ISAAC Conference, 23-27 April, 2007, Tbilisi, Georgia
Dedicated to the Centenary of I.Vekua

GENERAL THEOREMS IN THE THEORY OF MIXTURES WITH MEMORY

S. De Cicco and L. Nappa

Dipartimento di Costruzioni e Metodi Matematici in Architettura,
Università degli Studi di Napoli "Federico II",
Via Forno Vecchio 16, 80134 Napoli, Italy

The theory of mixtures has undergone significant developments in recent decades (see the review articles by Bedford and Drumheller (1983), Drumheller (2000) and the book by Rajagopal and Tao, (1985)). In this paper we consider the theory of viscoelastic composites modeled as interpenetrating solid continua with memory (see e.g., McCarthy and Tiersten, (1983), Iesan and Quintanilla (2002)).

We first introduce the basic equations for the mechanical behaviour of a binary mixture in the framework of the linearized dynamic theory, assuming that the constituents of the mixture are each viscoelastic bodies. An alternative characterization of the boundary-initial value problem in which the initial conditions are incorporated into the field equations is presented. Then a minimum principle, a variational theorem and an uniqueness theorem are established.

The minimum principle is obtained using the results of Reiss (1978) and Reiss and Haug (1978), but our procedure to take in account the initial data is different. Moreover, we consider a different type of boundary conditions.

The uniqueness question in the dynamic linear theory of viscoelasticity has been considered in various works (see e.g., Leitman and Fisher (1972), Edelstein and Gurtin (1964)). Uniqueness results in the theory of mixtures of elastic solids have been presented by Bowen (1976), Atkin, Chadwick and Steel (1967) and Knops and Steel (1969). In this paper we use the results established by Gurtin, McCamy and Murphy (1979) to derive an uniqueness theorem for the mixed problem. We remark that using Laplace transform techniques one can derive an uniqueness result from the minimum principle presented here, but our procedure avoids the use of Laplace transform.

Lastly we give a variational characterization of the solution. A Hu-Washizu type variational theorem is established. This result generalized the analogous theorem in the classical theory of viscoelasticity (see Leitman and Fisher (1972)).