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$\mathcal{L}$-FUZZY NONLINEAR APPROXIMATION THEORY WITH APPLICATION

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Abstract. In this paper, we show that cartesian product with a fixed compact set preserve various nonlinear approximation-theoretic properties in $\mathcal{L}$-fuzzy metric space. In fact, we show that for approximative compactness and proximinality, points can be replaced by compact sets (Theorem 2.4); also that cartesian product (Theorem 2.8), preserve the compactness hierarchy properties when we operate by cartesian product with a compact subset. Further, the set of points in an approximatively compact subset which minimize the distance to a given compact subset is itself compact (Theorem 2.9).

1. Preliminaries

The notion of fuzzy sets was introduced by Zadeh [13]. Using the idea of $\mathcal{L}$-fuzzy sets [5], the author introduced the notion of $\mathcal{L}$-fuzzy metric spaces with the help of continuous t-norms as a generalization of fuzzy metric space due to George and Veeramani [4].

In the sequel, we shall adopt usual terminology, notation and conventions of $\mathcal{L}$-fuzzy metric spaces introduced by Saadati et al. [1, 8, 11].

Definition 1.1. ([5]) Let $\mathcal{L} = (L, \leq_L)$ be a complete lattice, and U a non-empty set called universe. An $\mathcal{L}$-fuzzy set $\mathcal{A}$ on U is defined as a mapping $\mathcal{A} : U \longrightarrow L$. For each u in U, $\mathcal{A}(u)$ represents the degree (in L) to which u satisfies $\mathcal{A}$.

Lemma 1.2. ([3]) *Consider the set L^* and operation $\leq_{L^*}$ defined by:*

$$L^* = \{(x_1, x_2) : (x_1, x_2) \in [0, 1]^2 \text{ and } x_1 + x_2 \leq 1\},$$

$(x_1, x_2) \leq_{L^*} (y_1, y_2) \iff x_1 \leq y_1$ and $x_2 \geq y_2$, for every $(x_1, x_2), (y_1, y_2) \in L^*$. Then $(L^*, \leq_{L^*})$ is a complete lattice .

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Definition 1.3. ([2]) An intuitionistic fuzzy set $\mathcal{A}_{\zeta,\eta}$ on a universe U is an object $\mathcal{A}_{\zeta,\eta} = \{(\zeta_{\mathcal{A}}(u), \eta_{\mathcal{A}}(u)) : u \in U\}$, where, for all $u \in U$, $\zeta_{\mathcal{A}}(u) \in [0,1]$ and $\eta_{\mathcal{A}}(u) \in [0,1]$ are called the membership degree and the non-membership degree, respectively, of u in $\mathcal{A}_{\zeta,\eta}$, and furthermore satisfy $\zeta_{\mathcal{A}}(u) + \eta_{\mathcal{A}}(u) \leq 1$.

Classically, a triangular norm T on $([0,1], \leq)$ is defined as an increasing, commutative, associative mapping $T : [0,1]^2 \to [0,1]$ satisfying $T(1,x) = x$, for all $x \in [0,1]$. These definitions can be straightforwardly extended to any lattice $\mathcal{L} = (L, \leq_L)$. Define first $0_{\mathcal{L}} = \inf L$ and $1_{\mathcal{L}} = \sup L$.

Definition 1.4. A triangular norm (t-norm) on $\mathcal{L}$ is a mapping $\mathcal{T} : L^2 \to L$ satisfying the following conditions:

(i) $(\forall x \in L)(\mathcal{T}(x, 1_{\mathcal{L}}) = x)$; (boundary condition)
(ii) $(\forall (x,y) \in L^2)(\mathcal{T}(x,y) = \mathcal{T}(y,x))$; (commutativity)
(iii) $(\forall (x,y,z) \in L^3)(\mathcal{T}(x, \mathcal{T}(y,z)) = \mathcal{T}(\mathcal{T}(x,y), z))$; (associativity)
(iv) $(\forall (x,x',y,y') \in L^4)(x \leq_L x'$ and $y \leq_L y' \Rightarrow \mathcal{T}(x,y) \leq_L \mathcal{T}(x',y'))$. (monotonicity)

A t–norm $\mathcal{T}$ on $\mathcal{L}$ is said to be continuous if for any $x, y \in \mathcal{L}$ and any sequences $\{x_n\}$ and $\{y_n\}$ which converge to x and y we have

$$\lim_n \mathcal{T}(x_n, y_n) = \mathcal{T}(x,y)$$

For example, $\mathcal{T}(x,y) = \min(x,y)$ and $\mathcal{T}(x,y) = xy$ are two continuous t–norms on $[0,1]$.

A t-norm can also be defined recursively as an $(n+1)$-ary operation ($n \in \mathbf{N}$) by $\mathcal{T}^1 = \mathcal{T}$ and

$$\mathcal{T}^n(x_1, \cdots, x_{n+1}) = \mathcal{T}(\mathcal{T}^{n-1}(x_1, \cdots, x_n), x_{n+1})$$

for $n \geq 2$ and $x_i \in L$.

We say the continuous t-norm is *natural* and write $\mathcal{T}_N$ whenever $\mathcal{T}_N(a,b) = \mathcal{T}_N(c,d)$ and $a \leq_L c$ implies $b \geq_L d$.

Definition 1.5. ([3]) A t-norm $\mathcal{T}$ on L^* is called *t-representable* if and only if there exist a t-norm T and a t-conorm S on $[0,1]$ such that, for all $x = (x_1, x_2), y = (y_1, y_2) \in L^*$,

$$\mathcal{T}(x,y) = (T(x_1,y_1), S(x_2,y_2)).$$

Definition 1.6. A negation on $\mathcal{L}$ is any decreasing mapping $\mathcal{N} : L \to L$ satisfying $\mathcal{N}(0_{\mathcal{L}}) = 1_{\mathcal{L}}$ and $\mathcal{N}(1_{\mathcal{L}}) = 0_{\mathcal{L}}$. If $\mathcal{N}(\mathcal{N}(x)) = x$, for all $x \in L$, then $\mathcal{N}$ is called an involutive negation.

Definition 1.7. The 3-tuple $(X, \mathcal{M}, \mathcal{T})$ is said to be an *$\mathcal{L}$-fuzzy metric space* if X is an arbitrary (non-empty) set, $\mathcal{T}$ is a continuous t–norm on $\mathcal{L}$ and $\mathcal{M}$ is an $\mathcal{L}$-fuzzy set on $X^2 \times\,]0, +\infty[$ satisfying the following conditions for every x, y, z in X and t, s in $]0, +\infty[$:

(a) $\mathcal{M}(x,y,t) >_L 0_{\mathcal{L}}$;
(b) $\mathcal{M}(x,y,t) = 1_{\mathcal{L}}$ for all $t > 0$ if and only if $x = y$;
(c) $\mathcal{M}(x,y,t) = \mathcal{M}(y,x,t)$;
(d) $\mathcal{T}(\mathcal{M}(x,y,t), \mathcal{M}(y,z,s)) \leq_L \mathcal{M}(x,z,t+s)$;
(e) $\mathcal{M}(x,y,\cdot) :]0, \infty[\to L$ is continuous.

In this case $\mathcal{M}$ is called an *$\mathcal{L}$-fuzzy metric.* If $\mathcal{M} = \mathcal{M}_{M,N}$ is an intuitionistic fuzzy set (see Definition 1.3) then the 3-tuple $(X, \mathcal{M}_{M,N}, \mathcal{T})$ is said to be an *intuitionistic fuzzy metric space.*

Let $(X, \mathcal{M}, \mathcal{T})$ be an $\mathcal{L}$-fuzzy metric space. For $t \in]0, +\infty[$, we define the *open ball* $B(x, r, t)$ with center $x \in X$ and radius $r \in L \setminus \{0_{\mathcal{L}}, 1_{\mathcal{L}}\}$, as

$$B(x, r, t) = \{y \in X : \mathcal{M}(x, y, t) >_L \mathcal{N}(r)\}.$$

A subset $A \subseteq X$ is called *open* if for each $x \in A$, there exist $t > 0$ and $r \in L \setminus \{0_{\mathcal{L}}, 1_{\mathcal{L}}\}$ such that $B(x, r, t) \subseteq A$. Let $\tau_{\mathcal{M}}$ denote the family of all open subsets of X. Then $\tau_{\mathcal{M}}$ is called the *topology induced by the $\mathcal{L}$-fuzzy metric $\mathcal{M}$*. A subset A of X is said to be *$\mathcal{L}F$-bounded* if there exist $t > 0$ and $r \in L \setminus \{0_{\mathcal{L}}, 1_{\mathcal{L}}\}$ such that $\mathcal{M}(x, y, t) >_L \mathcal{N}(r)$ for each $x, y \in A$.

Example 1.8. ([12]) Let (X, d) be a metric space. Denote $\mathcal{T}(a, b) = (a_1 b_1, \min(a_2 + b_2, 1))$ for all $a = (a_1, a_2)$ and $b = (b_1, b_2)$ in L^* and let M and N be fuzzy sets on $X^2 \times (0, \infty)$ defined as follows:

$$\mathcal{M}_{M,N}(x, y, t) = (M(x, y, t), N(x, y, t)) = \left(\frac{t}{t + md(x, y)}, \frac{d(x, y)}{t + d(x, y)}\right),$$

in which $m > 1$. Then $(X, \mathcal{M}_{M,N}, \mathcal{T})$ is an intuitionistic fuzzy metric space.

Example 1.9. Let $X = \mathbf{N}$. Define $\mathcal{T}(a, b) = (\max(0, a_1 + b_1 - 1), a_2 + b_2 - a_2 b_2)$ for all $a = (a_1, a_2)$ and $b = (b_1, b_2)$ in L^* and let M and N be fuzzy sets on $X^2 \times (0, \infty)$ defined as follows:

$$\mathcal{M}_{M,N}(x, y, t) = (M(x, y, t), N(x, y, t)) = \begin{cases} (\frac{x}{y}, \frac{y-x}{y}) & if \quad x \leq y \\ (\frac{y}{x}, \frac{x-y}{x}) & if \quad y \leq x. \end{cases}$$

for all $x, y \in X$ and $t > 0$. Then $(X, \mathcal{M}_{M,N}, \mathcal{T})$ is an intuitionistic fuzzy metric space.

Definition 1.10. A sequence $\{x_n\}_{n \in \mathbf{N}}$ in an $\mathcal{L}$-fuzzy metric space $(X, \mathcal{M}, \mathcal{T})$ is called a *Cauchy sequence*, if for each $\varepsilon \in L \setminus \{0_{\mathcal{L}}\}$ and $t > 0$, there exists $n_0 \in \mathbf{N}$ such that for all $m \geq n \geq n_0$ ($n \geq m \geq n_0$),

$$\mathcal{M}(x_m, x_n, t) >_L \mathcal{N}(\varepsilon).$$

The sequence $\{x_n\}_{n \in \mathbf{N}}$ is said to be *convergent* to $x \in X$ in the $\mathcal{L}$-fuzzy metric space $(X, \mathcal{M}, \mathcal{T})$ (denoted by $x_n \xrightarrow{\mathcal{M}} x$) if $\mathcal{M}(x_n, x, t) = \mathcal{M}(x, x_n, t) \to 1_{\mathcal{L}}$ whenever $n \to +\infty$ for every $t > 0$.

Definition 1.11. Let $(X, \mathcal{M}, \mathcal{T})$ be an $\mathcal{L}$-fuzzy metric space. $\mathcal{M}$ is said to be continuous on $X \times X \times]0, \infty[$ if

$$\lim_{n \to \infty} \mathcal{M}(x_n, y_n, t_n) = \mathcal{M}(x, y, t)$$

whenever a sequence $\{(x_n, y_n, t_n)\}$ in $X \times X \times]0, \infty[$ converges to a point $(x, y, t) \in X \times X \times]0, \infty[$ i.e., $\lim_n \mathcal{M}(x_n, x, t) = \lim_n \mathcal{M}(y_n, y, t) = 1_{\mathcal{L}}$ and $\lim_n \mathcal{M}(x, y, t_n) = \mathcal{M}(x, y, t)$.

Lemma 1.12. *Let $(X, \mathcal{M}, \mathcal{T})$ be an $\mathcal{L}$-fuzzy metric space. Then $\mathcal{M}$ is continuous function on $X \times X \times]0, \infty[$.*

Proof. The proof is same as fuzzy metric spaces (see Proposition 1 of [9]). □

2. Main Results

Definition 2.1. Let $(X, \mathcal{M}, \mathcal{T})$ be an $\mathcal{L}$-fuzzy metric space and $A, B \subset X$. We define

$$\mathcal{M}(A, B, t) = \sup\{\mathcal{M}(a, b, t) : a \in A \text{ and } b \in B\}.$$

For $a \in X$, we write $\mathcal{M}(a, B, t)$ instead of $\mathcal{M}(\{a\}, B, t)$.

Definition 2.2. A sequence converges sub-sequentially if it has a convergent subsequence; the notation $x_n \gg x_{n'} \to x_0$ identifies the subsequence and the point to which it converges. Recall that a subset C of an $\mathcal{L}$-fuzzy metric space is compact if every sequence in C converges sub-sequentially to an element of C. Also, given sequences $x_n; y_n$, and a subsequence $x_{n'}$ of the first sequence, the corresponding subsequence of the second is denoted $y_{n'}$. A subset of a $\mathcal{L}$-fuzzy metric space is *$\mathcal{L}F$-boundedly compact* if every $\mathcal{L}F$-bounded sequence in the subset is sub-sequentially convergent. In the above notation, Y is $\mathcal{L}F$-boundedly compact if for any $\mathcal{L}F$-bounded sequence y_n in Y , there is a point x_0 (not necessarily in Y) for which $y_n \gg y_{n'} \to x_0$.

Definition 2.3. For an $\mathcal{L}$-fuzzy metric space $(X, \mathcal{M}, \mathcal{T})$ and nonempty subsets B and C, a sequence $b_n \in B$ is said to *converge in distance* to C if

$$\lim_{n\to\infty} \mathcal{M}(b_n, C, t) = \mathcal{M}(B, C, t).$$

The subset B is *approximatively compact* relative to C if every sequence $b_n \in B$ which converges in distance to C is sub-sequentially convergent to an element of B. We call B a subset of X approximatively compact if B is approximatively compact relative to each of the singletons of X; B is proximinal if for every $x \in X$ some element b in B satisfies the equation $\mathcal{M}(x, b, t) = \mathcal{M}(x, B, t)$.

The first result says that points can be replaced by compact subsets in the definition of approximative compactness.

Theorem 2.4. *Let B and C be nonempty subsets of a $\mathcal{L}$-fuzzy metric space $(X, \mathcal{M}, \mathcal{T})$. If B is approximatively compact and C is compact, then B is approximatively compact relative to C.*

Proof. Let $b_n \in B$ be any sequence converging in distance to C and let the sequence $c_n \in C$ satisfy

$$\lim_{n\to\infty} \mathcal{M}(b_n, c_n, t) \;=\; \mathcal{M}(B, C, t). \tag{2.1}$$

Since C is compact, $c_n \gg c_{n'} \to c_0 \in C$. Hence, for every $\epsilon \in L \setminus \{0_{\mathcal{L}}\}$ there exists n_0 such that for $n' > n_0$

$$\begin{aligned}\mathcal{M}(B, C, t) &\geq_L \mathcal{M}(b_{n'}, c_0, t)\\ &\geq_L \mathcal{T}(\mathcal{M}(b_{n'}, c_{n'}, t-\delta), \mathcal{M}(c_{n'}, c_0, \delta))\\ &\geq_L \mathcal{T}(\mathcal{M}(B, C, t-\delta), \mathcal{N}(\epsilon))\end{aligned}$$

for $\delta \in (0, t)$. Since $\epsilon \in L\backslash\{0_{\mathcal{L}}\}$ and δ were arbitrary, then $\lim_{n\to\infty} \mathcal{M}(b_{n'}, c_0, t) = \mathcal{M}(B, C, t)$. Therefore, $b_{n'}$ converges in distance to c_0 so, since B is approximatively compact, $b_n \gg b_{n'} \to b_0 \in B$, that is, b_n converges sub-sequentially to an element of B. □

Theorem 2.5. *Let B and C be nonempty subsets of a $\mathcal{L}$-fuzzy metric space $(X, \mathcal{M}, \mathcal{T})$. If B is approximatively compact and $\mathcal{L}F$-bounded, and C is IF-boundedly compact, then B is approximatively compact relative to C.*

Proof. Let $b_n \in B$ be any sequence converging in distance to C and let $c_n \in C$ satisfy (2.1). As b_n is $\mathcal{L}F$-bounded, so is c_n. Since C is $\mathcal{L}F$-boundedly compact, $c_n \gg c_{n'} \to c_0 \in X$. Now proceed as in the proof of last theorem. □

Theorem 2.6. ([7]) *Let B and C be nonempty subsets of a $\mathcal{L}$-fuzzy metric space $(X, \mathcal{M}, \mathcal{T})$. If B is closed and $\mathcal{L}F$-boundedly compact and C is $\mathcal{L}F$-bounded, then B is approximatively compact relative to C.*

Lemma 2.7. ([6, 10]) *Let $(X, \mathcal{M}, \mathcal{T})$ and $(Y, \mathcal{M}, \mathcal{T})$ be $\mathcal{L}$-fuzzy metric spaces. If we define*

$$\mathbf{M}((x, y), (x', y'), t) = \mathcal{T}(\mathcal{M}(x, x', t), \mathcal{M}(y, y', t)),$$

then $(X \times Y, \mathbf{M}, \mathcal{T})$ is a $\mathcal{L}$-fuzzy metric space and the topology induced on $X \times Y$ is the product topology.

Theorem 2.8. *Let S and P be nonempty subsets of $\mathcal{L}$-fuzzy metric spaces $(X, \mathcal{M}, \mathcal{T}_N)$ and $(Y, \mathcal{M}, \mathcal{T}_N)$, respectively. Suppose that P is compact. If S is $\mathcal{L}F$-boundedly compact or approximatively compact, then so is $S \times P$.*

Proof. If S is $\mathcal{L}F$-boundedly compact, we show that any sequence (s_n, p_n) in $S \times P$ which is $\mathcal{L}F$-bounded has a convergent subsequence. Indeed, by definition of the product $\mathcal{L}$-fuzzy metric, s_n is $\mathcal{L}F$-bounded and since S is $\mathcal{L}F$-boundedly compact, $s_n \gg s_{n'} \to s_0 \in X$. By compactness of P, $p'_n \gg p_{n''} \to p_0 \in P$. Hence, $(s_n, p_n) \gg (s_{n''}, p_{n''}) \to (s_0, p_0) \in X \times Y$.
If S is approximatively compact, let (x, y) be any element in $X \times Y$ and suppose that (s_n, p_n) is a sequence in $S \times P$ which converges in distance to (x, y), that is,

$$\lim_{n \to \infty} \mathbf{M}((s_n, p_n), (x, y), t) = \mathbf{M}(S \times P, (x, y), t).$$

By compactness of P, $p_n \gg p_{n'} \to p_0 \in P$. Hence, $\lim_{n \to \infty} \mathbf{M}((s_{n'}, p_0), (x, y), t) = \mathbf{M}(S \times P, (x, y), t)$ so

$$\lim_{n' \to \infty} \mathcal{T}_N(\mathcal{M}(s_{n'}, x, t), \mathcal{M}(p_0, y, t)) = \mathcal{T}_N(\mathcal{M}(S, x, t), \mathcal{M}(P, y, t)).$$

Since $\mathcal{M}(p_0, y, t) \leq_L \mathcal{M}(P, y, t)$ then $\lim_{n' \to \infty} \mathcal{M}(s_{n'}, x, t) \geq_L \mathcal{M}(S, x, t)$ which implies $\lim_{n' \to \infty} \mathcal{M}(s_{n'}, x, t) = \mathcal{M}(S, x, t)$. Hence, $s_{n'}$ converges in distance to x and since S is approximatively compact, $s_{n'} \gg s_{n''} \to s_0 \in S$. Therefore, $(s_n, p_n) \gg (s_{n''}, p_{n''}) \to (s_0, p_0) \in S \times P$, i.e., $S \times P$ is approximatively compact. □

Theorem 2.9. *Let B and C be nonempty subsets of a $\mathcal{L}$-fuzzy metric space $(X, \mathcal{M}, \mathcal{T})$. If B is approximatively compact and C is compact, then $K = \{b \in B : \exists\, c \in C,\ \mathcal{M}(b, c, t) = \mathcal{M}(B, c, t)\}$ is compact.*

Proof. Let (y_n) be a sequence in K and for every $n \in \mathbf{N}$ choose c_n in C so that y_n minimizes the distance from B to c_n. Since C is compact, $c_n \gg c_{n'} \to c_0 \in C$. Hence, for every $\epsilon \in L \setminus \{0_{\mathcal{L}}\}$, there exists n_0 such that for all $n' > n_0$, $\mathcal{M}(c_{n'}, c_0, t) \geq_L \mathcal{N}(\epsilon)$, therefore, for all $n' > n_0$,

$$\begin{aligned} \mathcal{M}(B, c_0, t) &\geq_L \mathcal{M}(y_{n'}, c_0, t) \\ &\geq_L \mathcal{T}^2(\mathcal{M}(B, c_0, t - 2\delta), \mathcal{M}(c_{n'}, c_0, \delta), \mathcal{M}_{M,N}(c_{n'}, c_0, \delta)) \\ &\geq_L \mathcal{T}^2(\mathcal{M}(B, c_0, t - 2\delta), \mathcal{N}(\epsilon), \mathcal{N}(\epsilon)) \end{aligned}$$

for every $\delta \in (0, t/2)$. Since $\epsilon \in L \setminus \{0_{\mathcal{L}}\}$ and $\delta \in (0, t/2)$ were arbitrary, then $\mathcal{M}(B, c_0, t) = \lim_{n' \to \infty} \mathcal{M}(y_{n'}, c_0, t)$. Therefore, $y_{n'}$ converges in distance to c_0, so it converges sub-sequentially. □

It follows that $\{b \in B : \mathcal{M}(b, C, t) = \mathcal{M}(B, c, t)\}$ is compact when C is compact and B is approximatively compact. Thus, in an $\mathcal{L}$-fuzzy metric space, the $\mathcal{L}$-fuzzy metric projection of a compact subset into an approximatively compact subset is compact.

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