

EXISTENCE AND NONEXISTENCE RESULTS FOR SECOND-ORDER NEUMANN BOUNDARY VALUE PROBLEM

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Abstract. In this paper some existence and nonexistence results for positive solutions are obtained for second-order boundary value problem

$$-u'' + Mu = f(t, u), \quad t \in (0, 1)$$

with Neumann boundary conditions

$$u'(0) = u'(1) = 0,$$

where $M > 0$, $f \in C([0, 1] \times \mathbb{R}^+, \mathbb{R}^+)$. By making use of fixed point index theory in cones, some new results are obtained.

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