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EXISTENCES AND BOUNDARY BEHAVIOR OF BOUNDARY BLOW-UP SOLUTIONS TO QUASILINEAR ELLIPTIC SYSTEMS WITH SINGULAR WEIGHTS

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ABSTRACT. Using the method of explosive sub and supper solution, the existence and boundary behavior of positive boundary blow up solutions for some quasilinear elliptic systems with singular weight function are obtained under more extensive conditions.

1. INTRODUCTION AND MAIN RESULTS

In this paper, we consider the existence and asymptotic behavior of positive solution to the following elliptic system

$$\begin{cases} \operatorname{div}(|\nabla u|^{m-2}\nabla u) = a(x)u^p v^q, & x \in \Omega, \\ \operatorname{div}(|\nabla v|^{n-2}\nabla v) = b(x)u^r v^s, & x \in \Omega, \end{cases} \tag{1.1}$$

subject to the boundary conditions

$$u = v = \infty, \quad x \in \partial\Omega. \tag{1.2}$$

where $p > m-1, s > n-1$, $q, r > 0$, $m, n > 1$, $(p-m+1)(s-n+1) - qr > 0$, Ω is bounded C^2 domain of $R^N, N \geq 1$, and the last condition (1.2) $u = v = \infty, x \in \partial\Omega$ means that $u \to \infty, v \to \infty$ as $d(x) := \operatorname{dist}(x, \partial\Omega) \to 0$, and the solution is called a large solutions or boundary blow-up solution. By a positive boundary blow-up solutions of (1.1), we mean that $(u, v) \in W^{1,p}_{\text{loc}}(\Omega) \bigcap C^1_{\text{loc}}(\Omega)$ and (u, v) satisfies

$$-\int_\Omega |\nabla u|^{m-2}\nabla u \nabla\varphi dx = \int_\Omega a(x)u^p v^q \varphi dx, \ \forall \varphi \in C_0^\infty(\Omega).$$

$$-\int_\Omega |\nabla v|^{n-2}\nabla v \nabla\varphi dx = \int_\Omega b(x)u^r v^s \varphi dx, \ \forall \varphi \in C_0^\infty(\Omega).$$

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and $u, v > 0$ in Ω, $u \to \infty, v \to \infty$ as $d(x) \to 0$,

The study of the elliptic systems is a classical topic that has attracted the attention of many researchers because of its interest in applications, which arises in the theory of quasi-regular and quasi-conformal mappings as well as in the study of non-Newtonian fluids, in non-Newtonian fluids, the pair (m, n) is a characteristic of the medium. Media with $(m, n) > (2, 2)$ are called dilatant fluids and those with $(m, n) < (2, 2)$ are called pseudoplastics. If $(m, n) = (2, 2)$, they are Newtonian fluids.

There is a large amount of literature on elliptic problems related to problem

$$\Delta u = b(x) f(u), x \in \Omega, \; x|_{\partial\Omega} = \infty. \tag{1.3}$$

for $b(x) = 1, f(u) = e^u$, the problem (1.3) was initiated by Bieberbach [1] for $\Omega \subset R^2$. Rademacher[26] extended the results of Bieberbach to $\Omega \subset R^3$. Later, Lazer and McKenna [21] generalized the results to the case of bounded domains in R^N and nonlinearities $b(x)e^u$. For $b \in C^{\alpha}_{loc}(\Omega), b > 0$ in Ω, and provided that b satisfies the following assumption: there exist constants $C_1, C_2 > 0$, $\kappa_2 \geq \kappa_1 > -2$ such that

$$C_2(d(x))^{\kappa_2} \leq b(x) \leq C_1(d(x))^{\kappa_1},$$

and $f(u)$ satisfies: $f \in C^1(R)$ is non-decreasing on R, $f(s) \leq C_1 e^{p_1 s}$ for all $s \in R$ and $f(s) \geq C_2 e^{p_2 s}$ for large $|s|$ with $p_1 \geq p_2 > 0$, C_1, C_2 are positive constants, García-Melián[13] showed that problem (1.3) has at least one solution $u \in C^2(\Omega)$such that

$$-m - (2 + \gamma_1)/p_1 \ln d(x)) \leq u(x) \leq M - (2 + \gamma_2)/p_2 \ln d(x), \quad \forall x \in \Omega.$$

where m, M are positive constants. Very recently, Zhang [33] and Yang[23] extended the above results to the problem (1.3) and gained some new results with nonlinear gradient terms. Problem (1.3) was discussed in a number of works; see,[2, 3, 4, 5, 9, 10, 11, 12, 13, 19, 23, 25, 34],

Now let us return to problem (1.1).

When $m = n = 2$, system (1.1) becomes

$$\begin{cases} \Delta u = a(x) u^p v^q, & x \in \Omega, \\ \Delta v = b(x) u^r v^s, & x \in \Omega, \end{cases} \tag{1.4}$$

in the paper [14], when $a(x) = 1, b(x) = 1$, under Dirichlet boundary conditions of three different types: both components of (u, v) are bounded on $\partial\Omega$ (finite case); one of them is bounded while the other blows up(semilinear case); or both components blow up simultaneously(infinite case), under the assumption that$(a-1)(e-1) > bc$, necessary and suffcient conditions for existence of positive solutions were found, and uniqueness or multiplicity were also obtained, together with the exact boundary behavior of solutions. In addition, they also treated some existence uniqueness and boundary behavior of solutions of systems (1.4) under the assumption

$$a(x) \sim C_1 d(x)^{\kappa_1}, \; b(x) \sim C_2 d(x)^{\kappa_2}, \tag{1.5}$$

when $d(x) \to 0$ for some positive constants C_1, C_2 and real numbers $\kappa_1, \kappa_2 > -2$. Problem(1.4) was later studied in [15] with general form

$$C_1 d(x)^{\kappa_1} \le a(x) \le C_2 d(x)^{\kappa_1},\ C_3 d(x)^{\kappa_2} \le b(x) \le C_4 d(x)^{\kappa_2},$$

for $x \in \Omega$, where $a(x), b(x) \in C^\theta(\Omega)$ for some $\theta \in (0,1)$, $\kappa_1, \kappa_2 > -2$, and $C_i, i = 1,2,3,4$, are positive constants. If the weights $a(x)$ and $b(x)$ satisfy the following two hypotheses:

(I) $a(x) \in C^\eta(\Omega), b(x) \in C^\eta(\Omega), \eta \in (0,1), a(x) > 0, b(x) > 0$;

(II) there exist constants $C_i > 0, i = 1,2,3,4$ and $\kappa_1 \ge \kappa_2 > -m, \kappa_3 \ge \kappa_4 > -n$ such that, for $x \in \Omega$,

$$C_1 d(x)^{\kappa_1} \le a(x) \le C_2 d(x)^{\kappa_2},\ C_3 d(x)^{\kappa_3} \le b(x) \le C_4 d(x)^{\kappa_4} \quad x \in \Omega,$$

Let us mention that under the hypothesis (I) and (II), the weight functions $a(x)$ and $b(x)$ may be singular near the boundary $\partial\Omega$. Huang [20] showed that problem (1.4) has unique large solution if and only if $\kappa_i \in R, \kappa_1 \ge \kappa_2 > -m, \kappa_3 \ge \kappa_4 > -n$ and

$$\frac{q}{s-n+1} < \frac{m+\kappa_1}{n+\kappa_4},\ \frac{m+\kappa_2}{n+\kappa_3} < \frac{p-m+1}{r}.$$

the solution verifies

$$D_1 d(x)^{-\alpha_1} \le u(x) \le D_2 d(x)^{-\alpha_2},\ D_3 d(x)^{-\beta_1} \le v(x) \le D_4 d(x)^{-\beta_2}.$$

where $D_i (i = 1,2,3,4)$ are positive constants , and

$$\alpha_1 = \frac{(m+\kappa_2)(s-n+1) - (n+\kappa_3)q}{(p-m+1)(s-n-1) - qr},\ \alpha_2 = \frac{(m+\kappa_1)(s-1) - (n+\kappa_4)q}{(p-m+1)(s-n-1) - qr},$$

$$\beta_1 = \frac{(m+\kappa_3)(p-m+1) - (n+\kappa_2)r}{(p-m+1)(s-n-1) - qr},\ \beta_1 = \frac{(n+\kappa_4)(p-m+1) - (m+\kappa_1)r}{(p-m+1)(s-n-1) - qr}.$$

Recently, Yang [31] studied the systems (1.1) and showed that if $a(x) = b(x) = 1$, $p > m-1$, $s > n-1$, $q, r > 0$, $m, n > 1$, $(p-m+1)(s-n+1) - qr > 0$, then systems (1.1) have boundary blow up solutions, and there exist constants A, B such that

$$Ad(x)^{-\alpha} \le u(x) \le Bd(x)^{-\alpha},\ Ad(x)^{-\beta} \le v(x) \le Bd(x)^{-\beta}.$$

where,

$$\alpha = \frac{m(s-q-n+1)}{(p-m+1)(s-n+1) - qr},\ \beta = \frac{n(p-r-m+1)}{(p-m+1)(s-n+1) - qr}.$$

Furthermore, they also obtained the existence and boundary behavior of solutions if $a(x), b(x)$ satisfies $a(x) \sim C_1 d(x)^{\kappa_1}, b(x) \sim C_2 d(x)^{\kappa_2}$, $\kappa_1 > -m, \kappa_2 > -n$.

More results to system with boundary blow up, we refer reader to [6, 7, 8, 16, 17, 18, 22, 24, 28, 29, 31, 32] and references therein.

The main purpose of the present paper is to investigate the influence of the weights $a(x)$ and $b(x)$ on the existence and boundary behavior of solutions of systems (1.1).

The main results of the present paper are the following.

Theorem 1.1. *Assume that $a(x), b(x)$ satisfy (I) and (II), $p > m-1, s > n-1$, $m > 1, n > 1$, $(p-m+1)(s-n+1) - qr > 0$, Then systems (1.1) has at least a positive solution (u, v) if and only if $\kappa_i \in R, \kappa_1 \geq \kappa_2 > -m, \kappa_3 \geq \kappa_4 > -n$ and*

$$\frac{q}{s-n+1} < \frac{m+\kappa_1}{n+\kappa_4}, \ \frac{m+\kappa_2}{n+\kappa_3} < \frac{p-m+1}{r}. \tag{1.6}$$

This solution verifies

$$D_1 d(x)^{-\alpha_1} \leq u(x) \leq D_2 d(x)^{-\alpha_2}, \ D_3 d(x)^{-\beta_1} \geq v(x) \geq D_4 d(x)^{-\beta_2}. \tag{1.7}$$

where $D_i (i = 1, 2, 3, 4)$ are positive constants, and

$$\alpha_1 = \frac{(m+\kappa_2)(s-n+1) - (n+\kappa_3)q}{(p-m+1)(s-n-1) - qr}, \ \alpha_2 = \frac{(m+\kappa_1)(s-n+1) - (n+\kappa_4)q}{(p-m+1)(s-n-1) - qr}, \tag{1.8}$$

$$\beta_1 = \frac{(m+\kappa_3)(p-m+1) - (n+\kappa_2)r}{(p-m+1)(s-n-1) - qr}, \ \beta_2 = \frac{(n+\kappa_4)(p-m+1) - (m+\kappa_1)r}{(p-m+1)(s-n-1) - qr}. \tag{1.9}$$

As a straight forward consequence, we obtain

Corollary 1.2. *If $d(x) \to 0$, $a(x) \sim C_1 d(x)^{\kappa_1}$, $b(x) \sim C_2 d(x)^{\kappa_2}$ then systems (1.1) have at least a positive solution (u, v) if and only if $\kappa_i \in R, \kappa_1 > -m, \kappa_2 > -n$ and*

$$\frac{q}{s-n+1} < \frac{m+\kappa_1}{n+\kappa_2} < \frac{p-m+1}{r}.$$

This solution verifies

$$D_1 d(x)^{-\alpha} \leq u(x) \leq D_2 d(x)^{-\alpha}, \ D_3 d(x)^{-\beta} \leq v(x) \leq D_4 d(x)^{-\beta}.$$

where $D_i (i = 1, 2, 3, 4)$ are positive constants, and

$$\alpha = \frac{(m+\kappa_1)(s-n+1) - (n+\kappa_2)q}{(p-m+1)(s-n-1) - qr}, \ \beta = \frac{(m+\kappa_2)(p-m+1) - (n+\kappa_1)r}{(p-m+1)(s-n-1) - qr}.$$

2. Proof of Theorem 1.1

We now ready to prove Theorem 1.1, whose proof will be split in several lemma. we begin by showing the definitions of blow up supper and subsolutions to systems (1.1).

Definition 2.1. $(\overline{u}, \overline{v}) \in (C^2(\Omega))^2$, *is called blow up upper solution of systems (1.1), provided that*

$$\begin{cases} div(|\nabla \overline{u}|^{m-2} \nabla \overline{u}) \leq a(x) \overline{u}^p \overline{v}^q, & x \in \Omega, \\ div(|\nabla \overline{v}|^{n-2} \nabla \overline{v}) \geq b(x) \overline{u}^r \overline{v}^s, & x \in \Omega, \\ \overline{u} = \overline{v} = \infty, & x \in \partial\Omega, \end{cases}$$

As always, a blow up subsolution $(\underline{u}, \underline{v})$ is defined by reversing the inequalities.

We now recall some already know results which will be used in the proof of Theorem 1.1. Consider

$$\operatorname{div}(|\nabla u|^{m-2} \nabla u) = d(x)^{\kappa} u^p, x \in \Omega, u = \infty, x \in \partial\Omega, \tag{2.1}$$

Here $d(x)$ stands for the distance of a point $x \in \Omega$ to the boundary $\partial\Omega$. This problem has been recently considered in [11], where all issues concerning existence, uniqueness and asymptotic behavior near the boundary of positive solutions were obtained. The following Lemma 2.1 contains the basic feature of problem (2.1), refer the reader to [11] for a proof.

Lemma 2.2. *Assume that $p > m-1$ and $\kappa > -m$, then problem (2.1) has a unique positive solution, which will be denoted by $U_{m,p,\kappa}$. Moreover,*

$$D_1 d(x)^{-\alpha} \le U_{m,p,\kappa} \le D_2 d(x)^{-\alpha}, \tag{2.2}$$

where $D_1, D_1 > 0$, $\alpha = (m+\kappa)/(p-m+1)$.

Lemma 2.3. *Assume that $(\overline{u}, \overline{v}) \in (C^2(\Omega))^2$, $(\underline{u}, \underline{v}) \in (C^2(\Omega))^2$ are upper solution and subsolution of systems*

$$\begin{cases} div(|\nabla u|^{m-2}\nabla u) = a(x)u^p v^q, & x \in \Omega, \\ div(|\nabla v|^{n-2}\nabla v) = b(x)u^r v^s, & x \in \Omega, \\ u = f(x), v = g(x), & x \in \partial\Omega, \end{cases} \tag{2.3}$$

and $\underline{u} \le f(x) \le \overline{u}$, $\underline{v} \ge g(x) \ge \overline{v}, x \in \partial\Omega$, $\underline{u} \le \overline{u}$, $\underline{v} \ge \overline{v}, x \in \Omega$, here, $f, g \in C^\eta(\Omega)(\eta \in (0,1))$. Then systems (2.3) has at least a solution (u, v), and $\underline{u} \le u \le \overline{u}$, $\underline{v} \ge v \ge \overline{v}, x \in \Omega$, in particular, $u = f(x), v = g(x), x \in \partial\Omega$.

Proof. Let u_1 (the existence and uniqueness of u_1 see Remark 3 of [11]) is the unique positive solution of

$$\operatorname{div}(|\nabla u|^{m-2}\nabla u) = a(x)\underline{v}^q u^p, \quad x \in \Omega, u = f(x), \quad x \in \partial\Omega, \tag{2.4}$$

Clearly, $\overline{u}$ and $\underline{u}$ are the upper solution and subsolution of (2.4), thanks to uniqueness of u_1 , we have $\overline{u} \le u_1 \le \underline{u}$. now assume that v_1 is the unique positive solution of

$$\operatorname{div}(|\nabla u|^{n-2}\nabla u) = b(x)u_1^r v^s, x \in \Omega, v = g(x), x \in \partial\Omega, \tag{2.5}$$

It following similarly that $\overline{v} \ge v_1 \ge \underline{v}$. We can continue in this way by defining u_n to be the unique solution to (2.4) whit v replaced by v_{n-1} and v_n the unique solution to (2.5) with u_1 replaced by u_n. We obtain unique positive solution sequences $\{u_n\}, \{v_n\}$, such that $\underline{u} \le u_n \le \overline{u}$, $\underline{v} \ge v_n \ge \overline{v}$. Moreover, u_n is increasing and $v_n$is decreasing. From standard regularity and compactly embedding theory, it following that there exist subsequence $\{u_{n_k}\}, \{v_{n_k}\}$, such that $u_{n_k} \to u(x), v_{n_k} \to v(x)$, $x \in C^\eta(\overline{\Omega}) \bigcap C^1_{loc}(\Omega)$, where $\{u(x), v(x)\}$ is the solution of (2.3), moreover, $u(x) = f(x), v(x) = g(x)$ and $\underline{u} \le u \le \overline{u}$, $\overline{v} \ge v \ge \underline{v}$. □

Lemma 2.4. *Set $(\overline{u}, \overline{v})$ and $(\underline{u}, \underline{v})$ are blow up upper solution and subsolution of (1.1) with $\underline{u} = \overline{u} = \underline{v} = \overline{v} = \infty, x \in \partial\Omega$ and $\underline{u} \le \overline{u}$, $\underline{v} \ge \overline{v}, x \in \Omega$. Then systems (1.1)+(1.2) have at least positive solution (u, v) such that $\underline{u} \le u \le \overline{u}$, $\underline{v} \ge v \ge \overline{v}, x \in \Omega$, in particular, $u = v = \infty, x \in \partial\Omega$.*

Proof. Set $\Omega_\delta = \{x \in \Omega : d(x) > \delta\}$, where $\delta > 0$, $d(x) := dist(x, \partial\Omega)$, and consider the problem

$$\begin{cases} \operatorname{div}(|\nabla u|^{m-2}\nabla u) = a(x)u^p v^q, & x \in \Omega_\delta, \\ \operatorname{div}(|\nabla v|^{n-2}\nabla v) = b(x)u^r v^s, & x \in \Omega_\delta, \\ u = f_\delta, v = g_\delta, & x \in \partial\Omega_\delta, \end{cases} \tag{2.6}$$

where f_δ, g_δ are are smooth functions defined on $\partial\Omega_\delta$ with $\underline{u} \le f_\delta \le \overline{u}$, $\underline{v} \ge g_\delta \ge \overline{v}, x \in \Omega$, in view of Lemma 2.2 and standard regularity theory, it following that systems (2.6) have a bounded solution (u_δ, v_δ) in $C^{1,\eta}_{\mathrm{loc}}(\Omega)$, so that we obtain subsequence $(u_{\delta_k}, v_{\delta_k}) \to (u, v), x \in C^1_{loc}(\Omega)$, where (u, v) is a positive solution of (1.1) with $\underline{u} \le u \le \overline{u}$, $\underline{v} \ge v \ge \overline{v}, x \in \Omega$ and $u = v = \infty, x \in \partial\Omega$. □

Lemma 2.5. *Assume $\kappa_i \in R, \kappa_1 \ge \kappa_2 > -m, \kappa_3 \ge \kappa_4 > -n$ and (1.6) holds, Then system (1.1)+(1.2) admits at least one positive solution.*

Proof. We use the method of blow up sub and super solution. Let $(\underline{u}, \underline{v}) = (\varepsilon U_{m,p,\tau_1}, \varepsilon^{-\delta} U_{n,s,\delta_1})$, where the functions $U_{m,p,\tau_1}, U_{n,s,\delta_1}$ are as introduced above, ε is small enough, τ_1, δ_1, δ are to be chosen such that $(\underline{u}, \underline{v})$ is the blow up sub solution of system (1.1).

Combining with (2.1), (2.2) and the definition of blow up sub and super solution, if we select

$$\tau_1 = (p - m + 1)\alpha_1 - m > -m,\ \delta_1 = (s - n + 1)\beta_1 - n > -n,$$
$$r/(s - n + 1) < \delta < (p - m + 1)/q.$$

α_1, β_1 are gave by (1.8) and (1.9). A simple calculation show that

$$\begin{aligned} &\mathrm{div}\left(|\nabla(\varepsilon U_{m,p,\tau_1})|^{m-2}\nabla(\varepsilon U_{m,p,\tau_1})\right) = \varepsilon^{m-1}\mathrm{div}\left(|\nabla U_{m,p,\tau_1}|^{m-2}\nabla U_{m,p,\tau_1}\right)\\ = \ &\varepsilon^{m-1} d(x)^{\tau_1} U^p_{m,p,\tau_1} \ge C_2 d(x)^{\kappa_2}\varepsilon^{p-q\delta} U^p_{m,p,\tau_1} U^q_{n,s,\delta_1},\\ &\mathrm{div}\left(|\nabla(\varepsilon^{-\delta} U_{n,s,\delta_1})|^{n-2}\nabla(\varepsilon^{-\delta} U_{n,s,\delta_1})\right) = \varepsilon^{-\delta(n-1)}\mathrm{div}\left(|\nabla U_{n,s,\delta_1}|^{n-2}\nabla U_{n,s,\delta_1}\right)\\ = \ &\varepsilon^{-\delta(n-1)} d(x)^{\delta_1} U^s_{n,s,\delta_1} \le C_3 d(x)^{\kappa_3}\varepsilon^{r-s\delta} U^r_{m,p,\tau_1} U^s_{n,s,\delta_1},\end{aligned}$$

which leads to $(\varepsilon U_{m,p,\tau_1}, \varepsilon^{-\delta} U_{n,s,\delta_1})$ is the sub solution of system (1.1). similarity, we can select $\tau_2 = (p-m+1)\alpha_2 - m > -m$, $\delta_2 = (s-n+1)\beta_2 - n > -n$, α_2, β_2 are gave by (1.8) and (1.9), if M is large enough, then $(\overline{u}, \overline{v}) = (MU_{m,p,\tau_2}, M^{-\delta} U_{n,s,\delta_2})$ is the super solution of system (1.1).

By $\kappa_1 \ge \kappa_2 > -m, \kappa_3 \ge \kappa_4 > -n$ and the definition of α_i, β_i, obtain $\alpha_1 \le \alpha_2$, $\beta_1 \ge \beta_2$, then we get $\underline{u} \le \overline{u}$ and $\underline{v} \ge \overline{v}, x \in \Omega$, according to Lemma 2.3, there exist a positive solution (u, v) of system (1.1)+(1.2) with $\underline{u} \le u \le \overline{u}$ and $\underline{v} \ge v \ge \overline{v}, x \in \Omega$, in particular, $u = v = \infty, x \in \partial\Omega$. □

Lemma 2.6. *Let (u, v) be a positive solution to system (1.1)+(1.2). Then there exist constants $D_i (i = 1, 2, 3, 4)$ such that (1.7) holds,*

Proof. By the definition of U_{m,p,τ_i}, U_{n,s,δ_i} and (2.2), we infer that there exist positive constants $E_i, F_i, E'_i, F'_i (i = 1, 2)$ such that

$$E_i d(x)^{-\alpha_i} \le U_{m,p,\tau_i} \le E'_i d(x)^{-\alpha_i},\ F_i d(x)^{-\beta_i} \le U_{n,s,\delta_i} \le F'_i d(x)^{-\beta_i}.$$

Since $\varepsilon U_{m,p,\tau_1} = \underline{u} \le u \le \overline{u} = MU_{m,p,\tau_2}$, $\varepsilon^{-\delta} U_{n,s,\delta_1} = \underline{v} \ge v \ge \overline{v} = M^{-\delta} U_{n,s,\delta_2}, x \in \Omega$ and $\alpha_1 \le \alpha_2$, $\beta_1 \ge \beta_2$, this implies that (1.7) holds. □

We finally show that the conditions $\kappa_i \in R, \kappa_1 \ge \kappa_2 > -m, \kappa_3 \ge \kappa_4 > -n$ and (1.6) are necessary for problem (1.1)+(1.2) to have a positive solution.

Lemma 2.7. *Assume problem (1.1)+(1.2) has a positive solution (u, v). Then $\kappa_i \in R, \kappa_1 \ge \kappa_2 > -m, \kappa_3 \ge \kappa_4 > -n$ and (1.6) holds.*

Proof. If $(m+\kappa_1)/(n+\kappa_4) \leq q/(s-n+1)$, by the definition of α_2, we have $\alpha_2 \leq 0$, then (1.7)implies u is bounded. Similarity, if $(m+\kappa_2)/(n+\kappa_3) \geq (p-m+1)/r$, we also obtain v is bounded by (1.7), which are contradiction to $u = v = \infty, x \in \partial\Omega$. This finishes the proof. $\square$

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