A COORDINATIZATION OF GENERALIZED QUADRANLGES OF ORDER $(s-1, s+1)$

Marijke de Soete

A generalized quadrangle is an incidence structure $S = (P, B, I)$ with P and B sets of objects called points and lines respectively, with a symmetric incidence relation I which satisfies:

(i) each point is incident with $1+t$ lines $(t \geq 1)$ and two distinct points are incident with at most one line;
(ii) each line is incident with $1+s$ points $(s \geq 1)$ and two distinct lines are incident with at most one point;
(iii) for each point x and each line L, $x \not{I} L$, there exists a unique pair $(y, M) \in P \times B$ such that $x \, I \, M \, I \, y \, I \, L$.

We call (s,t) the order of S.

Let us consider a generalized quadrangle S of order $(s-1, s+1)$ containing a spread R (i.e., a subset R of B such that each point is incident with a unique line of R). R is called a spread of symmetry for the generalized quadrangle S if the group G_R of automorphisms of S fixing R linewise acts transitively on each line of R. If S has a spread of symmetry, then from S there arises a generalized quadrangle S' of order s having a center of symmetry. So in this case we are able to give a coordinatization of S, using a planar ternary ring, and G_R, which is derived from the coordinatization of S' due to S. E. Payne [3].

We also investigate the converse problem. Given a planar ternary ring and a group G, which are the conditions to obtain a generazlied quadrangle of order $(s-1, s+1)$. Examples are given for the known models $AS(q)$, $T_2^*(O)$ and the dual of $P(T_2(O), x)$ with x a point of the oval O.

References

1. M. De Soete, J. A. Thas, *Acoordinatization of generalized quadrangles of order* $(s, s+2)$, J. Combin. Theory Ser. A **48** (1988), 1–11.
2. Stanley E. Payne, *Quadrangles of order* $(s-1,\ s+1)$, J. Algebra **22** (1972), 97–119.
3. Stanley E. Payne, *Generalized quadrangles of even order*, J. Algebra **31** (1974), 367–391.

Seminarie voor hogere Meetkunde, State University of Ghent, Krijgslaan 281, B-9000 Gent, Belguim

Typeset by $\mathcal{A}_{\mathcal{M}}\mathcal{S}$-TEX