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NEW SOLITONS AND PERIODIC SOLUTIONS FOR THE KADOMTSEV-PETVIASHVILI EQUATION

A. BORHANIFAR[1], H. JAFARI [2]* AND S. A. KARIMI[1]

Abstract. In this paper, the sine-cosine, the standard tanh and the extended tanh methods has been used to obtain solutions of the Kadomstev-Petviashvili(KP) equation. New solitons solutions and periodic solutions are formally derived. The change of parameters, that will drastically change characteristics of the equation, is examined.

1. Introduction and preliminaries

The investigation of exact traveling wave solutions to nonlinear evolution equations plays an important role in the study of nonlinear physical phenomena. The dynamics of nonlinear waves in higher-dimensional space have richer phenomena than one-dimensional case, since various localized structures may be considered as candidates for solitons. The two-dimensional generalization of the Korteweg-de Vries(KdV) equation was given by Kadomtsev and Petviashvili to discuss the stability of one-dimensional KdV soliton (line soliton in two dimensions) against transverse long-wave disturbances, which is known as the Kadomtsev-Petviashvili(KP) equation and is expressed as follows[7],

$$(u_t + 6uu_x + u_{xxx})_x + 3su_{yy} = 0, \ s = \pm 1. \tag{1.1}$$

The propagation property of solitons depend essentially on the sign of s in equation (1). In the media with negative dipersion (s=+1), line solitons are stable to long transverse perturbations. On the oyher hand, line solitons are unstable for positive dispersion (s=-1)[9].

The aim of the present paper is to extend the sine-cosine[3, 4, 5, 6], the standard

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* Corresponding author.

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tanh[1, 2] and the extended tanh[7, 8] methods to finding new solitons and periodic solutions for nonlinear KP equation.
The paper has been organized as follows. Section 2 gives analysis of methods. In section 3 application of the sine-cosine method is considered for the Kadomtsev-Petviashvili(KP) equation. section 4 gives application of standard tanh and extended tanh methods for the Kp equatin. Discussion and conclusion are presented in section 5.

2. Analysis of method

For the sine-cosine, standard tanh and extended tanh methods, we first unite the independent variable x, y and t into one wave variable $\xi = x + y - ct$, to carry out a PDE in two independent variables

$$P(u, u_t, u_x, u_y, u_{xx}, u_{yy}, ...) = 0 \quad or \quad R(u, u_{tt}, u_x, u_y, u_{xx}, u_{yy}, ...) = 0, \qquad (2.1)$$

into an ODE

$$Q(u, u', u'', u''', ...). \qquad (2.2)$$

Eq.(2.2) is then integrated as long as all terms contain derivatives. Usually the integration constants are considered to be zeros in view of the localized solutions.

2.1. **The sine-cosine method.** The solutions of the reduced ODE equation can be expressed in the form

$$u(x, y, t) = \begin{cases} \lambda cos^{\beta}(\mu\xi), & |\mu\xi| \leq \frac{\pi}{2\mu}, \\ otherwise, & 0, \end{cases} \qquad (2.3)$$

or in the form

$$u(x, y, t) = \begin{cases} \lambda sin^{\beta}(\mu\xi), & |\mu\xi| \leq \frac{\pi}{\mu}, \\ otherwise, & 0. \end{cases} \qquad (2.4)$$

Where λ, μ and β are parameters that will be determined, in Eqs.(2.3),(2.4) μ and c are the wave number and the wave speed respectively. The assumption (2.3) gives

$$u^2(\xi) = \lambda^2 cos^{2\beta}(\mu\xi), \qquad (2.5)$$

$$u''(\xi) = -\mu^2\beta^2\lambda cos^{\beta}(\mu\xi) + \mu^2\beta(\beta-1)\lambda cos^{\beta-2}(\mu\xi) \qquad (2.6)$$

where similar equations can be obtained for the sine assumption. Substituting the sine-cosine assumption and their derivatives into the reduce ODE gives a trigonometric equation of $cos^K(\mu\xi)$ or $sin^K(\mu\xi)$ terms. The parameters are then determined by first balancing the exponents of each pair of cosine or sine determine K. We next collect all coefficients of the same power in $cos^k(\mu\xi)$ or $sin^k(\mu\xi)$, where these coefficients have to vanish. This gives a system of algebraic equations among the unknowns c, λ, μ and ξ that will be determined. The solutions proposed in (2.3) and (2.4) follow immediately.

2.2. **The standard tanh and extended tanh methods.** The standard tanh method is developed by Malfliet[1, 2] where the tanh is introduced as a new variable, since all derivatives of a tanh are represented by a tanh itself. The tanh method introduces a new independent variable

$$Y = tanh(\mu\xi), \tag{2.7}$$

that leads to the changes of derivatives:

$$\frac{d}{d\xi} = \mu(1-Y^2)\frac{d}{dY} \quad , \quad \frac{d^2}{d\xi^2} = \mu^2(1-Y^2)(-2Y\frac{d}{dY} + (1-Y^2)\frac{d^2}{dY^2}). \tag{2.8}$$

The solutions for the standard tanh method can be proposed as a finite power series in Y in the form

$$u(\mu\xi) = S(Y) = \sum_{k=0}^{M} a_k Y^k. \tag{2.9}$$

The extended tanh method [7, 8] admits the use of the finite expansion

$$u(\mu\xi) = S(Y) = \sum_{k=0}^{M} a_k Y^k + \sum_{k=1}^{M} b_k Y^{-k}. \tag{2.10}$$

Where the parameter M is a positive , in most cases, that will be determined. To determine the parameter M, we usually balance the linear terms of highest order in the resulting equation with the highest order nonlinear terms.We then collect all coefficients of powers of Y in the resulting equation where these coefficients have to vanish. This will give a system of algebraic equations involving the parameters $a_k, (k = 0, ..., M)$, μ, ξ and c for the standard tanh method and the parameters $a_k, (k = 0, ..., M)$, $b_k(k = 1, ..., M)$, μ, ξ and c for the extended tanh method. Having determined these parameters we obtain an analytic solution $u(x, y, t)$ in a closed form.

3. Using the sine-cosine method

A nonlinear KP equation is given by

$$(u_t + 6uu_x + u_{xxx})_x + 3su_{yy} = 0, \;\; s = \pm 1. \tag{3.1}$$

Substituting $u(x, y, t) = u(\xi)$, where the wave variable is $\xi = x + y - ct$, carries out Eq.(3.1) to the ODE

$$(-cu' + 3(u^2)' + u''')' + 3su'' = 0. \tag{3.2}$$

Twice integrating (3.2), setting the constant of integrating to zero, we obtain

$$(3s - c)u + 3u^2 + u'' = 0. \tag{3.3}$$

Substituting cosine anstaz(2.4) into (3.3) gives

$$(3s-c)\lambda cos^{\beta}(\mu\xi) + 3\lambda^2 cos^{2\beta}(\mu\xi) - \lambda\mu^2\beta^2 cos^{\beta}(\mu\xi) + \lambda\mu^2\beta(\beta-1)cos^{\beta-2}(\mu\xi) = 0 \tag{3.4}$$

Equating the exponents of the second and the last cosine functions in above equation, collecting coefficients of each pair of cosine functions of like exponents, and setting it equal to zero, we obtain the following system of algebraic equations:

$$\begin{cases} \beta - 1 \neq 0, \\ \beta - 2 = 2\beta, \\ 3s - c = \mu^2\beta^2, \\ -3\lambda = \mu^2\beta(\beta - 1), \end{cases} \tag{3.5}$$

so that this gives

$$\begin{cases} \beta = -2, \\ \mu = \frac{1}{2}\sqrt{3s - c}, \\ \lambda = \frac{1}{2}(3s - c), \end{cases} \tag{3.6}$$

and c is any real number.
That can also be obtained by using the sine anstaz (2.4).
For $3s - c > 0$ this leads to the periodic solutions:

$$u_1(x, y, t) = \frac{1}{2}\ (3s - c)\ \sec^2\ \left[\frac{1}{2}\ \sqrt{3s - c}\ (x + y - ct)\right], \tag{3.7}$$

and for sine anstaz we obtain:

$$u_2(x, y, t) = \frac{1}{2}\ (3s - c)\ \csc^2\ \left[\frac{1}{2}\ \sqrt{3s - c}\ (x + y - ct)\right]. \tag{3.8}$$

However for $3s - c < 0$ we obtain the solitons solutions:

$$u_3(x, y, t) = \frac{1}{2}\ (3s - c)\ sech^2\ \left[\frac{1}{2}\ \sqrt{c - 3s}\ (x + y - ct)\right], \tag{3.9}$$

$$u_4(x, y, t) = -\frac{1}{2}\ (3s - c)\ csch^2\ \left[\frac{1}{2}\ \sqrt{3s - c}\ (x + y - ct)\right]. \tag{3.10}$$

4. Using the standard tanh and the extended tanh methods

We will employ the standard tanh method presented by Malfliet [1, 2] and the extended tanh method[7, 8].

4.1. Using the standard tanh method.

The KP equation given by

$$(u_t + 6uu_x + u_{xxx})_x + 3su_{yy} = 0,\ \ s = \pm 1, \tag{4.1}$$

as shown before, this equation can be transformed to the ODE

$$(3s - c)u + 3u^2 + u'' = 0. \tag{4.2}$$

Upon using the wave variable $\xi = x + y - ct$ and twice integrating the resulting equation.
Balancing u'' with u^2 gives

$$M + 2 = 2M, \tag{4.3}$$

so that

$$M = 2, \tag{4.4}$$

the standard tanh method assumes that finite expansion

$$u(\xi) = a_0 + a_1Y + a_2Y^2 \quad , \quad Y = tanh(\mu\xi), \tag{4.5}$$

Substituting (4.5) into (4.2) and using (2.8), collecting the coefficients of $Y^j, 0 \leq j \leq 4$, and equating this coefficients to zero and solving the system of algebraic for a_0, a_1, a_2, μ and c , we find the following set of solution:

$$\begin{cases} a_0 = 2\mu^2, \\ a_1 = 0, \\ a_2 = -2\mu^2, \\ c = 3s + 4\mu^2, \end{cases} \tag{4.6}$$

and μ is any real number.
Substituting (4.6), into (4.5)gives

$$u_5(x, y, t) = 2\mu^2 \{1 - tanh^2 [\mu (x + y - (3s + 4\mu^2) t)] \}, \tag{4.7}$$

Eq.(4.7) in turn gives the solitons solution

$$u_6(x, y, t) = 2\mu^2 sech^2 \{ \mu [x + y - (3s + 4\mu^2)t] \}, \tag{4.8}$$

4.2. **Using the extended tanh method.** As shown before, the equation as follows:

$$(3s - c)u + 3u^2 + u'' = 0, \tag{4.9}$$

is the transformed ODE of the KP equation with using the wave variable $\xi = x + y - ct$ and twice integrating the resulting equation.
Balancing u'' with u^2 gives

$$M + 2 = 2M, \tag{4.10}$$

so that

$$M = 2, \tag{4.11}$$

the extended tanh method assumes that finite expansion

$$u(\xi) = a_0 + a_1 Y + a_2 Y^2 + b_1 Y^{-1} b_2 Y^{-2} \quad , \quad Y = tanh(\mu\xi). \tag{4.12}$$

Substituting (4.12) into (4.9) and using the (2.8), collecting the coefficients of $Y^j, -4 \leq j \leq 4$ and equating this coefficients to zero and solving the system of $a_0, a_1, a_2, b_1, b_2, \mu$ and c we find the following set of solution:
$a_1 = b_1 = o$ and μ is any real number, after collecting,

$$a_0 = 2\mu^2 \ a_1 = 0 \ a_2 = -2\mu^2 \ b_1 = 0 \ b_2 = 0 \ c = 4\mu^2 + 3s, \tag{4.13}$$

$$a_0 = \frac{2}{3}\mu^2 \ a_1 = 0 \ a_2 = 0 \ b_1 = 0 \ b_2 = -2\mu^2 \ c = -4\mu^2 + 3s, \tag{4.14}$$

$$a_0 = -\frac{4}{3}\mu^2 \ a_1 = 0 \ a_2 = -2\mu^2 \ b_1 = 0 \ b_2 = -2\mu^2 \ c = -16\mu^2 + 3s. \tag{4.15}$$

The Eq.(4.13) gives:

$$u_7(x, y, t) = 2\mu^2 sech^2 \{ \mu[x + y - (3s + 4\mu^2) t] \}, \tag{4.16}$$

and the Eq.(4.14) gives:

$$u_8(x, y, t) = -2\mu^2 csch^2 \{ \mu[x + y - (3s + 4\mu^2) t] \}, \tag{4.17}$$

and then the Eq.(4.15) gives:

$$u_9(x,y,t) = 2\mu^2\{-\frac{2}{3}\tan^2[\mu(x+y-(3s+16\mu^2)t)]+\cot^2[\mu(x+y-(3s+16\mu^2)t)]\}. \tag{4.18}$$

Where the three of later equations are solitons solutions for Kadomtsev-Petviashvili(KP) equation.

5. Conclusion

The sine-cosine, the standard tanh and the extended tanh methods were used to investigate nonlinear KP equation. The work emphasized our belief the three methods are powerful technique to handle nonlinear depressive equation, hence these methods can be used in a wider context. The validity of these methods has been tested by applying to Kadomtsev-Petviashvili(KP) equation.
Finally it is a promising and powerful method for other nonlinear equations in mathematical physics.

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[1] Department of Mathematics, University of Mohaghegh Ardabili,Ardabil, Iran.
E-mail address: borhani@uma.ac.ir, karimi_s@math.com

[2] Department of Mathematics and Computer science, University of Mazandaran,P. O. BOX 47416-95447 Babolsar, Iran,
E-mail address: jafari@umz.ac.ir