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ON SOME NEW REPRESENTATIONS OF ANALYTIC FUNCTIONS IN LATTICED DOMAINS

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Various problems of Mathematical Physics are reduced to the boundary value problems for analytic functions in latticed domains domains. For example the theory of hydroturbins, the 3D motion of particles in a torus , the wave propagation in the sells [1,3,4].

Consider a complex z-plane ($\mathbb{C}$), and two numbers $\omega_1, \omega_2 > 0$.

Let line L is doubly-periodic line,i.e. i it is a union of a countable number of smooth non-intersected closed contours L_{mn} , $m, n = 0, \pm 1, \dots$ doubly-periodically distributed with periods $2\omega_1$ and $2i\omega_2$ in the whole z-plane

$$L = \bigcup_{m,n=-\infty}^{\infty} L_{mn},$$

By S^+ we denote an infinite region bounded by the contour L. The positive direction on L will be taken such that S^+ remains on the left.

The union of the finite domains S^-_{mn} contained in every L_{mn},$m, n = 0, \pm 1, \dots$, respectively will be denoted by S^-. Let us introduce a new class of functions [2]:

The function $\Phi(z)$ defined in the doubly-periodic domain is called exponentially doubly-quasi periodic if the following conditions are fulfilled

1. $\Phi(z + 2\omega_1) = \Phi(z)\exp(P_{k_1}(z))$,
2. $\Phi(z + 2i\omega_2) = \Phi(z)\exp(Q_{k_2}(z))$,

where P_{k_1} and Q_{k_2} are the definite polynomials of the k_1 and k_2 orders respectively.

This class of functions we denote by $P_e(k)$ [4],$k = \max(k_1, k_2)$

In this work the following problems are considered.

Problem 1. Find a function $\Phi(z)$ of $P_e(0)$ class with the jump line L sectionally- holomorphic everywhere , except possibly at countable number of points β_{mn},$m, n = 0, \pm 1, \dots$,satisfying the following boundary condition

$$\Phi^+(t_0) - \Phi^-(t_0) = \phi(t_0),\ t_0 \in L,$$

where $\phi(t_0)$ is the given Holder continuous on L_{00} of $P_e(0)$ class.

Problem 2. Find a function $\Psi(z)$ of $P_e(1)$ class with the jump line L sectionally- holomorphic everywhere , except possibly at countable number of points β_{mn},$m,n = 0, \pm1, \dots$,satisfying the following boundary condition

$$\Psi^+(t_0) = G(t_0)\Psi^-(t_0) + g(t_0),\ t_0 \in L,$$

where $G(t_0)$,$G(t_0) \neq 0$, and $g(t_0)$ are given functions Holder continuous on L_{00} of $P_e(1)$ class.

The effective solutions of this problems are given,hence representations of sectionally-holomorphic exponentially doubly quasi-periodic functions are obtained. Using this representations the following Dirichlet problem for harmonic functions is investigated

Problem 3. To find the real function $u(x, y)$, harmonic in S^+, continuous in $S^+ + L$ and satisfying the boundary condition

$$u^+ = f(t), \quad t \in L,$$

where $f(t)$ is real continuous function of $P_e(1)$ class given on L. $u(x, y)$ is also assumed to belong to the class $P_e(1)$.

This problem is investigated by means of the theory of conformal mapping and singular integral equations.

References

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