ISAAC Conference, 23-27 April, 2007, Tbilisi, Georgia
Dedicated to the Centenary of I.Vekua

ON THE APPLICATION OF RITZ'S EXTENDED METHOD TO APPROXIMATE SOLUTION OF INVERSE ND COMPUTING TOMOGRAPHY PROBLEMS

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Let H be a Hilbert space with the inner product $(\cdot,\cdot)$, and let $K : H \to H$ be a compact, selfadjoint operator having positive eigenvalues and everywhere dense range. We continue the study of the ill-posed problem $Ku = f$ initiated in [1]. It is assumed that the existence and uniqueness conditions are satisfied, but the stability is not. This means that the inverse operator is not continuous. Similarly A.N.Tikhonov [2], the equation $Ku = f$ is transfered in the Frechet space $D(K^{-\infty}) = \bigcap_{n=1}^{\infty} D(K^{-n+1})$, the Hilbert norms of which are given by the equalities $\|f\|_n^2 = \|f\|^2 + \|K^{-1}f\|^2 + \cdots + \|K^{-n+1}f\|^2$, where $\|\cdot\|^2 = (\cdot,\cdot)$. It is well-known, that the space $D(K^{-\infty})$ is isomorphic to the subspace of the Frechet space H^N. The operator $K^{-\infty}$ is defined by the equality $K^{-\infty}(x) = \{K^{-1}x, \ldots, K^{-n}x, \ldots\}$. Let us denote the operator $(K^{-\infty})^{-1}$ by K_∞. This operator maps the Frechet space $D(K^{-\infty})$ isomorphically onto and therefore the equation $K_\infty u = f$ has in the space $D(K^{-\infty})$ a unique and stable solution. More exactly, as a set, the Frechet space $D(K^{-\infty})$ is a part of the Hilbert space H and the operator K_∞ is selfadjoint operator on Frechet space $D(K^{-\infty})$ [3]. For approximate solution of this operator equation Ritz's extended method is applied [3].

These results are also applied to operators, mapping a separable Hilbert space into the same space and admitting a singular decomposition. In particular, the well-known Radon transformation admits the singular decomposition and therefore the application of Ritz's extended method to the computing tomography problem is possible.