# DESCRIPTION OF THE STRASBOURG GROUP

## A. The group itself.

In the field of Algebraic Combinatorics there are six people in Strasbourg enrolled in the EEC Human Capital and Mobility Programme, namely

Dominique Dumont, Dr Sc., maître de conférences;
Dominique Foata, Dr Sc., professor;
Guo-Niu Han, Dr, chargé de recherches C.N.R.S.;
Arthur Randrianarivony, Dr, research fellow;
Yaacob Akiba Slama, doctoral student;
Jean Zeng, Dr, maître de conférences.

To those six researchers we should add three other colleagues who attend the weekly Strasbourg Combinatorics seminar and also the biennial *Séminaire Lotharingien de Combinatoire* (Bayreuth, Erlangen, Strasbourg):

Abdallah Al Amrani, Dr Sc., maître de conférences,
Jean-Pierre Jouanolou, Dr Sc., professor
Raphaelle Supper, Dr, maître de conférences,

whose main research interests are centered around Algebraic Geometry for the first two and Classical Analysis for the last one.

During the two-year period 1994-95 the group has benefited from the EEC Human Capital and Mobility Programme with the invitations of Dr Einar Steingrìmsson (Goteborg), in Strasbourg for two months, and of Dr Christian Krattenthaler (Wien) who only stayed two weeks.

The group has also taken advantage of the invitations of Prof. Doron Zeilberger (Philadelphia), Mike Hirschhorn (Sydney) and Robert J. Clarke (Adelaide). The three of them have been appointed visiting professors in Strasbourg, respectively for one, two and four months. As will be noted below there had been a strong research interaction between them and the Strasbourg group.

## B. Research orientations.

The research of the group has four main directions : Combinatorics of classical numbers, Combinatorial Study of Special Functions, $q$-Eulerian Calculus, Analytical Tool Study. However, those topics are strongly interconnected.

1. *Combinatorics of classical numbers.* This study goes back ot the early seventies when several schools discovered the underlying combinatorial properties of classical numbers : Euler, tangent, Genocchi numbers... Several questions had remained unsolved, for instance, whether a joint combinatorial model for both Genocchi and median Genocchi numbers

could be found. The problem has been completely solved by Dumont and Randrianarivony [8]. The solution has suggested several extensions, in particular the study of new classes of polynomials that appear as generating polynomials for statistics on finite structures counted by those numbers, or simply as polynomials that specialize to those numbers and to other related classical numbers (see, e.g., [5, 9, 10, 20, 23, 24, 25, 27]).

In several instances, exponential generating functions for those numbers or their polynomial extensions cannot be evaluated, but *continued fraction* expansions for their ordinary generating functions can be evaluated. The combinatorial theory of formal continued fractions is well-known today, but in each example, an adequate calculation is to be found. The most interesting result is probably due to Randrianarivony [21] who could introduce true $q$-Genocchi polynomials and derive the continued fraction expansion for their generating function.

Other aspects of Combinatorial Theory of Numbers are the calculations of new *Seidel Triangles* [7] or the discovery of new geometric structures for interpreting the underlying calculus [8].

2. *Combinatorial Study of Special Functions.* The problem is to build up combinatorial structures together with their natural statistics, e.g., coloured derangements with their cycle or descent numbers, that account for analytical properties of the classical orthogonal polynomials. A thorough study of nonnegativity properties of the *linearization coefficients* had been solved earlier by Zeng for the class of Sheffer polynomials. An interesting joint study of the $q$-Laguerre polynomials and their moments has been derived by him [26]. In [14] a combinatorial device is proposed for summing series of products of orthogonal polynomials, essentially the Tchebychev polynomials of the two kinds.

3. *$q$-Eulerian Calculus.* Following the work of Denert on the calculation of zeta functions for hereditary orders of some central simple algebras Han [15] has introduced a new family of *partially commutative monoids* that enabled him to prove that the bivariate statistic (exc,den) was Euler-mahonian. This work has been extended to the case where the underlying alphabet is partitioned into two subclasses of letters, the small letters and the large letters, that lead to different classes of statistics (see [1, 2, 3, 4]). This also provides a natural extension of the classical statistical study of the symmetric group $\mathcal{S}_n$ towards the analogous study of the group $B_n$ of the *signed permutations.*

When going from $A_n$ to $B_n$ (from a single alphabet to a double-class alphabet) new tools are to be found or updated. The so-called *MacMahon Verfahren* is such a tool; it has been updated in [11] and applied in [3, 12, 13]. The study of equidistribution properties has led to the discovery of new relations, such as *bipartitional relations* [12] that need to be characterized or studied for their own sakes (see [17, 18, 19]).

4. *Analytical Tool Study.* In section we mention the study of polynomials defined by difference equations such as in [28], or the analytical study of combinatorial polynomials, such as the Eulerian polynomials, when the integers occurring in their definitions are replaced by their $q$-analogues [29] or by arbitrary complex numbers. The problem is to study under what conditions the other formulas involving those polynomials are preserved.

5. *Miscellaneous.* In this section are included the study of combinatorial constructions derived in the theory of *symmetric functions*. For instance, the construction by Kerov and his collaborators to interpret the Foulkes polynomials still needs a solid framework. A very natural geometric set-up to describe such a construction is due to Han [16].

Finally, new combinatorial structures are to be found to prove the identities of Classical Invariant Theory, such as the non-commutative extension of the *Capelli identity* found recently by Howe. The young Slama is preparing a memoir on the subject.

Not included in the following list, but already discussed in the weekly Strasbourg seminar, the *Combinatorial Applications of the resultant*, as proposed by Jouanolou, will soon appear as a basic tool in Algebraic Combinatorics.

### C. Publications.

Let us cite the papers of the group that have published or been accepted, or been submitted in 1994-95, under the EEC Algebraic Programme.


[1] Robert J. Clarke and Dominique Foata, Eulerian Calculus, I: univariable statistics, *Europ. J. Combinatorics*, vol. **15**, 1994, p. 345–362.

[2] Robert J. Clarke and Dominique Foata, Eulerian Calculus, II: an extension of Han's fundamental transformation, *Europ. J. Combinatorics*, vol. **16**, 1995, p. 221–252.

[3] Robert J. Clarke and Dominique Foata, Eulerian Calculus, III: the ubiquitous Cauchy formula, *Europ. J. Combinatorics*, to appear, 1995.

[4] Robert J. Clarke and Dominique Foata, Eulerian Calculus, IV: specializations, *Séminaire Lotharingien de Combinatoire*, **32**b, 1994, 9pp.

[5] Dominique Dumont, Conjectures sur des symétries ternaires liées aux nombres de Genocchi , *Disc. Math.*, vol. **139**, 1995, p. .

[6] Dominique Dumont, A. Ramamonjisoa, Grammaire de Ramanujan et Arbres de Cayley, *Electron. J. Math.*, 1995 (to appear).

[7] Dominique Dumont, Further Triangles of Seidel-Arnold type and continued fractions related to Euler and Springer numbers, *Adv. in Applied Math.*, 1995 (to appear).

[8] Dominique Dumont, Arthur Randrianarivony, Dérangements et nombres de Genocchi, *Disc. Math.*, vol. **132**, 1994, p. 37–49.

[9] Dominique Dumont and Arthur Randrianarivony, Sur une extension des nombres de Genocchi, *Europ. J. Combinatorics*, vol. **16**, 1995, p. 147–151.

[10] Dominique Dumont and Jean Zeng, Further results on the Euler and Genocchi numbers , *Aequat. Math.*, vol. **47**, 1994, p. 31–42.

[11] Dominique Foata, Les distributions Euler-Mahoniennes sur les mots, to appear in *Discrete Math.*, 1995.

[12] Dominique Foata and Doron Zeilberger, Graphical Major Indices, to appear in *J. of Computational and applied Math.*, 1995.

[13] Dominique Foata and Christian Krattenthaler, Graphical Major Indices, II, *Séminaire Lotharingien de Combinatoire*, **34**k, Works in Progress, 16 pp.

[14] Dominique Foata and Guoniu Han, Nombres de Fibonacci et polynômes orthogonaux, in *Leonardo Fibonacci: il tempo, le opere, l'eredità scientifica* [M. Morelli and M. Tangheroni, eds.], p. 179–208. Pacini, Roma.

[15] Guoniu Han, Une transformation fondamentale sur les réarrangements de mots, *Adv. in Math.*, vol. **105**, 1994, p. 26–41.

[16] Guoniu Han, Une version géométrique de la construction de Kerov-Kirillov-Reshetikhin, *Séminaire Lotharingien de Combinatoire*, **31**, Publ. IRMA Strasbourg, 1994/021, p. 71–85.

[17] Guoniu Han, The $k$-extension of a mahonian statistic, to appear in *Adv. in Appl. Math.*, 1995.

[18] Guoniu Han, Ordres bipartitionnaires et statistiques sur les mots, to appear in *Electronic J. Combinatorics*, 1995.

[19] Guoniu Han, Une démonstration "vérificative" d'un résultat de Foata-Zeilberger sur les relations bipartitionnaires, to appear in *J. of Computational and applied Math.*, 1995.

[20] Arthur Randrianarivony, Polynômes de Dumont-Foata généralisés, submitted for publication, 1995.

[21] Arthur Randrianarivony, Fractions continues, $q$-nombres de Catalan et $q$-polynômes de Genocchi, submitted for publication, 1995.

[22] Arthur Randrianarivony, $q, p$-analogues des nombres de Catalan, submitted for publication, 1995.

[23] Arthur Randrianarivony and Jean Zeng, Sur une extension des nombres d'Euler et les records des permutations alternantes, *J. Combin. Th. Ser. A*, vol. **68**, 1994, p. 86-99.

[24] Arthur Randrianarivony and Jean Zeng, Une famille de polynômes qui interpole plusieurs suites classiques de nombres, *Actes du 31e Séminaire Lotharingien*, 1994, p. 103-126, to appear in *Adv. Appl. Math.*

[25] Arthur Randrianarivony and Jean Zeng, Some equidistributed statistics on Genocchi permutations, to appear in *Electronic J. Combin.*, 1995.

[26] Jean Zeng, The $q$-Stirling numbers, continued fractions and the $q$-Charlier and $q$-Laguerre polynomials, *J. of Computational and applied Math.*, 1995 (to appear).
[27] Jean Zeng, Sur quelques propriétés de symétrie des nombres de Genocchi, *Proc. of the Fifth conference on FPSAC*, 1993, Florence, to appear in *Disc. Math.*, 1995.
[28] Jean Zeng, Multinomial convolution polynomials, to appear in *Disc. Math.*, 1995.
[29] Jean Zeng and Changui Zhang, A $q$-analog of Newton's series, Stirling functions and Eulerian functions, *Results in Math.*, vol. **25**, 1994, p. 370-391.

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