HOMOTOPY ANALYSIS METHOD FOR SOLVING KDV EQUATIONS

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Abstract. A scheme is developed for the numerical study of the Korteweg-de Vries (KdV) and the Korteweg-de Vries Burgers (KdVB) equations with initial conditions by a homotopy approach. Numerical solutions obtained by homotopy analysis method are compared with exact solution. The comparison shows that the obtained solutions are in excellent agreement.

Full text

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