ISAAC Conference, 23-27 April, 2007, Tbilisi, Georgia
Dedicated to the Centenary of I.Vekua

GENERALIZATION SPATIAL PARAMETRIZATION IN THE I.N. VEKUA'S SHELL THEORY

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Let's consider an shell which points are described by radius-vectors:

$$\vec{r}(x^1,x^2,x^3) = \overset{(-)}{\vec{r}}(x^1,x^2) + x^3\left(\overset{(+)}{\vec{r}}(x^1,x^2) - \overset{(-)}{\vec{r}}(x^1,x^2)\right) \quad (1)$$

Vector relations

$$\overset{(-)}{\vec{r}} = \overset{(-)}{\vec{r}}(x^1,x^2) \quad \text{and} \quad \overset{(+)}{\vec{r}} = \overset{(+)}{\vec{r}}(x^1,x^2) \quad (2)$$

define base surfaces $\overset{(-)}{s}$ and $\overset{(+)}{s}$ parametrization (1) of shell space. A vector

$$\vec{h}(x^1,x^2) = \overset{(+)}{\vec{r}}(x^1,x^2) - \overset{(-)}{\vec{r}}(x^1,x^2) \quad (3)$$

puts in conformity of a point of base surfaces with identical Gauss coordinates (x^1,x^2).

Entered (1) parametrization of shell space can be considered as generalization of the spatial coordinate systems normally connected to a base surface, entered by I.N.Vekua. Such systems turn out from the general case when the vector $\vec{h}$ is perpendicular surfaces $\overset{(-)}{s}$ and $\overset{(+)}{s}$. Entered parametrization is especially convenient by consideration of multilayered environments when acceptance as basic obverse surfaces of layers allows continuously on Gauss coordinates to pass from a layer to a layer. From group transformations of spatial coordinates we shall allocate transformations of a kind

$$x^{\alpha'} = x^{\alpha'}(x^1,x^2), x^{3'} = x^3,$$

which we shall name the generalized S - transformations. The sizes possessing tension properties concerning such transformations of coordinates, we shall name generalized S - tensors.

The three of vectors $(\vec{r}_1,\vec{r}_2,\vec{r}_3)$ form covariant mobile base of shell space. The two of vectors $(\vec{r}_{\underset{1}{-}},\vec{r}_{\underset{2}{-}})$ and $(\vec{r}_{\underset{1}{+}},\vec{r}_{\underset{2}{+}})$ form covariant mobile basis, corresponding base surfaces.

Metric tensor of shell spaces $g_{ij} = \vec{r}_i \cdot \vec{r}_j$ it is completely expressed through the value on one of base surfaces $g_{\bar{i}\bar{j}} = \vec{r}_{\underset{i}{-}} \cdot \vec{r}_{\underset{j}{-}}$ or $g_{\underset{i\,j}{+\,+}} = \vec{r}_{\underset{i}{+}} \cdot \vec{r}_{\underset{j}{+}}$ and metric carry tensor $g_{\underset{i\,j}{+\,-}} = \vec{r}_{\underset{i}{+}} \cdot \vec{r}_{\underset{j}{-}}$. Through components of these tensors factors of the second metric form of base shell surfaces, for example are expressed:

$$b_{\underset{\alpha\beta}{-\,-}} = \sqrt{g^{\overset{-\,-}{33}}}\left(g_{\underset{\alpha 3,\beta}{-}} - g_{\underset{\beta\alpha}{+\,-}} + g_{\underset{\alpha\beta}{-\,-}}\right) + \frac{1}{2}\frac{g^{\overset{-\,-}{\gamma 3}}}{\sqrt{g^{\overset{-\,-}{33}}}}\left(g_{\underset{\alpha\gamma,\beta}{-\,-}} + g_{\underset{\beta\gamma,\alpha}{-\,-}} - g_{\underset{\alpha\beta,\gamma}{-\,-}}\right)$$

For a determinant of the metric form g we have:

$$\sqrt{g} = \vec{r}_1\vec{r}_2\vec{r}_3 = \sqrt{\bar{g}}\left((1-x^3)^2 + x^3(1-x^3)(g_{\overset{+}{1}}^{\overset{-}{1}} + g_{\overset{+}{2}}^{\overset{-}{2}}) + (x^3)^2(g_{\overset{+}{1}}^{\overset{-}{1}} g_{\overset{+}{2}}^{\overset{-}{2}} - g_{\overset{+}{2}}^{\overset{-}{1}} g_{\overset{+}{1}}^{\overset{-}{2}}) \right)$$

Here $\bar{g}$ value of a determinant on a base surface $\overset{(-)}{s}$: $\sqrt{\bar{g}} = \vec{r}_{\overset{-}{1}}\vec{r}_{\overset{-}{2}}\vec{r}_{\overset{-}{3}} = \sqrt{g}\,|_{x^3=0}$.