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ON A CERTAIN SUBCLASS OF MEROMORPHIC UNIVALENT FUNCTIONS WITH FIXED SECOND POSITIVE COEFFICIENTS

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Abstract. In the present paper, we consider the subclass of meromorphic univalent functions $S_p^*[k, \alpha, \beta, c]$ with fixed second positive coefficient. The object of the present paper is to show coefficient estimates, convex liner combinations, some distortion theorems, and redii of starlikeness and convexity for $f(z)$ in the class $S_p^*[k, \alpha, \beta, c]$.

1 Introduction

For $0 < p < 1$, let S_p denote the class of functions f which are meromorphic and univalent in the unit disk $D = \{z : |z| < 1\}$ with the normalization $f(0) = 0, f'(0) = 1$ and $f(p) = \infty$.

Let $A(p)$ denote the set of function analytic in $D\backslash\{p\}$ with the topology given by uniform convergence on compact subsets of $D\backslash\{p\}$. Then $A(p)$ is locally convex linear topological space and S_p is a compact subset of $A(p)$ (cf. [17, p.55]). In the annulus $\{z : p < |z| < 1\}$ every function f in S_p has an expansion of the form

$$f(z) = \frac{\alpha}{z-p} + \sum_{n=1}^{\infty} a_n z^n, \tag{1.1}$$

where $\alpha = Res(f, p)$,with $0 < \alpha \leq 1$, $z \in D\backslash\{p\}$.

The function f given in (1.1) was studied by Jinxi Ma [11]. The functions f in S_p is said to be meromorphically starlike functions of order β if and only if

$$Re\left\{-\frac{zf'(z)}{f(z)}\right\} > \beta \qquad (z \in D\backslash\{p\}) \tag{1.2}$$

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for some $\beta(0 \leq \beta < 1)$. We denote by $S_p^*(\beta)$ the class of all meromorphically starlike functions of order β. Similarly , a function f in S is said to be meromorphically convex of order β if and only if

$$Re\left\{-1 - \frac{zf''(z)}{f'(z)}\right\} > \beta \qquad (z \in D\backslash\{p\}) \tag{1.3}$$

for some $\beta(0 \leq \beta < 1)$. We denote by $S_p^c(\beta)$ the class of all meromorphically convex functions of order β. We note that the class $S_0^*(\beta)$ and various other subclasses of $S_0^*(0)$ have been studied rather extensively by Nehari and Netanyahu[13], Clunie [5], Pommerenke [14, 15], Miller [12], Royster [16], and others (cf., e.g., Bajpai [3], Goel and Sohi [10], Uralegaddi and Ganigi [21], Cho et al [4], Aouf [1], and Uralegaddi and Somantha [22, 23]; see also Duren [6, pp. 29 and 137], Srivastava and Owa [19, pp.86 and 429].

Let $\mathbb{S}_p$ denote the subclass of S_p consisting of functions of the form:

$$f(z) = \frac{\alpha}{z-p} + \sum_{n=1}^{\infty} a_n z^n, \qquad (a_n \geq 0). \tag{1.4}$$

For the function f in the class S_p, we define the following:

$$I^0 f(z) = f(z),$$

$$I^1 f(z) = zf'(z) + \frac{\alpha(2z-p)}{(z-p)^2},$$

$$I^2 f(z) = z\left(I^1 f(z)\right)' + \frac{\alpha(2z-p)}{(z-p)^2},$$

and for k=1,2,3,... we can write

$$I^k f(z) = z\left(I^{k-1} f(z)\right)' + \frac{\alpha(2z-p)}{(z-p)^2}$$

$$= \frac{\alpha}{z-p} + \sum_{n=1}^{\infty} n^k a_n z^n. \tag{1.5}$$

The differential operator I^k studied extensively by Ghanim and Darus [8, 9] and for $p = 0$ and $\alpha = 1$, the differential operator I^k reduced to Frasin and Darus [7].

With the help of the differential operator I^k, we define the class $S_p^*(k, \alpha, \beta)$ as follow:

Definition 1. *The function $f \in S_p$ is said to be a member of the class $S_p^*(k,\alpha,\beta)$ if it satisfies*

$$\left|\frac{z\left(I^k f(z)\right)'}{I^k f(z)}+1\right| < \left|\frac{z\left(I^k f(z)\right)'}{I^k f(z)}+2\beta-1\right| \qquad (k \in N_0 = N \cup 0) \tag{1.6}$$

for some $\beta(0 \leq \beta < 1)$ and for all z in $D\backslash\{p\}$.

It is easy to check that $S_p^*(0,1,\beta)$ is the class of meromorphically starlike functions of order β and $S_p^*(0,1,0)$ gives the meromorphically starlike functions for all $z \in D\backslash\{p\}$.

Let us write

$$S_p^*[k,\alpha,\beta] = S_p^*(k,\alpha,\beta) \cap \mathbb{S}_p \tag{1.7}$$

where $\mathbb{S}_p$ is the class of functions of the form (1.4) that are analytic and univalent in $D\backslash\{p\}$.

Next, we will state some result which were studied previously by Ghanim and Darus [8].

2 Preliminary Results

For the class $S_p^*[k,\alpha,\beta]$, Ghanim and Darus [8] Showed:

Theorem 2. *Let the function f be defined by (1.4). If*

$$\sum_{n=1}^{\infty} n^k (n+\beta)(1-p)|a_n| \leq \alpha(1-\beta) \qquad (k \in N_0) \tag{2.1}$$

where $(0 \leq \beta < 1)$, then $f \in S_p^*[k,\alpha,\beta]$.

In the view of Theorem 2, we can see that the function f given by (1.4) is in the class $S_p^*[k,\alpha,\beta]$ which satisfies

$$a_n \leq \frac{\alpha(1-\beta)}{n^k(n+\beta)(1-p)}. \tag{2.2}$$

In view of (2.1), we can see that the functions f defined by (1.4) in the class $S_p^*[k,\alpha,\beta]$ satisfy the coefficient inequality

$$a_1 \leq \frac{\alpha(1-\beta)}{(1+\beta)(1-p)} \tag{2.3}$$

Hence we may take

$$a_1 = \frac{\alpha (1-\beta) c}{(1+\beta)(1-p)} \tag{2.4}$$

Making use of (2.4), we now introduce thee following class of functions:

Let $S_p^*[k, \alpha, \beta, c]$ denote the class of functions $f(z)$ in $S_p^*[k, \alpha, \beta]$ of the form

$$f(z) = \frac{\alpha}{z-p} + \frac{\alpha (1-\beta) c}{(1+\beta)(1-p)} z + \sum_{n=2}^{\infty} |a_n| z^n, \tag{2.5}$$

with $0 \leq c < 1$.

In this paper we obtain coefficient estimates, convex liner combinations, some distortion theorems, and redii of starlikeness and convexity for f in the class $S_p^*[k, \alpha, \beta, c]$.

There are many papers in which meromorphic functions with fixed second coefficient have been studied, see Aouf and Darwish [2], Silverman and Silvia [18], and Uralegaddi [20], and we use the same techniques to prove our results.

3 Coefficient Inequalities

Theorem 3. *A function f defined by (2.5) is in the class $S_p^*[k, \alpha, \beta, c]$, if and only if,*

$$\sum_{n=2}^{\infty} n^k \left[(n+\beta)(1-p)\right] |a_n| \leq \alpha (1-\beta)(1-c). \tag{3.1}$$

The result is sharp.

Proof. By putting

$$a_1 = \frac{\alpha (1-\beta) c}{(1+\beta)(1-p)}, \qquad (0 < c < 1) \tag{3.2}$$

in (2.1), the result is easily derived. the result is sharp for function

$$f_n(z) = \frac{\alpha}{z-p} + \frac{\alpha (1-\beta) c}{(1+\beta)(1-p)} z$$

$$+ \frac{\alpha (1-\beta)(1-c)}{n^k \left[(n+\beta)(1-p)\right]} z^n, \qquad n \geq 2. \tag{3.3}$$

□

Corollary 4. *Let the function* $f(z)$ *given by (2.5) be in the class* $S_p^*[k,\alpha,\beta,c]$*, then*

$$a_n \leq \frac{\alpha(1-\beta)(1-c)}{n^k[(n+\beta)(1-p)]}, \qquad n \geq 2. \tag{3.4}$$

Corollary 5. *If* $0 \leq c_1 \leq c_2 \leq 1$*, then*

$$S_p^*[k,\alpha,\beta,c_2] \subseteq S_p^*[k,\alpha,\beta,c_1]. \tag{3.5}$$

4 Growth and Distortion Theorems

A growth and distortion property for function f to be in the class $S_p^*[k,\alpha,\beta,c]$ is given as follows:

Theorem 6. *If the function* f *defined by (2.5) is in the class* $S_p^*[k,\alpha,\beta,c]$ *for* $0 < |z| = r < 1$*, then we have*

$$\frac{\alpha}{|z-p|} - \frac{\alpha(1-\beta)c}{(1+\beta)(1-p)}|z| - \frac{\alpha(1-\beta)(1-c)}{[(2+\beta)(1-p)]}|z|^2 \leq |f(z)|$$

$$\leq \frac{\alpha}{|z-p|} + \frac{\alpha(1-\beta)c}{(1+\beta)(1-p)}|z| + \frac{\alpha(1-\beta)(1-c)}{[(2+\beta)(1-p)]}|z|^2.$$

Where $\alpha = Res(z,p)$*, with* $0<\alpha\leq 1$, $0 \leq \beta < 1$, $0<c<1$, $z \in D\backslash\{p\}$*.*

Proof. Since $f \in S_p^*[k,\alpha,\beta,c]$, Theorem 3 yields the inequality

$$a_n \leq \frac{\alpha(1-\beta)(1-c)}{n^k[(n+\beta)(1-p)]}, \qquad n \geq 2. \tag{4.1}$$

Thus, for $0<|z|=r<1$, and note that for $k=0$, we have

$$|f(z)| \leq \frac{\alpha}{|z-p|} + \frac{\alpha(1-\beta)c}{(1+\beta)(1-p)}|z| + \sum_{n=2}^{\infty} a_n |z|^n$$

$$\leq \frac{\alpha}{|z-p|} + \frac{\alpha(1-\beta)c}{(1+\beta)(1-p)}|z| + |z|^2\sum_{n=2}^{\infty} a_n$$

$$\leq \frac{\alpha}{|z-p|} + \frac{\alpha(1-\beta)c}{(1+\beta)(1-p)}|z| + \frac{\alpha(1-\beta)(1-c)}{[(2+\beta)(1-p)]}|z|^2$$

and

$$|f(z)| \geq \frac{\alpha}{|z-p|} - \frac{\alpha(1-\beta)c}{(1+\beta)(1-p)}|z| - \sum_{n=2}^{\infty} a_n |z|^n$$

$$\geq \frac{\alpha}{|z-p|} - \frac{\alpha(1-\beta)c}{(1+\beta)(1-p)}|z| - |z|^2\sum_{n=2}^{\infty} a_n$$

$$\geq \frac{\alpha}{|z-p|} - \frac{\alpha(1-\beta)c}{(1+\beta)(1-p)}|z| - \frac{\alpha(1-\beta)(1-c)}{[(2+\beta)(1-p)]}|z|^2.$$

Thus the proof of the theorem is complete. □

Theorem 7. *If the function f defined by (2.5) is in the class $S_p^*[k,\alpha,\beta,c]$ for $0 < |z| = r < 1$, then we have*

$$\frac{\alpha}{|z-p|^2} - \frac{\alpha(1-\beta)c}{(1+\beta)(1-p)} - \frac{\alpha(1-\beta)(1-c)}{[(2+\beta)(1-p)]}|z| \leq |f'(z)|$$

$$\leq \frac{\alpha}{|z-p|^2} + \frac{\alpha(1-\beta)c}{(1+\beta)(1-p)} + \frac{\alpha(1-\beta)(1-c)}{[(2+\beta)(1-p)]}|z|$$

Proof. From Theorem 3, it follows that

$$na_n \leq \frac{\alpha(1-\beta)(1-c)}{n^{k-1}[(n+\beta)(1-p)]}, \qquad n \geq 2. \tag{4.2}$$

Thus, for $0 < |z| = r < 1$, and making use of (4.2) with noting that $k = 1$, we have

$$|f'(z)| \leq \left|\frac{-\alpha}{(z-p)^2}\right| + \frac{\alpha(1-\beta)c}{(1+\beta)(1-p)} + \sum_{n=2}^{\infty} na_n|z|^{n-1}$$

$$\leq \frac{\alpha}{|z-p|^2} + \frac{\alpha(1-\beta)c}{(1+\beta)(1-p)} + |z|\sum_{n=2}^{\infty} na_n$$

$$\leq \frac{\alpha}{|z-p|^2} + \frac{\alpha(1-\beta)c}{(1+\beta)(1-p)} + \frac{\alpha(1-\beta)(1-c)}{[(2+\beta)(1-p)]}|z|$$

and

$$|f'(z)| \geq \left|\frac{-\alpha}{(z-p)^2}\right| - \frac{\alpha(1-\beta)c}{(1+\beta)(1-p)} - \sum_{n=2}^{\infty} na_n|z|^{n-1}$$

$$\geq \frac{\alpha}{|z-p|^2} - \frac{\alpha(1-\beta)c}{(1+\beta)(1-p)} - |z|\sum_{n=2}^{\infty} na_n$$

$$\geq \frac{\alpha}{|z-p|^2} - \frac{\alpha(1-\beta)c}{(1+\beta)(1-p)} - \frac{\alpha(1-\beta)(1-c)}{[(2+\beta)(1-p)]}|z|.$$

The proof is complete. □

5 Radii of Starlikeness and Convexity

The radii of starlikeness and convexity for the class $S_p^*[0, \alpha, \beta, c]$ is given by the following theorem:

Theorem 8. *If the function f defined by (2.5) in the class $S_p^*[k, \alpha, \beta, c]$, then f is starlikeness of order $\delta (0 \leq \delta \leq 1)$ in the disk $|z-p| < |z| < r_1(k, \alpha, \beta, c)$ where $r_1(k, \alpha, \beta, c, \delta)$ is the largest value for which*

$$\frac{\alpha (3-\delta) (1-\beta) c}{(1+\beta) (1-p)} r^2 + \frac{\alpha (n+2-\alpha) (1-\beta) (1-c)}{n_o^k [(n+\beta) (1-p)]} r^{n+1} \leq \alpha (1-\delta)$$

for $n \geq 2$. The result is sharp for function $f_n(z)$ given by (3.3).

Proof. It suffice to show that

$$\left| \frac{z f'(z)}{f(z)} + 1 \right| \leq 1 - \delta.$$

Note that

$$\left| \frac{z f(z)'}{f(z)} + 1 \right| \leq \left| \frac{\frac{2\alpha(1-\beta)c}{(1+\beta)(1-p)} z + \sum_{n=2}^{\infty} (n+1) a_n z^n}{\frac{\alpha}{z-p} - \frac{\alpha(1-\beta)c}{(1+\beta)(1-p)} z - \sum_{n=2}^{\infty} a_n z^n} \right| \leq 1 - \delta. \tag{5.1}$$

Hence for $|z-p| < |z| < r$, (5.1) hold true if

$$\frac{2\alpha (1-\beta) c}{(1+\beta) (1-p)} r^2 + \sum_{n=2}^{\infty} (n+1) a_n r^{n+1}$$

$$\leq (1-\delta) \left(\alpha - \frac{\alpha (1-\beta) c}{(1+\beta) (1-p)} r^2 - \sum_{n=2}^{\infty} a_n r^{n+1} \right) \tag{5.2}$$

or

$$\frac{\alpha (3-\delta) (1-\beta) c}{(1+\beta) (1-p)} r^2 + \sum_{n=2}^{\infty} (n+2-\delta) a_n r^{n+1} \leq \alpha (1-\delta) \tag{5.3}$$

and it follow that from (3.1), we may take

$$a_n \leq \frac{\alpha (1-\beta) (1-c) \lambda_n}{n^k [(n+\beta) (1-p)]}, \qquad (n \geq 2). \tag{5.4}$$

where $\lambda_n \geq 0$ and $\sum\limits_{n=2}^{\infty} \lambda_n \leq 1$.
For each fixed r, we choose the positive integer $n_o = n_o(r)$ for which

$$\frac{n+2-\alpha}{n^k\left[(n+\beta)(1-p)\right]} r^{n+1}, \qquad \textit{is maximal.}$$

Then it follows that

$$\sum_{n=2}^{\infty} (n+2-\delta)\, a_n r^{n+1} \leq \frac{\alpha\,(n_o+2-\alpha)(1-\beta)(1-c)}{n_o^k\left[(n_o+\beta)(1-p)\right]} r^{n_o+1}. \tag{5.5}$$

Then f is starlike of order δ in $|z-p| < |z| < r_1(k,\alpha,\beta,c)$ provided that

$$\frac{\alpha\,(3-\delta)(1-\beta)\,c}{(1+\beta)(1-p)} r^2$$

$$+ \frac{\alpha\,(n_o+2-\alpha)(1-\beta)(1-c)}{n_o^k\left[(n_o+\beta)(1-p)\right]} r^{n_o+1} \leq \alpha\,(1-\delta)\,. \tag{5.6}$$

We find the value $r_o = r_o(k,\alpha,\beta,c,\delta,n)$ and the corresponding integer $n_o(r_o)$ so that

$$\frac{\alpha\,(3-\delta)(1-\beta)\,c}{(1+\beta)(1-p)} r_o^2 +$$

$$\frac{\alpha\,(n_o+2-\alpha)(1-\beta)(1-c)}{n_o^k\left[(n_o+\beta)(1-p)\right]} r^{n_o+1} = \alpha\,(1-\delta)\,. \tag{5.7}$$

Then this value is the radius of starlikeness of order δ for function f belong to class $S_p^*[k,\alpha,\beta,c]$. □

Theorem 9. *If the function f defined by (2.5) in the class $S_p^*[k,\alpha,\beta,c]$ then f is convexity of order $\delta(0 \leq \delta \leq 1)$ in the disk $|z-p| < |z| < r_2(0,\alpha,\beta,c,\delta)$ where $r_2(k,\alpha,\beta,c,\delta)$ is the largest value for which*

$$\frac{\alpha\,(3-\delta)(1-\beta)\,c}{(1+\beta)(1-p)} r^2 + \frac{\alpha\,(n+2-\alpha)(1-\beta)(1-c)}{n_o^{k-1}\left[(n+\beta)(1-p)\right]} r^{n+1} \leq \alpha\,(1-\delta)\,.$$

The result is sharp for function f_n given by (3.3).

Proof. By using the same technique in the proof of Theorem 8 we can show that

$$\left|\frac{(z-p)\,f''(z)}{f'(z)} + 2\right| \leq (1-\delta)$$

for $|z-p| < |z| < r_2$ with the aid of Theorem 3. Thus, we have the assertion of Theorem 9. □

6 Convex Linear Combination

Our next result involves a linear combination of function of the type (2.5).

Theorem 10. *If*

$$f_1(z) = \frac{\alpha}{z-p} + \frac{\alpha(1-\beta)c}{(1+\beta)(1-p)} z \tag{6.1}$$

and

$$f_n = \frac{\alpha}{z-p} + \frac{\alpha(1-\beta)c}{(1+\beta)(1-p)} z$$

$$+ \frac{\alpha(1-\beta)(1-c)}{n^k[(n+\beta)(1-p)]} z^n, \qquad n \geq 2. \tag{6.2}$$

Then $f \in S_p^*[k,\alpha,\beta,c]$ if and only if it can expressed in the form

$$f(z) = \sum_{n=2}^{\infty} \lambda_n f_n(z) \tag{6.3}$$

where $\lambda_n \geq 0$ and $\sum\limits_{n=2}^{\infty} \lambda_n \leq 1$.

Proof. from (6.1), (6.2) and (6.3), we have

$$f(z) = \sum_{n=2}^{\infty} \lambda_n f_n(z)$$

so that

$$f(z) = \frac{\alpha}{z-p} + \frac{\alpha(1-\beta)c}{(1+\beta)(1-p)} z + \sum_{n=2}^{\infty} \frac{\alpha(1-\beta)(1-c)\lambda_n}{n^k[(n+\beta)(1-p)]} z^n. \tag{6.4}$$

Since

$$\sum_{n=2}^{\infty} \frac{\alpha(1-\beta)(1-c)\lambda_n}{n^k[(n+\beta)(1-p)]} \cdot \frac{n^k[(n+\beta)(1-p)]}{\alpha(1-\beta)(1-c)}$$

$$= \sum_{n=2}^{\infty} \lambda_n = 1 - \lambda_1 \leq 1.$$

It follows from Theorem 3 that the function $f \in S_p^*[k,\alpha,\beta,c]$.

Conversely, let us suppose that $f \in S_p^*[k,\alpha,\beta,c]$. Since

$$a_n \le \frac{\alpha(1-\beta)(1-c)}{n^k[(n+\beta)(1-p)]}, \qquad (n \ge 2).$$

Setting

$$\lambda_n = \frac{n^k[(n+\beta)(1-p)]}{\alpha(1-\beta)(1-c)}a_n.$$

and

$$\lambda_1 = 1 - \sum_{n=2}^{\infty}\lambda_n$$

It follow that

$$f(z) = \sum_{n=2}^{\infty}\lambda_n f_n(z)$$

Thus completes the proof of theorem. □

Theorem 11. *The class $S_p^*[k,\alpha,\beta,c]$ is closed under linear combinations.*

Proof. Suppose that the function f be defined by (2.5), and let the function g be defined

$$g(z) = \frac{\alpha}{z-p} + \frac{\alpha(1-\beta)c}{(1+\beta)(1-p)}z + \sum_{n=2}^{\infty}|b_n|z^n, \qquad b_n \ge 2.$$

Assuming that f and g are in the class $S_p^*[k,\alpha,\beta,c]$, it is sufficient to prove that the function H defined by

$$H(z) = \lambda f(z) + (1-\lambda)g(z) \qquad (0 \le \lambda \le 1)$$

is also in the class $S_p^*[k,\alpha,\beta,c]$.
Since

$$H(z) = \frac{\alpha}{z-p} + \frac{\alpha(1-\beta)c}{(1+\beta)(1-p)}z + \sum_{n=2}^{\infty}|a_n\lambda + (1-\lambda)b_n|z^n,$$

we observe that

$$\sum_{n=2}^{\infty}n^k[(n+\beta)(1-p)]|a_n\lambda + (1-\lambda)b_n| \le \alpha(1-\beta)(1-c)$$

with the aid of Theorem 3.
Thus $H \in S_p^*[k,\alpha,\beta,c]$. Hence the theorem. □

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References

[1] M. K. Aouf, *On a certain class of meromorphic univalent functions with positive coefficients*, Rend. Mat. Appl., **7** 11 (1991), no. 2, 209-219. MR1122335. Zbl 0745.30011.

[2] M. K. Aouf and H. E. Darwish, *Certain meromorphically starlike functions with positive and fixed second coefficients*, Tr. J. of Mathematics **21** (1997), 311- 316. Zbl 0887.30014.

[3] S. K. Bajpai, *A note on a class of meromorphic univalent functions*, Rev. Roum. Math. Pures Appl. **22** (1977), 295 -297. MR447546. Zbl 0354.30006.

[4] N. E. Cho, S. H. Lee, and S. Owa, *A class of meromorphic univalent functions with positive coefficients*, it Kobe J. Math. **4** (1987), 43-50. Zbl 0639.30010 Zbl 0639.30010.

[5] J. Clunie, *On meromorpic schlicht functions*, J. Lond. Math.Soc. **34** (1959), 215-216. MR215737. Zbl 0087.07704.

[6] P. L. Duren, *Univalent Functions*, New York, Springer-Verlag, 1983.

[7] B. A. Frasin and M. Darus, *On certain meromorphic functions with positive coefficients*, Southeast Asian Bull. Math. **28** (2004), 615-623. MR2086021. Zbl 1069.30016.

[8] F. Ghanim and M. Darus, *On certain subclass of meromorphic univalent functions with fixed residue* α, Far East J. Math. Sci. (FJMS), **26** (2007), 195-207. Zbl 1133.30305.

[9] F. Ghanim and M. Darus, *A new subclass of uniformly starlike and convex functions with negative coefficients. II*, Int. J. Pure Appl. Math. **45** (4) (2008), 559-572. MR2426233. Zbl 1151.30316.

[10] R. M. Goel and N. S. Sohi, *On a class of meromorphic functions*, Glas. Mat., III. Ser. **17** (37) (1982), 19-28. Zbl 0504.30005.

[11] Jinxi Ma, *Extreme points and minimal outer area problem for meromorphic univalent functions*, J. Math. Anal. Appl. **220 (2)** (1998), 769-773. MR1614960. Zbl 0911.30009.

[12] J. Miller, *Convex meromorphic mappings and related functions*, Proc. Am. Math. Soc. **25** (1970), 220-228. Zbl 0196.09202.

[13] Z. Nehari and E.Netanyahu, *On the coefficients of meromorphic schlicht functions*, Proc. Am. Math. Soc. **8** (1957), 15-23. Zbl 0079.10004.

[14] Ch. Pommerenke, *Über einige Klassen meromorpher schlichter Funktionen*, Math. Z. **78** (1962), 263-284. #1285).

[15] Ch. Pommerenke, *On meromorphic starlike functions*, Pac. J. Math. **13** (1963), 221-235. MR0150279(27 #280). Zbl 0116.05701.

[16] W. C. Royster, *Meromorphic starlike multivalent functions*, Trans. Am. Math. Soc. **107** (1963), 300-308. MR0148895(26 #6392). Zbl 0112.05101.

[17] G. Schober, *Univalent Functions - Selected Topics*, Lecture Notes 478, Berlin-Heidelberg - New York: Springer, 1975. MR0507770(58 #22527).

[18] H. Silverman and E. M. Silvia, *Fixed coefficients for subclasses of starlike functions*, Houston J. Math. **7** (1981), 129-136. MR0628498(83d:30014). Zbl 0461.30010.

[19] H. M. Srivastava and S. Owa, *Current Topics In Analytic Functions Theory*, World Scientific, Singapore, 1992. MR1232424(94b:30001).

[20] B. A. Uralegaddi, *Meromorphically starlike functions with positive and fixed second coefficients*, Kyungpook Math. J. **29 (1)** (1989), 64-68. Zbl 0707.30008.

[21] B. A. Uralgaddi and M. D. Ganigi, *A certain class of meromorphically starlike functions with positive coefficients*, Pure Appl. Math. Sci. **26** (1987), 75-81. Zbl 0638.30014.

[22] B. A. Uralegaddi and C. Somanatha, *Certain differential operators for meromorphic functions*, Houston J. Math. **17** (1991), 279-284. MR1115149(92j:30019). Zbl 0749.30016.

[23] B. A. Uralegaddi and C. Somanatha, *New criteria for meromorphic starlike univalent functions*, Bull. Austral. Math. Soc. **43** (1991), 137-140. MR1086726(92j:30018).

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