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STABILITY OF AN ADDITIVE–QUADRATIC FUNCTIONAL EQUATION OF TWO VARIABLES IN F–SPACES

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ABSTRACT. In this paper, we achieve the Hyers-Ulam-Rassias stability of the following system of functional equations

$$\begin{cases} f(x_1+x_2,y)=f(x_1,y)+f(x_2,y), \\ \\ f(x,y_1+y_2)+f(x,y_1-y_2)=2f(x,y_1)+2f(x,y_2) \end{cases}$$

in F-spaces.

1. INTRODUCTION AND PRELIMINARIES

The stability problem of functional equations originated from a question of Ulam [26] in 1940, concerning the stability of group homomorphisms. Let $(G_1,.)$ be a group and let $(G_2,*)$ be a metric group with the metric $d(.,.)$. Given $\epsilon>0$, dose there exist a $\delta>0$, such that if a mapping $h:G_1\longrightarrow G_2$ satisfies the inequality $d(h(x.y),h(x)*h(y))<\delta$ for all $x,y\in G_1$, then there exists a homomorphism $H:G_1\longrightarrow G_2$ with $d(h(x),H(x))<\epsilon$ for all $x\in G_1$? In the other words, under what condition does there exist a homomorphism near an approximate homomorphism? The concept of stability for functional equation arises when we replace the functional equation by an inequality which acts as a perturbation of the equation. In 1941, D. H. Hyers [10] gave a first affirmative answer to the question of Ulam for Banach spaces. Let $f:E\longrightarrow E'$ be a mapping between Banach spaces such that

$$\|f(x+y)-f(x)-f(y)\|\leq\delta$$

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for all $x, y \in E$, and for some $\delta > 0$. Then there exists a unique additive mapping $T : E \longrightarrow E'$ such that

$$\|f(x) - T(x)\| \le \delta$$

for all $x \in E$. Moreover if $f(tx)$ is continuous in t for each fixed $x \in E$, then T is linear. In 1978, Th. M. Rassias [19]provided a generalization of Hyers' Theorem which allows the Cauchy difference to be unbounded (see also [20]).
In 1990, Th.M. Rassias [20] during the 27th International Symposium on Functional Equations asked the question whether such a theorem can also be proved for $p \ge 1$. In 1991, Gajda [8] gave an affirmative solution to this question for $p > 1$ by following the same approach as in Rassias' paper [19]. It was proved by Gajda [8], as well as by Th.M. Rassias and Šemrl [22] that one cannot prove a Rassias type theorem when $p = 1$. In 1994, P. Găvruta [9] provided a generalization of Rassias' theorem in which he replaced the bound $\varepsilon(\|x\|^p + \|y\|^p)$ in ([19]) by a general control function $\varphi(x, y)$. The paper of Th.M. Rassias [19] has provided a lot of influence in the development of what we now call Hyers–Ulam–Rassias stability of functional equations. During the last decades several stability problems for various functional equations have been investigated by many mathematicians; we refer the reader to the monographs [3, 5, 6, 7, 11, 12, 13, 16, 21, 24].

The functional equation

$$f(x + y) + f(x - y) = 2f(x) + 2f(y), \tag{1.1}$$

is called the quadratic functional equation and every solution of the quadratic equation (1.1) is said to be a quadratic function. It is well known that a function f between two real vector spaces is quadratic if and only if there exists a unique symmetric bi-additive function B such that $f(x) = B(x, x)$ for all x, where

$$B(x, y) = \frac{1}{4}(f(x + y) - f(x - y)), \tag{1.2}$$

(see [17]).

The Hyers–Ulam stability problem for the quadratic functional equation was solved by Skof [25] and, independently, by Cholewa [4]. An analogous result for quadratic stochastic processes was obtained by Nikodem [18]. In [2], Czerwik proved the generalized Hyers–Ulam stability of the quadratic functional equation. Jung [15] dealt with stability problems for the quadratic functional equation of Pexider type.

Jun and Kim [14] introduced the following functional equation

$$f(2x + y) + f(2x - y) = 2f(x + y) + 2f(x - y) + 12f(x), \tag{1.3}$$

and established the general solution and the generalized Hyers–Ulam–Rassias stability for functional equation (1.3). Obviously, the $f(x) = x^3$ satisfies functional equation (1.3), so it is natural to call (1.3) the cubic functional equation. Every solution of the cubic functional equation is said to be a cubic function. Jun and Kim proved also that a function f between two real vector spaces X and Y is a solution of (1.3) if and only if there exists a unique function $C : X \times X \times X \longrightarrow Y$ such that $f(x) = C(x, x, x)$ for all $x \in X$, moreover, C is symmetric for each fixed one variable and is additive for fixed two variables. Later a number of mathematicians worked on the stability of some types of the cubic equation [23].

Let X, Y and Z be vector spaces on $\mathbb{R}$ or $\mathbb{C}$. We say that a mapping $f : X \times Y \to Z$ is additive–quadratic if f satisfies the following system of functional equations:

$$\begin{cases} f(x_1 + x_2, y) = f(x_1, y) + f(x_2, y), \\ \\ f(x, y_1 + y_2) + f(x, y_1 - y_2) = 2f(x, y_1) + 2f(x, y_2) \end{cases} \tag{1.4}$$

for all $x, x_1, x_2 \in X$ and $y, y_1, y_2 \in Y$. See the following examples.

- Let $X = Y = Z = \mathbb{R}$. We define f by $f(x, y) = xy^2$. It is easy to see that f is an additive–quadratic map.
- Let X be a normed space, and let $Y = Z = X$. Suppose $f(x, y) = x\|y\|^2$. Then f is an additive–quadratic map.
- Let A be an algebra, and let X be a right $A-$module. Set $Z = X$ and $Y = A$. Suppose $f(x, a) = xa^2$. Then f is an additive–quadratic map.

We recall some basic facts concerning $F-$spaces. In functional analysis, an $F-$space is a vector space V over the real or complex numbers together with a metric $d : V \times V \to \mathbb{R}$ so that scalar multiplication in V is continuous with respect to d and the standard metric on $\mathbb{R}$ or $\mathbb{C}$. Addition in V is continuous with respect to d. The metric is translation-invariant, i.e. $d(x + a, y + a) = d(x, y)$ for all x, y and a in V. The metric space (V, d) is complete.

Some authors call these spaces "Frchet spaces", but usually the term Frchet space is reserved for locally convex $F-$spaces. The metric may or may not necessarily be part of the structure on an $F-$space; many authors only require that such a space be metrizable in a manner that satisfies the above properties. Trivially, every Banach space is a $F-$space as the norm induces a translation invariant metric and the space is complete with respect to this metric. The L_p spaces are $F-$spaces for all $p > 0$ and for $p = 1$ they are locally convex and thus Frchet spaces and even Banach spaces. So for example, $L_{\frac{1}{2}}([0, 1])$ is a $F-$space, which is not a Banach space.

2. Main results

We start our work with the following result, which explain the relation between additive–quadratic maps and cubic maps.

Theorem 2.1. *Let X, Z be vector spaces. A mapping $g : X \to Z$ is cubic if and only if there exists an additive–quadratic mapping $f : X \times X \to Z$ such that g(x)=f(x,x) and that*

$$f(y, 2x + y) - f(y, 2x - y) = 2(f(y, x + y) - f(y, x - y))$$

for all $x, y \in X$.

Proof. Let $g : X \to Z$ be a cubic mapping. Then there exists a mapping $C : X \times X \times X \to Z$ such that C is symmetric for each fixed one variable and is additive for fixed two variables. Define $f : X \times X \to Z$ by $f(x, y) = C(x, y, y)$. One can show that f is additive–quadratic mapping satisfies $f(y, 2x+y) - f(y, 2x-y) = 2(f(y, x+y) - f(y, x-y))$ for all $x, y \in X$. For the converse, let $f : X \times X \to Z$ be an additive–quadratic mapping such that $f(y, 2x+y) - f(y, 2x-y) = 2(f(y, x+y) - f(y, x-y))$ for all $x, y \in X$. Then it is easy to see that the mapping $g : X \to Z$, defined by $g(x) = f(x, x)$ for all $x \in X$, is cubic. □

We now investigate the generalized Hyers-Ulam-Rassias stability problem for system of functional equations (1.4). From now on, let X be a real vector space and Y be a real $F-$space by metric d. Let $f : X \times X \to Y$ be a function then we define $\Delta_f, D_f : X \times X \times X \to \mathbb{R}$ by

$$D_f(x_1, x_2, y) = d(f(x_1 + x_2, y), f(x_1, y) + f(x_2, y)),$$
$$\Delta_f(x, y_1, y_2) = d(f(x, y_1 + y_2) + f(x, y_1 - y_2), 2f(x, y_1) + 2f(x, y_2))$$

for all $x, x_1, x_2, y, y_1, y_2 \in X$.

Theorem 2.2. *Let $\phi, \psi : X \times X \times X \to [0, \infty)$ be mappings satisfying*

$$\sum_{i=0}^{\infty} \frac{\psi(2^{i+1}x, 2^i y, 2^i y) + \phi(2^i x, 2^i x, 2^i y)}{8^i} < \infty$$

for all $x, y \in X$, and

$$\lim \frac{\psi(2^n x, 2^n y, 2^n z) + \phi(2^n x, 2^n y, 2^n z)}{8^n} = 0$$

for all $x, y, z \in X$. If $f : X \times X \to Y$ is a mapping such that

$$D_f(x_1, x_2, y) \leq \phi(x_1, x_2, y), \tag{2.1}$$

$$\Delta_f(x, y_1, y_2) \leq \psi(x, y_1, y_2) \tag{2.2}$$

for all $x, y, x_1, x_2, y_2, y_2 \in X$, then there exists a unique additive–quadratic mapping $T : X \times X \to Y$ satisfying (1.4) and

$$d(f(x, y), T(x, y)) \leq \frac{1}{8} \sum_{i=0}^{\infty} \frac{\psi(2^{i+1}x, 2^i y, 2^i y)}{8^i} + \frac{1}{2} \sum_{i=0}^{\infty} \frac{\phi(2^i x, 2^i x, 2^i y)}{8^i}, \tag{2.3}$$

for all $x, y \in X$.

Proof. Putting $x_1 = x_2 = x$ in (2.1), we get

$$d(f(2x, y), 2f(x, y)) \leq \phi(x, x, y). \tag{2.4}$$

Replacing y_1, y_2 by y in (2.2) to obtain

$$d(f(x, 2y), 4f(x, y)) \leq \psi(x, y, y). \tag{2.5}$$

Replacing x by $2x$ in (2.5), yields

$$d(f(2x, 2y), 4f(2x, y)) \leq \psi(2x, y, y). \tag{2.6}$$

Combining (2.4) and (2.6), we lead to

$$d(\frac{1}{8}f(2x, 2y), f(x, y)) \leq \frac{1}{2}\psi(x, x, y) + \frac{1}{8}\psi(2x, y, y). \tag{2.7}$$

From the inequality (2.7) we use iterative methods and induction on n to prove our next relation:

$$d(\frac{1}{8^n}f(2^nx,2^ny),f(x,y))\leq\frac{1}{2}\sum_{i=0}^{n-1}\psi(2^nx,2^nx,2^ny)+\frac{1}{8}\sum_{i=0}^{n-1}\frac{1}{8^n}\psi(2^{n+1}x,2^ny,2^ny). \tag{2.8}$$

We divide (2.8) by 8^m and replace x by 2^mx to obtain that

$$d(\frac{1}{8^{m+n}}f(2^{m+n}x,2^{m+n}y),\frac{1}{8^m}f(2^mx,2^my))\leq\frac{1}{2}\sum_{i=0}^{n-1}\psi(2^{m+n}x,2^{m+n}x,2^{m+n}y)$$
$$+\frac{1}{8}\sum_{i=0}^{n-1}\frac{1}{8^{n+m}}\psi(2^{m+n+1}x,2^{m+n}y,2^{m+n}y). \tag{2.9}$$

This shows that $\{\frac{1}{8^n}f(2^nx,2^ny)\}$ is a Cauchy sequence in Y by taking the limit $m\to\infty$. Since Y is a Banach space, it follows that the sequence $\{\frac{1}{8^n}f(2^nx,2^ny)\}$ converges. We define $T:X\times X\to Y$ by $T(x,y)=\lim_{n\to\infty}\frac{1}{8^n}f(2^nx,2^ny)$ for all $x,y\in X$. It follows from (2.1) that

$$D_T(x_1,x_2,y)=\lim_{n\to\infty}\frac{1}{8^n}D_f(2^nx_1,2^nx_2,2^ny)\leq\lim_{n\to\infty}\frac{1}{8^n}\phi(2^nx_1,2^nx_2,2^ny)=0$$

for all $x_1,x_2,y\in X$. Also it follows from (2.2) that

$$\Delta_T(x,y_1,y_2)=\lim_{n\to\infty}\frac{1}{8^n}\Delta_f(2^nx,2^ny_1,2^ny_2)\leq\lim_{n\to\infty}\frac{1}{8^n}\phi(2^nx,2^ny_1,2^ny_2)=0$$

for all $x,y_1,y_2\in X$. This means that T is additive–quadratic. It remains to show that T is unique. Suppose that there exists another additive–quadratic mapping $T':X\times X\to Y$ which satisfies (1.4) and (2.3). Since $\frac{1}{8^n}T(2^nx,2^ny)=T(x,y)$, and $\frac{1}{8^n}T'(2^nx,2^ny)=T'(x,y)$ for all $x,y\in X$, we conclude that

$$\begin{aligned}d(T(x,y),T'(x,y))&=\frac{1}{8^n}d(T(2^nx,2^ny),T'(2^nx,2^ny))\\&\leq\frac{1}{8^n}d(T(2^nx,2^ny),f(2^nx,2^ny))+d(f(2^nx,2^ny),T'(2^nx,2^ny))\\&\leq 2[\frac{1}{8}\sum_{i=0}^{\infty}\frac{\psi(2^{n+i+1}x,2^{n+i}y,2^{n+i}y)}{8^{n+i}}+\frac{1}{2}\sum_{i=0}^{\infty}\frac{\phi(2^{n+i}x,2^{n+i}x,2^{n+i}y)}{8^{n+i}}]\end{aligned}$$

for all $x,y\in X$. By letting $n\to\infty$ in this inequality, it follows that $T(x,y)=T'(x,y)$ for all $x,y\in X$, which gives the conclusion. □

Let X, Y and Z be vector spaces on $\mathbb{R}$ or $\mathbb{C}$. We say that a mapping $f:X\times Y\to Z$ is quadratic-additive if f satisfies

$$\begin{cases}f(x_1+x_2,y)+f(x_1-x_2,y)=2f(x_1,y)+2f(x_2,y),\\ \\ f(x,y_1+y_2)=f(x,y_1)+f(x,y_2)\end{cases}$$

for all $x,x_1,x_2\in X$ and $y,y_1,y_2\in Y$.

Theorem 2.3. *Let $\phi, \psi : X \times X \times X \to [0, \infty)$ be mappings satisfying*

$$\sum_{i=0}^{\infty} \frac{\psi(2^i x, 2^i x, 2^{i+1} y) + \phi(2^i x, 2^i y, 2^i y)}{8^i} < \infty$$

for all $x, y \in X$, and

$$\lim \frac{\psi(2^n x, 2^n y, 2^n z) + \phi(2^n x, 2^n y, 2^n z)}{8^n} = 0$$

for all $x, y, z \in X$. If $f : X \times X \to Y$ is a mapping such that

$$d(f(x_1 + x_2, y) + f(x_1 - x_2, y), 2f(x_1, y) + 2f(x_2, y)) \leq \psi(x_1, x_2, y),$$

$$d(f(x, y_1 + y_2), f(x, y_1) + f(x, y_2)) \leq \phi(x, y_1, y_2)$$

for all $x, y, x_1, x_2, y_2, y_2 \in X$, then there exists a unique additive–quadratic mapping $T : X \times X \to Y$ satisfying (1.4) and

$$d(f(x, y), T(x, y)) \leq \frac{1}{2} \sum_{i=0}^{\infty} \frac{\psi(2^i x, 2^i y, 2^i y)}{8^i} + \frac{1}{8} \sum_{i=0}^{\infty} \frac{\phi(2^i x, 2^i x, 2^{i+1} y)}{8^i} \tag{2.10}$$

for all $x, y \in X$.

Proof. Put $f_0(x, y) = f(y, x), \phi_0(x, y, z) = \phi(z, y, x)$ and $\psi_0(x, y, z) = \psi(z, y, x)$, for all $x, y, z \in X$. Then by above theorem, there exists a unique additive–quadratic mapping $T_0 : X \times X \to Y$ satisfies

$$d(f_0(x, y), T_0(x, y)) \leq \frac{1}{8} \sum_{i=0}^{\infty} \frac{\psi_0(2^{i+1} x, 2^i y, 2^i y)}{8^i} + \frac{1}{2} \sum_{i=0}^{\infty} \frac{\phi_0(2^i x, 2^i x, 2^i y)}{8^i}$$

for all $x, y \in X$. Now, we put $T(x, y) = T_0(y, x)$ for all $x, y \in X$. Hence, T is a unique quadratic-additive map satisfies (2.10). □

Corollary 2.4. *Let X be a vector space and Y be a Banach space. Suppose the mappings $\phi, \psi : X \times X \times X \to [0, \infty)$ satisfying*

$$\sum_{i=0}^{\infty} \frac{\psi(2^{i+1} x, 2^i y, 2^i y) + \phi(2^i x, 2^i x, 2^i y)}{8^i} < \infty$$

for all $x, y \in X$, and

$$\lim \frac{\psi(2^n x, 2^n y, 2^n z) + \phi(2^n x, 2^n y, 2^n z)}{8^n} = 0$$

for all $x, y, z \in X$. If $f : X \times X \to Y$ is a mapping such that

$$\|f(x_1 + x_2, y) - f(x_1, y) + f(x_2, y)\| \leq \phi(x_1, x_2, y),$$

$$\|f(x, y_1 + y_2) + f(x, y_1 - y_2) - 2f(x, y_1) - 2f(x, y_2)\| \leq \psi(x, y_1, y_2)$$

for all $x, y, x_1, x_2, y_2, y_2 \in X$, then there exists a unique additive–quadratic mapping $T : X \times X \to Y$ satisfying (1.4) and

$$\|f(x, y) - T(x, y)\| \leq \frac{1}{8} \sum_{i=0}^{\infty} \frac{\psi(2^{i+1} x, 2^i y, 2^i y)}{8^i} + \frac{1}{2} \sum_{i=0}^{\infty} \frac{\phi(2^i x, 2^i x, 2^i y)}{8^i}$$

for all $x, y \in X$.

Proof. It follows from theorem 2.2. by Putting $d(a,b) = \|a-b\|$ for all $a, b \in Y$. □

We are going to investigate the Hyers-Ulam -Rassias stability problem for system of functional equations (1.4).

Corollary 2.5. *Let* $\epsilon > 0, p < 3$, *and let* X, Y *be a normed space a Banach space, respectively. If* $f : X \times X \to Y$ *is a mapping such that*

$$\begin{aligned} Max\{&\|f(x, y_1 + y_2) + f(x, y_1 - y_2) - 2f(x, y_1) - 2f(x, y_2)\| \\ &, \|f(x_1 + x_2, y) - f(x_1, y) + f(x_2, y)\|\} \\ &\leq \epsilon(Min\{\|x_1\|^p + \|x_2\|^p + \|y\|^p, \|x\|^p + \|y_1\|^p + \|y_2\|^p\}) \end{aligned}$$

for all $x, y, x_1, x_2, y_2, y_2 \in X$, *then there exists a unique additive–quadratic mapping* $T : X \times X \to Y$ *satisfying (1.4) and*

$$\|f(x,y) - T(x,y)\| \leq \frac{\epsilon}{8 - 2^p}((2^p + 9)\|x\|^p + 5\|y\|^p)$$

for all $x, y \in X$.

Proof. It follows from corollary 2.4. by Putting $\phi(a,b,c) = \psi(a,b,c) = \|a\|^p + \|b\|^p + \|c\|^p$ for all $a, b, c \in X$. □

By Corollary 2.5, we solve the following Hyers-Ulam stability problem for system of functional equations (1.4).

Corollary 2.6. *Let* $\epsilon > 0$, *and let* X, Y *be a normed space a Banach space, respectively. If* $f : X \times X \to Y$ *is a mapping such that*

$$\begin{aligned} Max\{&\|f(x, y_1 + y_2) + f(x, y_1 - y_2) - 2f(x, y_1) - 2f(x, y_2)\| \\ &, \|f(x_1 + x_2, y) - f(x_1, y) + f(x_2, y)\|\} \\ &\leq \epsilon \end{aligned}$$

for all $x, y, x_1, x_2, y_2, y_2 \in X$, *then there exists a unique additive–quadratic mapping* $T : X \times X \to Y$ *satisfying (1.4) and*

$$\|f(x,y) - T(x,y)\| \leq \frac{15\epsilon}{7}$$

for all $x, y \in X$.

Similarly we can prove the Hyers-Ulam-Rassias stability and the Hyers-Ulam stability problems for quadratic-additive maps.

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