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Algebraic representation of continua

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Abstract. Using the duality between the category whose objects are the representations of Hausdorff quotients of Cantor spaces and the category whose objects are the Cantor ring endowed with a link relation (this duality is a particular case of an extension of the Stone duality obtained in [3]), we obtain algebraic representations of the following continua: the unit interval $I = [0, 1]$, the unit circle S^1, the Sierpiński triangular curve and the simple triod.

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Resumen. Usando la dualidad entre la categoría cuyos objetos son las representaciones de cocientes Hausdorff de espacios de Cantor y la categoría cuyos objetos son el anillo de Cantor dotado con una relación de ligazón (esta dualidad es un caso particular de una extensión de la dualidad de Stone obtenida en [3]), obtenemos representaciones algebraicas de los siguientes continuos: el intervalo unidad $I = [0, 1]$, el círculo unitario S^1, la curva triangular de Sierpiński y el triodo simple.

1. Introduction

In 1920 Moore [2], proved essentially that if K is a metric continuum with exactly two noncut points, then K is homeomorphic to the unit interval I, and that if K is a metric continuum such that for any two points a and b, $K \smallsetminus \{a, b\}$ is not connected, then K is homeomorphic to the unit circle. In 1916 W. Sierpiński [5], presented his triangular curve and in 1990 W. Debski and J. Mioduszewski [1] studied some topological properties of this curve, they proved that the Sierpiński triangular curve is *non-sewable* (they call a compactum *sewable* if it can be mapped by a simple map onto a plane subset having non-empty interior). These results about continua are obtained from a topological point of view.

In [3] we established a duality between two categories, extending the well-known Stone duality between totally disconnected compact Hausorff spaces (Stone spaces) and boolean rings with a unit ([6], [7]). Furthermore in [3], we studied connectedness from an algebraic point of view, in the context of the proposed generalized Stone duality, and we established an algebraic characterization of continua in general. In this work, by means of that extension of the Stone duality, we present algebraic characterizations of some continua. This type of characterizations are relevant because they establish a direct connexion between two large fields of mathematics, namely, Algebra and Topology. In this case in particular, a meticulous correlation is obtained between the topological properties of continua and the algebraic properties of certain boolean ring equipped with a relation. For instance, by using the algebraic representations introduced in Section 3 and by means of Proposition 3.3 of [3], the connectivity of the represented topological spaces is obtained.

In this paper, continua are the compact, connected metric spaces. If A is a boolean ring, then $Spec(A)$ denotes the set of ultrafilters in A, endowed with the topology whose basic open sets are $\mathbb{D}(a) =: \{U \in Spec(A) \mid a \in U\}$, $a \in A$, and if X is a Stone space then $\mathbb{A}(X)$ denotes the boolean ring of the clopen subsets of X.

On the other hand, it is known that the Cantor space is a Stone space and that the boolean ring that corresponds to the Cantor space is the unique (up to isomorphism) boolean ring with unit, denumerable and without atoms. We call this ring the *Cantor's ring* and we denote it by $\mathbb{K}$.

The main definitions and results required in this paper were presented in [3] and they are summarized in Section 2. In Section 3, as another application of the main result of [3], we present an algebraic representations of the unit interval $I = [0, 1]$, the unit circle S^1, the Sierpiński triangular curve and the simple triod.

2. Preliminaries

In [3] we defined the following concepts:

(i) Let X be a set and α be a relation on X. The relations R_α and R^α in any subfamily of PX are defined as follows: If C and D are subsets of X,

$$CR_\alpha D \iff (\exists x)(\exists y)(x \in C,\ y \in D,\ \wedge\ x\alpha y)$$

$$CR^\alpha D \iff (\forall x)(\forall y)(x \in C,\ y \in D \Longrightarrow x\alpha y).$$

(ii) The category **BRLR** (*boolean rings with unit and with a link relation*) as the category of pairs (A, α) where A is a boolean ring with unit and α is a relation on $A - \{0\}$, that satisfies:

(L1) α is reflexive;

(L2) α is symmetric;
(L3) $(\forall c, d \in A)$ $(c\alpha d,\ c \leq a,\ d \leq b \Longrightarrow a\alpha b)$;
(L4) $(\forall a, b, c \in A)$ $(c\ \alpha\ a \vee b \implies c\alpha a$ or $c\alpha b)$;
(L5) R^{α} is transitive in $Spec(A)$.

Such relation α is called a *link relation.* The morphisms $f : (A, \alpha) \longrightarrow (A', \alpha')$ are the morphisms of boolean ring with 1, such that $f(c)\alpha' f(d)$ implies $c\alpha d$, $\forall c, d \in A$.

(iii) The category **RHQS** (*Representations of Hausdorff quotients of Stone spaces*) is the category of pairs (X, γ), where X is a Stone space, γ is an equivalence relation on X, and γ is closed (that is, γ is a closed subset of $X \times X$). The morphisms: $f : (X, \gamma) \longrightarrow (X', \gamma')$ are continuous functions such that $x\gamma y$ implies $f(x)\gamma' f(y)$, $\forall x, y \in X$.

(iv) The contravariant functors $\mathbb{S}$: **BRLR** $\to$ **RHQS** and $\mathbb{A}$: **RHQS** $\to$ **BRLR** are defined as follows. If (A, α) is an object in **BRLR** and (X, γ) is an object of **RHQS**, then $\mathbb{S}(A, \alpha) =: (Spec(A), R^{\alpha})$, $\mathbb{A}(X, \gamma) =: (\mathbb{A}(X), R_{\gamma})$, $\mathbb{S}(f) =: f^!$, $\mathbb{A}(f) =: f^!$, where if $f : A \longrightarrow A'$ then $f^! : Spec\,(A') \longrightarrow Spec(A)$ is defined by: $f^!(U) = f^{-1}(U)$ and, similarly, if $f : X \longrightarrow X'$ then $f^! : \mathbb{A}\,(X') \longrightarrow \mathbb{A}(X)$ is defined by $f^!(C) = f^{-1}(C)$.

In [3] it was shown that the contravariant functors $\mathbb{S}$ and $\mathbb{A}$ establish a duality between the categories **BRLR** and **RHQS**, and that this duality reduces to Stone's duality when restricted to certain subcategories isomorphic to the classical categories *boolean rings with unit-boolean ring morphisms* and *Stone spaces-continuous functions.* Furthermore, if **CRLR** (*Cantor's ring with a link relation*) denotes the full subcategory of **BRLR** whose objects are the pairs $(\mathbb{K}, \alpha)$, and if **RHQC** (*representations of Hausdorff quotients of the Cantor space*) denotes the full subcategory of **RHQS** whose objects are the pairs $(X, \sim)$, where X is the Cantor space, then it is immediate that **CRLR** and **RHQC** are equivalent categories.

(v) Let A be a boolean ring, $t \in A$, $t \neq 0$ and $n \in \mathbb{Z}^+$, we will call $C \subseteq A$ a *n-partition* of t if: (a) $|C| = n$; (b) $(\forall c \in C)(c < t)$; (c) $\bigvee_{c \in C} c = t$; (d) $0 \notin C$; (e) $(\forall c, d \in C)(c \neq d \Longrightarrow cd = 0)$.

(vi) Let A be boolean ring; $E \subseteq A$ is called a *n-ary tree of top t* if $E = \bigcup_{n=0}^{\infty} N(n)$ where (a) $N(0) = \{t\}$; (b) $(\forall m \in \mathbb{N})\big(N(m+1) = \cup\{Q(v) \mid v \in N(m)\}\big)$, being $Q(v)$ some n-partition of v. $N(m)$ is called the *m level* of E. If $n = 2$ then E is called a *dichotomic tree.*

In [4], Proposition 2.6.5 we proved that for the Cantor's ring $\mathbb{K}$, and for all $n \in \mathbb{N}$, there exists a n-ary tree $\mathbb{P}$ of top 1, such that the subring of $\mathbb{K}$ generated by $\mathbb{P}$, is $\mathbb{K}$. The elements of the tree $\mathbb{P}$ can be labeled with "words" on the alphabet $\Sigma = \{a_1, a_2, \cdots, a_n\}$, in this case we will denote: $\mathbb{P} =< a_1, a_2, \cdots, a_n >$. Hence $\mathbb{P}$ can be identified with Σ^*, the set of finite words on the alphabet Σ, including the empty word. If $w \in \Sigma^*$ them $[w)$ denotes the set of words that begin with w.

3. Algebraic representations of some continua

Since X is a continuum if and only if it is homeomorphic to a Hausdorff connected quotient of the Cantor space, from the equivalence of categories **CRLR** and **RHQC**, arises the idea of establishing algebraic representations of particular continua. In this section we consider the unit interval I, the circle S^1, the Sierpiński triangular curve and the simple triod. In the following proposition $\left(N(m), \alpha_|\right)$ denotes the $N(m)$ level of the tree $\mathbb{P}$ with the relation α restricted to $N(m)$.

Proposition 3.1. *Let* $(\mathbb{K}, \alpha)$ *and* $(\mathbb{K}, \alpha')$ *be objects of* **CRLR**. *Let*

$$\mathbb{P} = \langle a_1, \cdots, a_n\rangle \quad \text{and} \quad \mathbb{P}' = \langle b_1, \cdots, b_n\rangle$$

be n-ary trees which generate $\mathbb{K}$. *If for every* $m \in \mathbb{N}$ *there exists*

$$f_m : \left(N(m), \alpha_|\right) \longrightarrow \left(N'(m), \alpha'_|\right)$$

such that,

(i) f_m *is bijective and preserves the relations in two senses, that is:* $p\alpha p'$ *iff* $f_m(p)\alpha' f_m(p')$;
(ii) *If* $p \in N(m)$ *then* $f_{m+1}(pa_1) \vee \cdots \vee f_{m+1}(pa_n) = f_m(p)$,

then $(\mathbb{K}, \alpha)$ *and* $(\mathbb{K}, \alpha')$ *are isomorph.*

Proof. It is sufficient to consider the case $n = 2$, (the general case is totally similar). Let $\mathbb{P} =< a, b >$ and $\mathbb{P}' =< a', b' >$ be dichotomic trees. For each $m \in \mathbb{N}$, let $< N(m) >$ be the finite boolean ring generated by the $N(m)$ level, $< N(m) >$ is (isomorphic to) the ring $P2^m$. Let

$$f^m :< N(m) > \longrightarrow < N'(m) >$$

be the function defined by $f^m(p_1 \vee \cdots \vee p_k) =: f_m(p_1) \vee \cdots \vee f_m(p_k)$. Then f^m is an isomorphism of boolean rings with a unit, which preserves the relations in the two senses.

Now, if $p \in N(m)$ then $p = pa \ \vee \ pb$ and $pa, pb \in N(m+1)$, hence for each $m \in \mathbb{N}$, $< N(m) >\subseteq< N(m+1) >$. Furthermore $f^m \subseteq f^{m+1}$ (that is $f^{m+1}_{|_{<N(m)>}} = f^m$). Since $\bigcup_{m\in\mathbb{N}} < N(m) > = \mathbb{K}$, then $f =: \cup_{m\in\mathbb{N}} f^m : (\mathbb{K}, \alpha) \longrightarrow (\mathbb{K}, \alpha')$ is an isomorphism of **CRLR**. ☑

3.1. The unit interval I.

Definition 3.2. *Let X be a set and α be a relation in X. For every $a \in X$ the set $\dot{a}$ is defined by* $\dot{a} =: \{x \in X - \{a\} \mid (a\alpha x) \vee (x\alpha a)\}$.

Definition 3.3. *Let X be a set and α be a relation in X. If the following two conditions are satisfied:*

(i) *there exist* $a, b \in X$, $a \neq b$, *such that* $|\dot{a}| = |\dot{b}| = 1$;
(ii) *for every* $x \in X - \{a, b\}$, $|\dot{x}| = 2$,

then α is called a **linear relation with extremes** a **and** b*, or* (X,α) *is called* **linear with extremes** a **and** b.

Definition 3.4. *Let* $(\mathbb{K},\alpha)$ *be an object of* **CRLR** *and* $\mathbb{P} =< a,b >$ *a dichotomic tree of top* 1*,*

1. $(\mathbb{P},\alpha|_{\mathbb{P}})$ *is called* linear *if it satisfies the following two conditions:*
   (i) $(\forall p,p' \in \mathbb{P})\,(\ p\alpha p' \Longrightarrow ap\ \alpha\ ap'$ *and* $bp\ \ \alpha\ bp')$*;*
   (ii) $\forall n \in \mathbb{N}$, $\big(N(n),\alpha_{|}\big)$ *is linear with extremes* a^n *and* b^n *(* a^n *denotes the* n *letters word* $aa\cdots a$*);*
2. $(\mathbb{K},\alpha)$ *is called a* linear object *if it admits a linear dichotomic tree that generates it.*

Proposition 3.5. *If* $(\mathbb{K},\alpha)$ *and* $(\mathbb{K},\alpha')$ *are linear objects of* **CRLR***, then they are isomorphic.*

Proof. Let $\mathbb{P} = \langle a,b\rangle$ and $\mathbb{P}' = \langle a',b'\rangle$ be linear trees that generate $(\mathbb{K},\alpha)$ and $(\mathbb{K},\alpha')$ respectively and such that $\forall m \in \mathbb{N}$, $\big(N(m),\alpha_{|}\big)$ and $\Big(N'(m),\alpha'_{|}\Big)$ are linear with extremes a^m, b^m and ${a'}^m$, ${b'}^m$ respectively. It suffices to define $f_m : N(m) \longrightarrow N'(m)$ by $f_m(p) = p' =$: the word that is obtained from p when each a is replaced by a' and each b is replaced by b' and then apply Proposition 3.1. ☑

Proposition 3.6. *The unit interval* $I = [0,1]$ *is a Hausdorff quotient of the Cantor space which admits a linear algebraic representation in the category* **CRLR**.

Proof. Let $\Sigma = \{0,1\}$ be an alphabet with two symbols. In $\{0,1\}^{\mathbb{N}}$ we define the $\sim$ relation by:

$$x \sim y \Longleftrightarrow (x=y) \vee \big(x,y \in \big\{w0\bar{1},\ w1\bar{0}\big\}\,,\ w \in \Sigma^*\ \big)\,.$$

Then $\sim$ is a closed equivalence relation on the Cantor space $\Sigma^{\mathbb{N}}$ and $\Sigma^{\mathbb{N}}/\sim\ \simeq I$. The dichotomic tree of Figure 1 generates the ring $\mathbb{A}(\Sigma^{\mathbb{N}})$, and then $(\mathbb{A}(\Sigma^{\mathbb{N}}),R_{\sim})$ is a linear object of **CRLR**. ☑

As an immediate consequence of propositions 3.5, 3.6 and the equivalence between the categories **CRLR** and **RHQC**, we have:

Corollary 3.1. *I is the unique (up isomorphism) topological space that admits a linear algebraic representation in* **CRLR**.

Similarly an algebraic representation of the circle S^1, the Sierpiński triangular curve and the simple triod can be obtained.

The circle S^1.

Definition 3.7. *Let* X *be a set and* α *be a relation in* X*. If for every* $x \in X$, $|\dot{x}| = 2$ *then* α *is called a* **circular relation***, or* (X,α) *is called* **circular**.

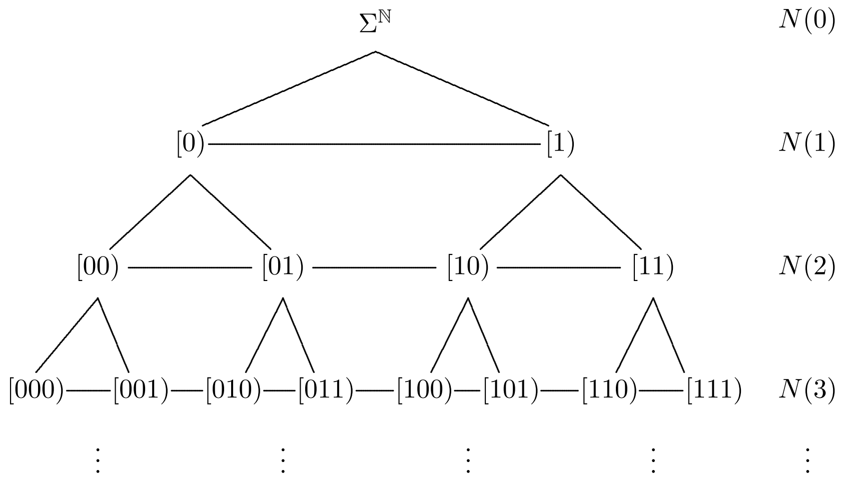

FIGURE 1

Definition 3.8. *Let* $(\mathbb{K},\alpha)$ *be an object of* **CRLR** *and* $\mathbb{P} =< a, b >$ *a dichotomic tree of top* 1*,*

1. $(\mathbb{P},\alpha|_{\mathbb{P}})$ *is called* **circular** *if it satisfies the following three conditions:*
   (i) $a\alpha b$;
   (ii) $(\forall p,p' \in \mathbb{P})\,(\,p\alpha p' \Longrightarrow ap\ \alpha\ ap'$ *and* $bp\ \alpha\ bp')$;
   (iii) $\forall n \geq 2$, $\big(N(n),\alpha_{|}\big)$ *is circular.*
2. $(\mathbb{K},\alpha)$ *is called a* **circular object** *if it admits a dichotomic circular tree which generates it.*

It follows:

Proposition 3.9. *If* $(\mathbb{K},\alpha)$ *and* $(\mathbb{K},\alpha')$ *are circular objects of* **CRLR***, then they are isomorphic.*

Proposition 3.10. S^1 *is a Hausdorff quotient of the Cantor space which admits a circular algebraic representation in the category* **CRLR***.*

Proof. In $\{0,1\}^{\mathbb{N}}$ the relation $\sim$ is defined by:

$$x \sim y \Longleftrightarrow (x = y) \vee \big(x, y \in \big\{w0\bar{1},\ w1\bar{0}\big\}\,,\ w \in \Sigma^*\ \big) \vee \big(x, y \in \{\bar{0},\bar{1}\}\big)\,.$$

The quotient space obtained is the circle S^1.

In the dichotomic tree of Figure 2, which generates the ring $\mathbb{A}(\Sigma^{\mathbb{N}})$, we have that $(\mathbb{A}(\Sigma^{\mathbb{N}}), R_{\sim})$ is a circular object of **CRLR**. ☑

Corollary 3.2. S^1 *is the unique (up to homeomorphism) topological space which admits a circular algebraic representation in* **CRLR***.*

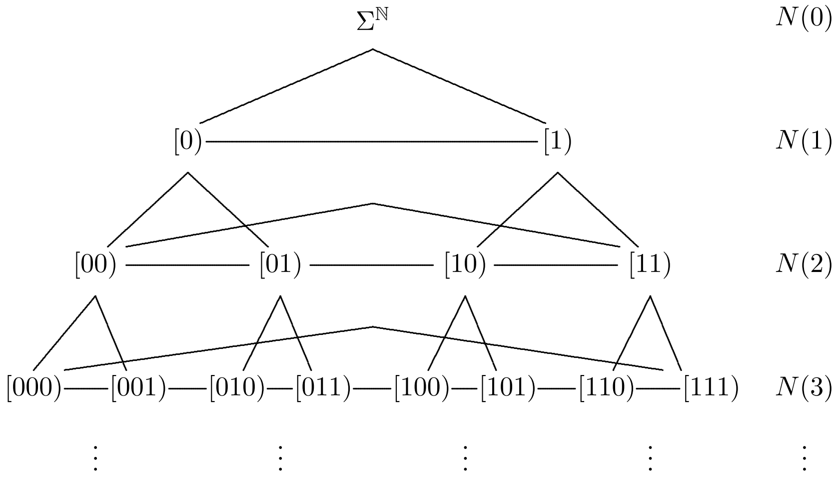

FIGURE 2

The Sierpiński triangular curve. Let K be an equilateral triangle on the plane. Divide K into four congruent triangles and remove the interior of the middle one. The remaining triangles have diameters equal to half of that of K and the union K_1 of them is connected. Apply the same dividing procedure to the remaining triangles, and then to the triangles obtained recursively in this way. We get at the $n-th$ stage of the procedure 3^n congruent triangles of diameters equal to the $\left(\frac{1}{2^n}-th\right)$ of the diameter of K. The union K_n of the triangles obtained at the $n-th$ stage is connected and thus a plane continuum. The intersection

$$\mathcal{S} = K \cap K_1 \cap K_2 \cap \cdots$$

is the *Sierpiński triangular curve* that was described by Sierpiński in [5]. $\mathcal{S}$ can be obtained as follows: in $\{0,1,2\}^{\mathbb{N}}$ the relation $\sim$ is defined by:

$$\begin{aligned} x \sim y \Longleftrightarrow (x=y) &\vee \left(x,y \in \left\{w0\overline{1},\ w1\overline{0}\right\},\ w \in \Sigma^*\right) \\ &\vee \left(x,y \in \left\{w0\overline{2},\ w2\overline{0}\right\},\ w \in \Sigma^*\right) \\ &\vee \left(x,y \in \left\{w1\overline{2},\ w2\overline{1}\right\},\ w \in \Sigma^*\right). \end{aligned}$$

The quotient space obtained is the Sierpiński triangular curve.

Definition 3.11. *Let X be a set and α be a relation in X. If the following two conditions are satisfied:*

(i) *there exist a, b, c, three different elements of X, such that $|\dot{a}| = |\dot{b}| = |\dot{c}| = 2$;*
(ii) *for every $x \in X - \{a,b,c\}$, $|\dot{x}| = 3$,*

then α is called a **Sierpiński relation with extremes** a, b **and** c, *or (X,α) is called* **of Sierpiński with extremes** a, b **and** c.

Figure 3 shows some graphs of Sierpiński relations.

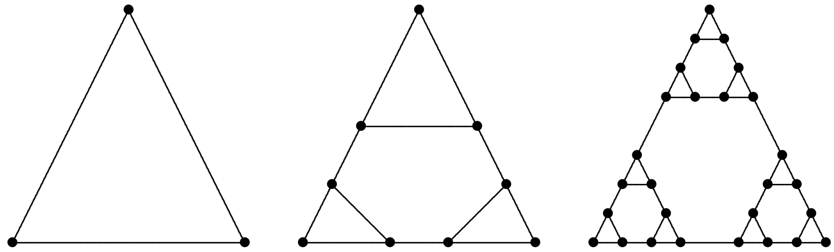

FIGURE 3

Definition 3.12. *Let* $(\mathbb{K}, \alpha)$ *be an object of* **CRLR** *and* $\mathbb{P} =< a, b, c >$ *a ternary tree of top* 1*,*

1. $(\mathbb{P}, \alpha|_{\mathbb{P}})$ *is called* **Sierpiński** *if it satisfies the following two conditions:*
   (i) $(\forall p, p' \in \mathbb{P})\,(\ p\alpha p' \Longrightarrow ap\ \alpha\ ap',\quad bp\ \ \alpha\ bp'$ *and* $cp\ \alpha\ cp')$*;*
   (ii) $\forall n \geq 2,\ \big(N(n), \alpha_{|}\big)$ *is Sierpiński.*
2. $(\mathbb{K}, \alpha)$ *is called a* **Sierpiński object** *if it admits a Sierpiński tree that generates it.*

Then we have:

Proposition 3.13. *If* $(\mathbb{K}, \alpha)$ *and* $(\mathbb{K}, \alpha')$ *are Sierpiński objects of* **CRLR***, then they are isomorphic.*

Using the quotient relation $\sim$ defined above and a ternary tree which generates the ring $\mathbb{A}\left(\Sigma^{\mathbb{N}}\right)$, we have that $(\mathbb{A}(\Sigma^{\mathbb{N}}), R_{\sim})$ is a Sierpiński object of **CRLR**. That is,

Proposition 3.14. *The Sierpiński triangular curve is a Hausdorff quotient of the Cantor space which admits a Sierpiński algebraic representation in the category* **CRLR***.*

Corollary 3.3. *The Sierpiński triangular curve is the unique (up to homeomorphism) topological space which admits a Sierpiński algebraic representation in* **CRLR***.*

The simple triod. A **simple triod** $\mathcal{T}$ is the continuum that is homeomorphic to the symbol T.

In $\{0,1\}^{\mathbb{N}}$ the $\sim$ relation is defined by:

$$\begin{aligned} x \sim y \Longleftrightarrow (x = y) &\vee \left(x, y \in \left\{00\overline{1},\ 1\overline{0},\ 01\overline{0}\right\}\right) \\ &\vee \left(x, y \in \left\{w00\overline{1},\ w01\overline{0}\right\},\ w \in \Sigma^*\right) \\ &\vee \left(x, y \in \left\{w10\overline{1}, w11\overline{0}\right\},\ w \in \Sigma^*\right). \end{aligned}$$

The quotient space obtained is the simple triod $\mathcal{T}$.

Definition 3.15. *Let X be a finite set and α be a linear relation in X. We call* **length** *of α the cardinal of X.*

Definition 3.16. *Let X be a set and α be a relation in X. If the following two conditions are satisfied:*

(i) $|X| = 2^n$, $n \in \mathbb{N}$;
(ii) *there exist a, b, c three different elements of X, such that α restricted to $\{a, b, c\}$ is circular;*
(iii) *a, b, c are extremes of three linear relations mutually disjoint, two of them of length 2^{n-2} and the third of length 2^{n-1},*

then α is called a **triodal relation**, *or (X, α) is called* **triodal**.

Figure 4 shows some graphs of triodal relations.

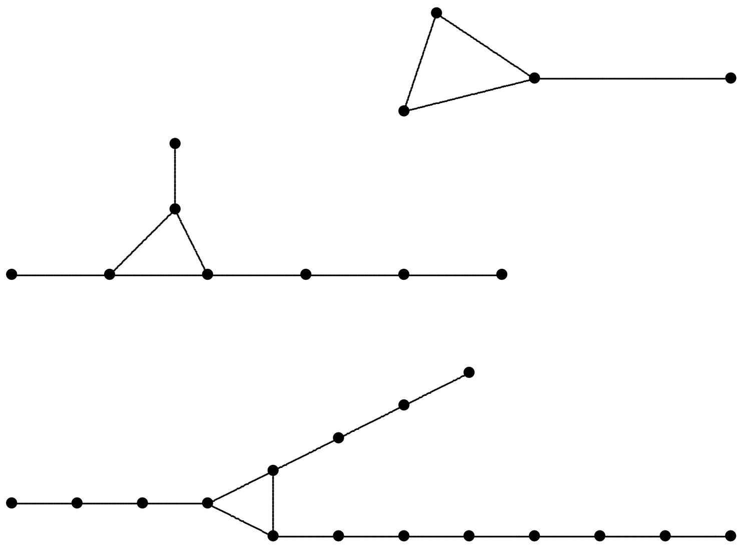

FIGURE 4

Definition 3.17. *Let $(\mathbb{K}, \alpha)$ be an object of* **CRLR** *and $\mathbb{P} =< a, b >$ a dichotomic tree of top 1,*

1. *$(\mathbb{P}, \alpha|_{\mathbb{P}})$ is called* **triodal** *if it satisfies the following three conditions:*
   (i) $a\alpha b$
   (ii) $(\forall p, p' \in \mathbb{P})\,(\, p\alpha p' \Longrightarrow ap \;\alpha\; ap',\ and\ bp \;\; \alpha\; bp')$;
   (iii) $\forall n \geq 2$, $\big(N(n), \alpha_|\big)$ *is triodal.*
2. *$(\mathbb{K}, \alpha)$ is called a* **triodal object** *if it admits a dichotomic triodal tree that generates it.*

Hence we have:

Proposition 3.18. *If* $(\mathbb{K}, \alpha)$ *and* $(\mathbb{K}, \alpha')$ *are triodal objects of* **CRLR***, then they are isomorphic.*

Using the quotient relation $\sim$ defined above and a dichotomic tree which generates the ring $\mathbb{A}\left(\Sigma^{\mathbb{N}}\right)$, we have that $\left(\mathbb{A}\left(\Sigma^{\mathbb{N}}\right), R_{\sim}\right)$ is a triodal object of **CRLR**. That is,

Proposition 3.19. *The simple triod is a Hausdorff quotient of the Cantor space which admits a triodal algebraic representation in the category* **CRLR**.

And finally,

Corollary 3.4. *The simple triod is the unique (up to homeomorphism) topological space which admits a triodal algebraic representation in* **CRLR**.

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