeling they can use DERIVE adequately.

In both classes, better understanding is more likely to be noticed (line 7, Table 1)

Roughly, only a quarter of the pupils in the survey agree that they learnt something hard with the help of the software. Unlike teachers, few pupils appreciate the support that DERIVE can bring for learning and understanding mathematics. It is an interesting outcome that they are significantly more numerous in the two grade 9 classes, where the emphasis was put on this dimension of DERIVE's use especially in algebra.

Similarities and differences between the two classes

Pupils in the two classes had already encountered technology in the teaching of mathematics, using computer packages like spreadsheets or interactive geometry, and using their hand-held calculators. However, they are less likely than the entire population to think that it is easy to learn to use DERIVE for simple tasks. This fact tends to confirm the hypothesis we expressed above, i.e. that the use of DERIVE, even for simple tasks, can be difficult for pupils with poor knowledge in algebra. Obviously, these difficulties do not prevent them from seeing DERIVE as a help for understanding mathematics, more than the entire population. There are evident differences between the two classes with respect to the familiarity gained with DERIVE; they seem coherent with our hypothesis, but it is worth noticing that they do not influence students' opinion about the help provided by DERIVE.

3.3 Pupils' attitudes towards DERIVE in the entire population and in the grade 9 classes

The pupils' questionnaire included a subset of 16 questions concerned with attitudes towards DERIVE. In Table 2 below, pupils' possible answers are presented. The questions were accompanied by a four-way Likert scale answer grid: Strongly disagree / disagree / agree / strongly agree. Pupils' responses were weighted 0, $\frac{1}{3}$, $\frac{2}{3}$, and 1 respectively. Columns 3 and 4 show the respective average ratings for the pupils in the entire population, and in the grade 9 classes.

Question number		Average ratings		
		Totality	A	B
"Using DERIVE is a worthless activity"				
6	DERIVE is complicated and provides little help in learning mathematics	0.30	0.33	0.45
2	When working with DERIVE I do not need to learn how to compute because the computer does everything	0.41	0,62	0,51
3	DERIVE does not help me as, in exams, I must write out the calculations and reasoning	0.48	0,57	0,60
"DERIVE is an unfailing mine of information"				
7	If DERIVE gives no answer, then the problem has no solution	0.31	0,32	0,35
1	DERIVE helps people who have difficulties with algebra to still be able to do Mathematics	0.59	0,67	0,41
16	When the input data is correct, then I can fully trust DERIVE's output	0.68	0,75	0,47
"DERIVE is a positive change in the learning of mathematics"				
10	DERIVE is a useful support in discovering algebraic rules	0.44	0,49	0,49
4	DERIVE increases my desire to do mathematics	0.46	0,48	0,51
5	DERIVE helps me to understand mathematics	0.51	0,69	0,54
13	Using DERIVE I can solve problems instead of doing repetitive exercises	0.61	0,46	0,53
14	With DERIVE we think of mathematics in a different way	0.64	0,63	0,67
15	DERIVE is useful because we can plot a graph while working on equations	0.75	0,71	0,73
"Derive is a tool to control or facilitate calculations"				
8	DERIVE helps me to get an idea of the result of a calculation before doing it	0.72	0,63	0,51
12	DERIVE helps me solve problems without getting lost in calculations	0.72	0,74	0,50
11	DERIVE is especially helpful with lengthy and boring calculations	0.83	0,72	0,65
9	DERIVE is a useful tool to check the result of a calculation	0.85	0,54	0,65

Table 2

We proceeded to a deeper statistical analysis of the results. For this purpose, we used implicative analysis (Gras & Larher, 1993), a multidimensional statistical method, to establish links in the set of opinions and obtained four collections of opinions. These four collections appear in Table 2 and can be seen as four distinct attitudes towards DERIVE with respect to mathematics learning.

In this paragraph, we present the main characteristics of each attitude and the way grade 9 pupils agree with them. We also look for possible differences in the average ratings between the grade 9 classes and the entire population as well as between the two classes.

Grade 9 pupils are more likely to take the attitude "Using DERIVE is a worthless activity"

The strongest opinion in this attitude is number 3: DERIVE is seen as a waste of time as the only important thing is assessment. Some pupils in agreement with this attitude experienced difficulties when using the software, and thus (opinion 6) reject DERIVE as an obstacle to mathematics learning. Other pupils consider that DERIVE is an effective means for doing mathematics (opinion 2) and reject DERIVE because, in their view, if they use this software, they will have nothing more to learn.

The three opinions in this attitude have a stronger average rating in the grade 9 classes, even if they do not meet strong agreement. This outcome is deceiving, because, during the classroom observation, we saw no pupils rejecting DERIVE. But we can admit that, for these 9 grades pupils, using DERIVE had remained something unusual when they completed the questionnaire at the end of the year. This may be different with older pupils who use sophisticated calculators more often.

In addition, the reasons for a rejection of DERIVE are different in the two classes, as shown by the respective rating of opinions 2 and 6.

The rating of opinion 2 is 0.11 higher in class A. We can think that in this class, where the ability for mathematics is higher, some pupils reach an average level in mathematics mainly by performing mechanistic rules. Tall and Thomas (1991) call this practice of algebra "manipulative activity". Their concern may be that the emergence of CAS would have a negative effect on their skills in algebra.

The rating of opinion 6 is 0.12 higher in class B. As stated before, grade 9 pupils have the same familiarity with DERIVE as compared with the average pupil. Nevertheless, they are more likely to think that DERIVE is complicated. We can hypothesise that this attitude is linked with the difficulties they have encountered. Pupils in this class are not skilled enough in algebra to really understand what they are doing with DERIVE. For instance, in the observation reported above, we see pupils wondering why developing and simplifying give the same result on a composite equation.

Pupils of class B are less confident in "DERIVE as an unfailing mine of information"

Pupils who agree with opinions 7, 1 and 16 may feel that they need not do hand calculations or use their mathematical knowledge to confirm the DERIVE's output and that DERIVE makes a calculation or an investigation by hand unnecessary.

The rating of the three opinions are clearly lower in class B as compared with both the entire population and with class A. We are inclined to assume that, in their relatively long experience, these pupils encountered difficulties with

DERIVE as well as with algebra. As a consequence, this experience made them less confident in DERIVE's output and in the help DERIVE could provide for overcoming their conceptual difficulties in algebra.

Grade 9 pupils take the attitude "DERIVE is a positive change in the learning of mathematics" roughly to the same extent than the entire population, but they put more emphasis on understanding

Opinions 10, 4, 5, 13, 14 and 15 are positive towards using DERIVE. They stress a change in mathematical practice, or a better understanding of mathematics, when using DE-RIVE. They are consistent with the hypotheses of teachers reported above that the use of CAS can help pupils solve new problems and lead them to a better understanding of mathematics. Opinions 5 and 13 express two different reasons which could explain why pupils like using DERIVE.

In this attitude, the only opinions differing more than .05 are opinions 5 and 13. In class A, the rating of opinion 5 increases from 0.5 in the entire population to nearly 0.7, and opinion 13 decreases nearly to the same extent. In class B, the ratings move in the same directions, but to a lower extent.

Therefore, grade 9 pupils tend to view DERIVE's help for understanding as a reason for a "good opinion" on the use of this software. This confirms the outcome stated above. Furthermore, this accent on understanding places less emphasis on the use of DERIVE for problem solving.

We cannot link this change with a more extensive integration of the software in these two classes because, in the entire population, integration tends to induce an emphasis on problem solving (Lagrange, 1996). It seems more reasonable to link such an emphasis on understanding with teacher's priorities as expressed above: understanding algebra rather than using algebra to solve unusual or complex problems.

In contrast, many teachers in the survey developed the use of DERIVE for modelling real-life processes in a problem solving process, e.g. Aldon (1996). Hence preferences of a teacher regarding the help that DERIVE provides clearly influences pupils' view of the software. However, these preferences may depend on the topic: the purpose of Aldon's real life problems is to introduce elementary calculus notions, like the derivative. In addition, this emphasis is better accepted by pupils of class A, who have better mathematical skill.

Grade 9 pupils are less likely to take the attitude "DE-RIVE is a tool to control or facilitate calculations"

Opinions 8, 12, 11 and 9 are also positive towards using DERIVE. In contrast with the previous attitude, they stress the support DERIVE gives in doing calculations. Pupils in this opinion focus on calculation and training rather than on problem solving or understanding.

In the entire population, this attitude was obviously dominant. The ratings of the opinions in this attitude range from 0.72 to 0.85, which is very high. Most pupils express that they see DERIVE as a means for "double check". However, by observing pupils' work, we could see that this double check was rarely performed except when the output of DERIVE and the ordinary pencil and paper were

closely similar. In many occasions, pupils encountered serious difficulties when trying this double check, because the output of DERIVE did not match their pencil and paper result: they were not able to decide whether or not DERIVE's output and their result denoted the same expression. In the grade 9 classes the ratings of these opinions are weaker. Especially opinion 9 shifts down by 0.31 points in class A, and 0.2 in class B. Once more, we can see here the influence of teacher's opinions. In contrast, teachers focusing on DERIVE as a tool for calculations in non-specific activities or for specific problem solving possibly induced a view of the software mainly as a means for double checking.

4. Conclusion

Results obtained with these 9 grade students are compatible with the global results of the research project. They tend to confirm that using CAS in a well-thought-out context can support the learning of mathematics and help to liven up school mathematics with interesting activities, even for beginners in algebra. They also tend to prove that students' assumptions about DERIVE are strongly dependent on teachers' didactic choices as well as on the students' mathematical abilities. Nevertheless, as stressed Artigue (forthcoming) or Lagrange (1996), it is not so easy to utilize the theoretical potential of CAS for mathematics teaching and learning in current classroom sessions. Didactic situations have to be carefully designed and managed as the use of CAS can as well reduce students' mathematical activity, by taking over their usual technical work without moving them to higher levels of activity. We would like to point out that, from this point of view, common literature devoted to CAS may be misleading as it often neglects to identify the conditions required for making full use of the theoretical potential of CAS.

5. References

Aldon, G. (1996): DERIVE for 16–18 year old students. – In: The International DERIVE Journal, 3 (N°3), p. 13–20

Artigue, M.; Abboud, M.; Drouhard, J. P.; Lagrange, J. B. (1995): Une recherche sur le logiciel DERIVE.- Cahier de DIDIREM n°3 (special issue). Paris: IREM Paris 7

Artigue, M. (1995): Une approche didactique de l'intégration des EIAO à l'enseignement. – In: D. Guin; J. F. Nicaud; D. Py (Eds.), Environnements Interactifs d'Apprentissage avec Ordinateur. Paris: Eyrolles, p. 17–28

Artigue, M. (1996): Using computer algebraic systems to teach mathematics. – In: ADIREM (Ed.), The relationship between Knowledge, Curriculum and Practice. Pont à Mousson: Topiques Edition, p. 223–239

Artigue, M. (to appear): Le logiciel DERIVE comme révélateur de phénomènes didactiques liés à l'apprentissage des mathématiques. – In: Educational Studies in Mathematics

Balacheff, N. (1994): Didactique et intelligence artificielle. – In: Recherches en Didactique des Mathématiques 14 (N°1.2), p. 9–42

Brousseau, G. (1986): Fondements et méthodes de la didactique des mathématiques. – In: Recherches en Didactique des Mathématiques 7 (N°2), p. 33–115

Brousseau, G. (1988): Le contrat didactique: le milieu. – In: Recherches en Didactique des Mathématiques 9 (N°3), p. 309–336

Chevallard, Y. (1985): Passage de l'arithmétique à l'algèbre dans l'enseignement des mathématiques au collège: l'évolution de la transposition didactique. – In: Petit x 5, p. 51–94

Chevallard, Y. (1989): Passage de l'arithmétique à l'algèbre dans

l'enseignement des mathématiques au collège: perspectives curriculaires, la notion de modélisation. – In: Petit x 19, p. 43–75

Chevallard, Y. (1990): Passage de l'arithmétique à l'algèbre dans l'enseignement des mathématiques au collège: voies d'attaque et problèmes didactiques. – In: Petit x 23, p. 5–38

Gras, R.; Larher, A. (1993): L'implication statistique, une nouvelle méthode d'analyse de données. – In: Mathématiques, Informatique et Sciences Humaines 120

Hirlimann, A. (Ed.) (1994): Enseignement des mathématiques et logiciels de calcul formel. – Paris: Ministère de l'Education Nationale, DITEN B2

Hirlimann, A. (1996): Computer algebra systems in French secondary schools. – In: International DERIVE Journal 3 (N°3), p. 1–4

Hirlimann, A. (Ed.) (to appear): Enseigner des mathématiques avec des systèmes de mathématiques symboliques. – Paris: Ministère de l'Education Nationale, DISTN

Kieran, C. (1992): The learning and teaching of school algebra. – In: D. A. Grouws (Ed.), Handbook of research on mathematics teaching and learning New York: Macmillan, p. 390–419

Lagrange, J. B. (1996): Analysing actual use of a computer algebra system in the teaching and learning of mathematics: an attitudinal survey of the use of DERIVE in French classrooms. – In: The International DERIVE Journal 3 (N°3), p. 91–107

Rousselet, M. (1996): Can DERIVE ease the learning of elementary algebra? . – In: The International DERIVE Journal 3 (N°3), p. 5–12

Sfard, A. (1995): The development of algebra – Confronting historical and psychological perspectives. – In: C. Kieran (Ed.), New perspectives on school algebra: Papers and discussions of the ICME-7 algebra working group (special issue). Journal of Mathematical Behaviour 14, p. 15–39

Tall, D.; Thomas, M. (1991): Algebraic thinking with a computer. – In: Educational Studies in Mathematics 22, p.125–147

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