Surveys in Mathematics and its Applications

ISSN 1842-6298 (electronic), 1843-7265 (print)
Volume **5** (2010), 35 – 47

SOME APPLICATIONS OF GENERALIZED RUSCHEWEYH DERIVATIVES INVOLVING A GENERAL FRACTIONAL DERIVATIVE OPERATOR TO A CLASS OF ANALYTIC FUNCTIONS WITH NEGATIVE COEFFICIENTS I

Waggas Galib Atshan and S. R. Kulkarni

Abstract. For certain univalent function f, we study a class of functions f as defined by making use of the generalized Ruscheweyh derivatives involving a general fractional derivative operator, satisfying

$$Re\left\{\frac{z(\mathcal{J}_1^{\lambda,\mu}f(z))'}{(1-\gamma)\mathcal{J}_1^{\lambda,\mu}f(z)+\gamma z^2(\mathcal{J}_1^{\lambda,\mu}f(z))''}\right\}>\beta.$$

A necessary and sufficient condition for a function to be in the class $A_\gamma^{\lambda,\mu,\nu}(n,\beta)$ is obtained. In addition, our paper includes distortion theorem, radii of starlikeness, convexity and close-to-convexity, extreme points. Also, we get some results in this paper.

Full text

References

[1] E. S. Aqlan, *Some problems connected with geometric function theory*, Ph.D. Thesis (2004), Pune University, Pune.

[2] P. L. Duren, *Univalent Functions*, Grundelheren der Mathematischen Wissenchaften **259**, Springer-Verlag, New York, Berlin, Heidelberg, Tokyo, 1983. MR708494 (85j:30034). Zbl 0514.30001.

[3] S. P. Goyal and Ritu Goyal, *On a class of multivalent functions defined by generalized Ruscheweyh derivatives involving a general fractional derivative operator*, Journal of Indian Acad. Math. **27(2)** (2005), 439-456. MR2259538 (2007d:30005). Zbl 1128.30008.

[4] S. Kanas and A. Wisniowska, *Conic regions and k-uniformly convexity II*, Folia Sci. Tech. Reso. **170** (1998), 65-78. MR1693661 (2000e:30017). Zbl 0995.30013.

2000 Mathematics Subject Classification: 30C45.
Keywords: Distortion theorem; Radii of starlikeness; Extreme points.

[5] R. K. Raina and T. S. Nahar, *Characterization properties for starlikeness and convexity of some subclasses of analytic functions involving a class of fractional derivative operators*, Acta Math. Univ. Comen., New Ser. **69**, No.1, 1-8 (2000). ISSN 0862-9544. MR1796782 (2001h:30014). Zbl 0952.30011.

[6] V. Ravichandran, N. Sreenivasagan, and H. M. Srivastava, *Some inequalities associated with a linear operator defined for a class of multivalent functions*, JIPAM, J. Inequal. Pure Appl. Math. 4, No. **4**, Paper No. 70, 12 p., electronic only (2003). ISSN 1443-5756. MR2051571 (2004m:30022). Zbl 1054.30013.

[7] T. Rosy, K. G. Subramanian and G. Murugusundaramoorthy, *Neighbourhoods and partial sums of starlike functions based on Ruscheweyh derivatives*, JIPAM, J. Inequal. Pure Appl. Math. 4, No. 4, Paper No. 64, 8 p., electronic only (2003). ISSN 1443-5756. MR2051565. Zbl 1054.30014.

[8] S. Shams and S. R. Kulkarni, *Certain properties of the class of univalent functions defined by Ruscheweyh derivative*, Bull. Calcutta Math. Soc. **97** (2005), 223-234. MR2191072. Zbl 1093.30012.

[9] H. Silverman, *Univalent functions with negative coefficients*, Proc. Amer. Math. Soc. **51** (1975), 109-116. MR0369678 (51 #5910). Zbl 0311.30007.

[10] H. M. Srivastava, *Distortion inequalities for analytic and univalent functions associated with certain fractional calculus and other linear operators* (In Analytic and Geometric Inequalities and Applications eds. T. M. Rassias and H. M. Srivastava), Kluwar Academic Publishers, **478** (1999), 349-374. MR1785879 (2001h:30016). Zbl 0991.30007.

[11] H. M. Srivastava and R. K. Saxena, *Operators of fractional integration and their applications*, Applied Mathematics and Computation, **118** (2001), 1-52. MR1805158 (2001m:26016). Zbl 1022.26012.

[12] A. Tehranchi and S. R. Kulkarni, *Study of the class of univalent functions with negative coefficients defined by Ruscheweyh derivatives (II)*, J. Rajasthan Acad. Phy. Sci., **5(1)** (2006), 105-118. MR2214020. Zbl 1138.30015.

Waggas Galib Atshan
Department of Mathematics,
College of Computer Science and Mathematics,
University of AL-Qadisiya, Diwaniya, Iraq.
e-mail: waggashnd@yahoo.com

S. R. Kulkarni
Department of Mathematics,
Fergusson College, Pune - 411004,
India.
e-mail: kulkarni_ferg@yahoo.com