PUBLICATIONS DE L'INSTITUT MATHÉMATIQUE
Nouvelle série tome 54 (68), 1993, 47–54

# ON REGULAR AND ABSOLUTELY REGULAR DISTRIBUTIONS IN $K'_M$

**Saleh Abdullah**

**Abstract**. We characterize spaces of regular and absolutely regular distributions of rapid growth and study their topological structure.

**Introduction.** Pilipović [**5**] characterized regular $K'_M\{M_p\}$ distributions for some sequences $M_p$ of functions which satisfy some suitable conditions. In this paper we study absolutely regular and regular $K'_M$ distributions. The space $K'_M$ is a $K'_M\{M_p\}$ space with $M_p(x) = e^{pM(x)}$, where $M(x)$ is a function. This sequence satisfies the conditions (1)–(5) and (N) of [**5**]. The distributions studied in [**5**] here will be called absolutely regular. We will distinguish them from what we will call regular $K'_M$ distributions. We utilize an example of Pilipović to show that in $K'_M$ the absolutely regular distributions are properly contained in the regular ones. It turns out that the space of absolutely regular distributions in $K'_M$ is not stable under differentiation. Moreover, we provide the space $(K'_M)_{ar}$ of absolutely regular distributions in $K'_M$ with several topologies, and find relations between them. Finally we characterize the space $(K'_M)_r$ of regular distributions in $K'_M$, provide it with two topologies and show that they are equal.

## Notation and preliminary results

We will use the standard notation as in [**4**]. The space $K_M$ of test functions consists of all $C^\infty$-functions $\varphi$ such that

$$\nu_k(\varphi) = \sup_{\substack{x\in\mathbf{R}^n \\ |\alpha|\leq k}} e^{M(kx)}\,|D^\alpha\varphi(x)| < \infty;\;\; k = 0,1,2,\dots\;.$$

The function $M(x)$ is even, incereasing and convex. An example of such function is $M(x) = |x|^p/p;\; p > 1$, and $M(x) = e^x$. We provide $K_M$ with the topology generated by the family of seminorms $\{\nu_k;\; k = 0,1,2,\dots\}$. The space $K_M$

*AMS Subject Classification* (1980): Primary 46 F 05

is Frechet, nuclear, barreled, reflexive, bornologic, Montel and a normal space of distributions. It turns out that $K_M$ coincides with the intersection of the spaces $e^{-M(kx)}S$; $k = 0, 1, 2, \dots$, provided with the projective limit topology. By $K'_M$ we denote the space of continuous linear functionals on $K_M$ provided with the strong dual topology, which will be denoted by $\beta(K'_M, K_M)$. It turns out that $K'_M$ coincides with the space which consists of the union of the spaces $e^{M(kx)}S'$; $k = 0, 1, 2, \dots$, provided with the inductive limit topology. For more details on the spaces $K_M$ and $K'_M$ we refer the reader to [**1**]. We denote by $O_c(K'_M; K'_M)$ the union of the spaces $e^{M(kx)}S$; $k = 0, 1, 2, \dots$, provided with inductive limit topology. The space $O_c(K'_M; K'_M)$ is boronologic, and it coincides with the strong dual of $O'_c(K'_M; K'_M)$ of convolution operators in $K'_M$. The folowing theorem which characterizes the elements of $K'_M$ will be used later in the lext.

THEOREM A [**1**, Theorem 4]. *Let $T$ be any distribution; the following statements are equivalent.*

(a) *$T$ is in $K'_M$;*

(b) *$T = D^\alpha[e^{M(kx)}f]$, for some multi-index $\alpha$, a positive integer $k$ and a bounded continuous function $f$;*

(c) *For any $\varphi \in K_M$, there exists a positive integer $k_1$ such that* *$D^\alpha(T^*\varphi)(x) = O(e^{M(k_1 x)})$, for all multi-indices $\alpha$.*

Following [**5**] we denote by $(K_M)_\infty$ the vector space of functions $f$ defined almost everywhere on $\mathbf{R}^n$ for which the seminorms

$$q_k(f) = \operatorname{ess\,sup}\{e^{M(kx)}|f(x)|;\ x \in \mathbf{R}^n\};\ \ k = 0, 1, 2, \dots ,$$

are finite. We equip this space with the topology defined by the family of seminorms $q_k$; $k = 0, 1, 2, \dots$ . By $L^1_{\text{loc}}$ we denote the space of all locally integrable functions on $\mathbf{R}^n$, provided with the topology generated by the family of seminorms $\|u\|_{1,K} = \int_K |u|dx$; $K$ any compact subset of $\mathbf{R}^n$. The space $L^1_{\text{loc}}$ is contained in $D'$, the space of Schwartz distributions.

## Regular and absolutely regular distributions in $K'_M$

We give first the definitions of regular and absolutely regular distributions in $K'_M$.

*Definition* 1. A distrubution $u \in K'_M$ is said to be absolutely regular if $u \in L^1_{\text{loc}}$ and $u\varphi \in L^1$ for every $\varphi \in K_M$. The set of all absolutely regular distributions in $K'_M$ will be denoted by $(K'_M)_{ar}$.

We should mention that in [**5**] the space $(K'_M)_{ar}$ was called the space of regular $K'_M$ distrubutions. We called the elements of this space absolutely regular to distinguish them from the elements of $K'_M$ which satisfy the following defintion.

*Definition* 2. A distribution $u \in K'_M$ is said to be regular if $u \in L^1_{\text{loc}}$, i.e. $(K'_M)_r = K'_M \cap L^1_{\text{loc}}$.

After I finished writting the paper, I found out that the above distinction between regular and absolutely regular distributions, in the general space $K'\{M_p\}$, was given independently by Pilipović [**6**].

*Example.* We utilize the following example of Pilipović [**5**] to show that $(K'_M)_{ar}$ is properly contained in $(K'_M)_r$. Let $\psi$ be the element of $K_M$ which was given in [**5**, p. 33]. $\psi$ is nonnegative, and $f(x)\psi(x)$ does not belong to $L^1$ for any nonnegative continuous function $f$. Let $F(x) = \int_0^x f(t)dt$, and let $S = e^{iF(x)}$. For $\varphi \in K_M$ we define

$$\langle S, \varphi\rangle = \int e^{iF(x)}\varphi(x)dx, \text{ for all } \varphi \in K_M.$$

Since $|\langle S,\varphi\rangle| \leq \int |\varphi(x)|dx$, it follows that $S \in K'_M$. Moreover, $S \in L^1_{\text{loc}}$ since it is a bounded continuous function. For any $\varphi \in K_M$ one has

$$\left|\int S\varphi dx\right| \leq \int \left|e^{iF(x)}\varphi(x)\right| dx \leq \int |\varphi(x)|dx < \infty.$$

Thus $S$ is absolutely regular. Let $T$ be the derivative of $S$. Since $S$ is differentiable as a function, namely $S' = if(x)e^{iF(x)}$, its derivative in the sense of distributions coincides with its derivative in the usual sense. We claim that $S'$ does not belong to $(K'_M)_{ar}$, but $S'$ is in $(K'_M)_r$. Indeed, it was shown in [**5**] that $S'$ is not in $(K'_M)_{ar}$. But on the other hand $S'$ is a bounded continuous function, hence $T \in L^1_{\text{loc}}$.

*Definition* 3. A sequence $(\psi_k;\ k \in \mathbf{N})$ in $D$ is called an approximate unit in $K_M$ if the following conditions hold:

(i) $\lim_{k\to\infty} \psi_k = 1$ in $\mathcal{E}$;

(ii) The set $\{\psi_k : k \in \mathbf{N}\}$ is bounded in $B$, i.e. for any $\alpha \in \mathbf{N}^n$ there exists a constant $C_\alpha > 0$ such that $|D^\alpha\psi_k(x)| \leq C_\alpha$ for all $x$ in $\mathbf{R}^n$ and all $k \in \mathbf{N}$.

This definition is due to, Dierolf [**2**] who gave an example of such sequences. Deirolf's example was given for Schwartz space $S$ of rapidly decreasing functions. The same example do for $K_M$. Moreover, $\lim_{k\to\infty} \psi_k\varphi = \varphi$ in $K_M$ for all $\varphi$ in $K_M$. Indeed, given any positive integer $m$, one has

$$\begin{aligned}
\nu_m(\psi_k\varphi - \varphi) &= \sup_{\substack{|\alpha|\leq m\\ x\in\mathbf{R}^n}} e^{M(mx)}\left|D^\alpha((\psi_k - 1)\varphi(x)\right| \\
&\leq \sum_{\beta\leq\alpha} \sup_{\substack{x\in\mathbf{R}^n\\ |\alpha|\leq m}} e^{-M(mx)}\left|D^\beta(\psi_k - 1)(x)\right| e^{M(2mx)}\left|D^{\alpha-\beta}\varphi(x)\right| \\
&\leq \sum_{\beta\leq\alpha} \sup_{\substack{x\in\mathbf{R}^n\\ |\alpha|\leq m}} e^{-M(mx)}\left|D^\beta(\psi_k - 1)(x)\right| \sup_{\substack{x\in\mathbf{R}^n\\ |\alpha|\leq m}} e^{M(2mx)}\left|D^{\alpha-\beta}\varphi(x)\right| \\
&\leq C_1 \sup_{\substack{|\beta|\leq m\\ x\in\mathbf{R}^n}} e^{-M(mx)}\left|D^\beta(\psi_k - 1)(x)\right|,
\end{aligned} \tag{1}$$

for some constant $C_1$. Since the set $\{\psi_k;\ k \in \mathbf{N}\}$ is bounded in $B$ and $e^{-M(mx)}$ goes to zero as $x$ goes to infinity, it follows that the supremum on the right-hand side of (1) occurs when $x$ is bounded. Thus we can find a compact subset $K$ of $\mathbf{R}^n$ such that

$$\sup_{\substack{x\in\mathbf{R}^n \\ |\alpha|\leq m}} e^{-M}(mx)\left|D^{\beta}(\psi_k-1)(x)\right| \leq \sup_{\substack{|\beta|\leq m \\ x\in K}} e^{-M(mx)}\left|D^{\beta}(\psi_k-1)(x)\right|.$$

Since $\lim_{k\to\infty}\psi_k = 1$ in $\xi$, from (1) it follows that

(2) $\nu_m((\psi_k\varphi-\varphi)) \leq C\sup_{\substack{\beta\leq m \\ x\in K}}\left|D^{\beta}(\psi_k-1)(x)\right| = C\|\psi_k-1\|_{m,K}.$

Since the right-hand side of (2) converges to zero as $k$ goes to infinity, this completes the proof of the claim.

The following theorem characterizes the elements of $(K'_M)_{ar}$; it is a combination of Theorem 2 of [**5**] and Proposition 1.1 of [**2**].

THEOREM 1. *Let $f\in L^1_{\text{loc}}$. The following statements are equivalent:*

(i) *$f$ is in $(K'_M)_{ar}$;*

(ii) *There exists a positive integer $k$ such that $e^{-M(kx)}f\in L^1(\mathbf{R}^n)$;*

(iii) *The map $\varphi\longrightarrow f\varphi$ from $K_M$ into $L^1$ is continuous;*

(iv) *The map $\varphi\longrightarrow f\varphi$ from $(K_M)_\infty$ into $L^1$ is continuous;*

(v) *For every $g\in L^\infty(\mathbf{R}^n)$ the linear form*

$$\Lambda:\varphi\longrightarrow\int_{\mathbf{R}}^{n} g(x)f(x)\varphi(x)dx,\quad \varphi\in D$$

*is continuous on $D$ with the topology induced by $K_M$.*

*Proof.* We will prove the equivalence of (ii), (iii) and (v). For the proof of the remaining statements see [**5**] and [**7**].

The implication (ii) $\Rightarrow$ (iii). Let $k$ be a positive integer such that

$$e^{-M(kx)}f\in L^1(\mathbf{R}^n).$$

For any $\varphi\in K_M$ we have

$$\begin{aligned}\int|f\varphi|dx &= \int e^{M(kx)}|\varphi(x)|e^{-M(kx)}|f(x)|dx \\ &\leq \nu_k(\varphi)\int e^{-M(kx)}|f(x)|dx \leq C\nu_k(\varphi).\end{aligned}\tag{3}$$

Since $K_M$ is bornologic the, inequality (3) implies that $f\varphi\in L^1$ and the mapping $\varphi\longrightarrow f\varphi$ from $K_M$ into $L^1(\mathbf{R}^n)$ is continuous.

The implication (iii) $\Rightarrow$ (v). Given $g\in L^\infty(\mathbf{R}^n)$, for any $\varphi\in D$ it follows form (iii) that

$$\begin{aligned}\int|g(x)f(x)\varphi(x)|dx &\leq \|g\|_\infty\int|f(x)||\varphi(x)|dx \\ &\leq C_1\|g\|_\infty\nu_k(\varphi) = C\nu_k(\varphi),\end{aligned}$$

which completes the proof of the implication.

The implication (v) $\Rightarrow$ (ii). The proof is similar to the proof of the implication (a) $\Rightarrow$ (b) of Proposition 1.1 of [**2**] and is omitted.

The following theorem characterizes the space $(K'_M)_r$ of regular distributions in $K'_M$.

THEOREM 2. *Let $f \in L^1_{\text{loc}}(\mathbf{R}^n)$; the following statements are equivalent:*

(1) *$f \in (K'_M)_r$;*

(2) *$f^*\varphi \in O_c$ for all $\varphi \in K_M$;*

(3) *There exists an approximate unit in $K_M$, $(\psi_k;\ k \in \mathbf{N})$, such that for all $\varphi$ in $K_M$ the sequence $(\int_{\mathbf{R}^n} f(x)\psi_k(x)\varphi(x)dx;\ k \in \mathbf{N})$ is convergent.*

*Proof.* Equivalence of (1) and (2) follows from the structure theorem of rapidly increasing distributions (Theorem A). Next we prove equivalence of (1) and (3). Let the sequence $(\psi_k;\ k \in \mathbf{N})$ be the approximate unit in $K_M$. For every $k \in \mathbf{N}$ one has $\psi_k \in O_c \subset O_M$, where $O_M$ is the space of multipliers of $K_M$. It was shown (after Definition 3) that $\psi_k\varphi \longrightarrow \varphi$ in $K_M$ for every $\varphi \in K_M$. Hence $\psi_k \longrightarrow 1$ in $O_M$. Thus, for any $f \in (K'_M)_r$ the sequence $(\psi_k f)$ converges to $f$ in $K'_M$. Hence $\langle \psi_k f, \varphi\rangle \longrightarrow \langle f, \varphi\rangle$ for all $\varphi \in K_M$, i.e. $\langle f, \psi_k\varphi\rangle = \int f(x)\psi_k(x)\varphi(x)dx \longrightarrow \int f(x)\varphi(x)dx$. Conversely, suppose that the sequence $(\langle f\psi_k, \varphi\rangle) = (\int f(x)\psi_k(x)\varphi(x)dx)$ converges for every $\varphi \in K_M$. Define $\langle f, \varphi\rangle = \lim_{k\to\infty}\langle f\psi_k, \varphi\rangle, \varphi \in K_M$. Thus $f$ is a well-defined linar functional on $K_M$. Since $K_M$ is bornologic, continuity of $f$ is equivalent to sequential continuity. Let $(\varphi_j;\ j \in \mathbf{N})$ be a sequence in $K_M$ converging to $O$ in $K_M$. Since $\psi_k \longrightarrow 1$ in $O_M$, it follows that $\psi_k\varphi_j \longrightarrow \varphi_j$ uniformly in $j$. Thus

$$\lim_{j\to\infty}\langle f, \varphi_j\rangle = \lim_{j\to\infty}\lim_{k\to\infty}\langle f\psi_k, \varphi_j\rangle = \lim_{k\to\infty}\lim_{j\to\infty}\langle f, \psi_k\varphi_j\rangle = \lim_{k\to\infty}\lim_{j\to\infty}\langle f, \psi_k\varphi_j\rangle.$$

On the other hand since $f \in L^1_{\text{loc}}$, one has for each fixed $k$

$$\begin{aligned}|\langle f, \psi_k\varphi_j\rangle| &\le \int_{\sup p\psi_k}|f(x)||\psi_k(x)||\varphi_j(x)|dx \\ &\le \sup_{x\in\mathbf{R}^n}|\varphi_j(x)|\int_{\sup p\psi_k}|f(x)||\psi_k(x)|dx \le C_k\sup_{x\in\mathbf{R}^n}|\varphi_j(x)|.\end{aligned}$$

Therefore

(4) $\lim_{j\to\infty}\langle f, \varphi_j\rangle \le \lim_{j\to\infty}|\langle f, \varphi_j\rangle| \le \lim_{k\to\infty} C_k \lim_{j\to\infty}\sup_{x\in\mathbf{R}^n}|\varphi_j(x)|$.

Since $\varphi_j \longrightarrow O$ in $K'_M$, the right hand-side of (4) converges to zero. This completes the proof of the theorem.

From theorems $A$ and 2 it follows that every regular distribution in $K'_M$ is the derivative (in the sense of distributions) of an absolutely regular distribution in $K'_M$. This, together with the example following Definition 2 motivates the following question. Is it true that the distributional derivative of every absolutely regular distribution in $K'_M$ is regular? The following proposition which gives an affirmative answer to the question is due to S. Pilipović. The proof which is provided here is different from the one suggested by him.

Proposition 1. *The derivative of every absolutely regular distribution in $K'_M$ is regular.*

*Proof.* Let $u$ be any absolutely regular distribution in $K'_M$ and $\beta$ any multi-index. Since $u \in L^1_{\text{loc}}$, Theorem 2.3.1 of Hormander [**3**], integration by parts, and induction on the order of the derivative imply that $D^\beta u$ is in $L^1_{\text{loc}}$. From part (b) of Theorem $A$ it follows that $D^\beta u$ is in $K'_M$. Thus $D^\beta u \in (K'_M)_{ar}$.

### Topologies of the spaces $(K'_M)_{ar}$ and $(K'_M)_r$

The structure theorems of $(K'_M)_{ar}$ and $(K'_M)_r$ (Theorems 1 and 2 respectively) suggest several topologies on these spaces. Both $(K'_M)_{ar}$ and $(K'_M)_r$ are contained in $K'_M \cap L^1_{\text{loc}}$. In each of those topologies for each of these spaces we require continuity of the map $f \longrightarrow (f, f)$ from $(K'_M)_{ar}$ or $(K'_M)_r$ into $K'_M x L^1_{\text{loc}}$.

We first start by considering topologies of the space $(K'_M)_{ar}$. The first topology $\tau_1$ of $(K'_M)_{ar}$ is the one induced by $\beta(K'_M;\ K_M)$, the strong dual topology on $K'_M$, i.e. the topology of uniform convergence on bounded subsets of $K_M$. Property (ii) of Theorem 1 suggests the inductive limit topology $\tau_2$ defined by the union $\bigcup_{k\in\mathbf{N}} L^1_{-k}$, where

$$L^1_{-k} = \{f \in (K'_M)_{ar} \text{ such that } e^{-M(kx)} f \in L^1\},$$

provided with the topology generated by the norm $\|f\|_{-k} = \int e^{-M(kx)} |f(x)| dx$. Property (iii) of the same theorem suggests the topology $\tau_3$ induced by the space $L(K_M, L^1)$ of continuous linear maps from $K_M$ into $L^1$ provided with the topology of uniform convergence on bounded subsets of $K_M$. Property (iv) of the same theorem suggests the topology $\tau_4$ induced by the space $L((K_M)_\infty, L^1)$ of continuous linear maps from $(K_M)_\infty$ into $L^1$ provided with the topology of uniform convergence on bounded subsets of $(K_M)_\infty$. We have the following

Theorem 3. *The topologies $\tau_1, \tau_2, \tau_3$ and $\tau_4$ of $(K'_M)_{ar}$ satisfy the relation $\tau_1 \le \tau_3 \le \tau_4 \le \tau_2$.*

*Proof.* $\tau_1$ is weaker than $\tau_3$: Let $U(B^o) = \{f \in (K'_M)_{ar}: |\langle f, \varphi\rangle| < 1 \text{ for all } \varphi \in B\}$, where $B$ is a bounded subset of $K_M$, be a member of 0-neighborhood base in $\tau_1$. One can see easily that the set

$$V(B, \|\ \|_{1,1}) = \{f \in (K'_M)_{ar}: \|f\varphi\|_1 < 1 \text{ for all } \varphi \in B\},$$

is a member of 0-neighborhood base in $\tau_3$, and is contained in $U(B^o)$.

$\tau_3$ is weaker than $\tau_4$: A member of 0-neighborhood base in $\tau_4$ is of the form $W(B_1, r_1) = \{f \in (K'_M)_{ar}: \int |f\varphi| < r_1 \text{ for all } \varphi \text{ in } B_1\}$, where $B_1$ is a bounded subset of $(K_M)_\infty$ and $r_1$ is a positive real number. Let $V(B, \|\ \|_{1,5})$, where $B$ is a bounded subset of $K_M$ and $s$ is a positive number, be a member of 0-neighborhood base in $\tau_3$. To prove the assertion it suffices to show that $B$ is a bounded subset of $(K_M)_\infty$. Let $W_1$ be a neighborhood of 0 in $(K_M)_\infty$. Without loss of generality we can assume that $W_1 = \{f \in (K_M)_\infty: q_k(f) < \epsilon\}$, for some positive integer $k$ and $a$ positive $\epsilon$. Let

$$V_1 = \{\psi \in K_M: \sup_{\substack{|\alpha|\le k \\ x\in\mathbf{R}^n}} e^{M(kx)} |D^\alpha \psi(x)| < \epsilon\}.$$

$V_1$ is a member of 0-neighborhood base in the topology of $K_M$. Since $B$ is bounded in $K_M$, it follows that there exists a positive number $\lambda$ such that $\lambda B \subset V_1 \subset W_1$. Hence $B$ is bounded in $(K_M)_\infty$.

$\tau_4$ is weaker than $\tau_2$: Let $W(B_1, r_1)$ be any member of 0-neighborhood base in $\tau_4$. Let $k$ be any positive integer. Since $B_1$ is bounded in $(K_M)_\infty$, there exists a positive constant $C_k$ such that $q_k(\theta) < C_k$ for all $\theta$ in $B_1$, i.e. $|\theta(x)| \le C_k e^{-M(kx)}$ for almost all $x$. Let

$$V(k, \|\ \|_{1,r_1/C_k}) = \{f \in (K'_M)_{ar}\colon \int e^{-M(kx)} |f(x)| dx < r_1/C_k\}.$$

It is clear that $V(k, \|\ \|_{1,r_1/C_k})$ is contained in $W(B_1, r_1)$ for all $k$. Hence $\bigcup_{k=1}^{\infty} V(k, \|\ \|_{1,r/C_k})$ is contained in $W(B_1, r_1)$. Since without loss of generality we can assume that $W(B_1, r_1)$ is convex, it follows that $\Gamma(\bigcup_{k=1}^{\infty} V(k, \|\ \|_{1,r_1/C_k})$ is contained in $W(B_1, r_1)$, where $\Gamma$ denotes the convex hull.

Next we study the topologies of $(K'_M)_r$. The first topology $\tau_s$ is the topology of $(K'_M)_r$ induced by $\beta(K'_M, K_M)$, the strong dual topology. The second topology $\tau_c$ is the topology induced by $L(K_M, O_c)$ of uniform convergence on bounded subsets of $K_M$. A member of 0-neighborhood base in $\tau_c$ is of the form

$$V(B, U) = \{f \in K'_M \cap L^1_{\text{loc}}\colon f^*\varphi \in U \ \text{ for all } \ \varphi \in B\},$$

where $B$ is a bounded subset of $K_M$ and $U$ is a neighborhood of 0 in $O_c$. Without loss of generality we can assume that $U = (B'_c)^o$; the polar of $B'_c$ a bounded subset of $O'_c$. We have the following

THEOREM 4. *On $(K'_M)_r$ the topologies $\tau_s$ and $\tau_c$ are equal.*

*Proof.* We show first that $\tau_c$ is weaker than $\tau_s$. Let $V(B, U)$ be a member of 0-neighborhood base in $\tau_c$. We have

$$V(B, U) = \{f \in K'_M \cap L^1_{\text{loc}}\colon |\langle f^*\varphi, S\rangle| = |\langle f, \check{\varphi}^* S\rangle| < 1, \ \text{ for all } \ \varphi \in B \ \text{ and } \ S \in B'_c\}$$

where $B$ is a bounded subset of $K_M$ and $B'_c$ is a bounded subset of $O'_c$. Since the whole set

$$\{\varphi \in K_M\colon \ \text{the orbit } \ \{\varphi^* S\colon S \in B'_c\} \ \text{ is bounded in } \ K_M\}$$

is of $K_M$, which is of second category, it follows from the uniform boundedness principle that the set $L = \{\Lambda_s\colon K_M \longrightarrow K_M;\ \Lambda_s(\varphi) = \varphi^* S,\ S \in B'_c\}$ is equicontinuous. Hence there exists a bounded subset $B_2$ of $K_M$ such that $\check{B} * B'_c \subset B_2$. Thus $\check{B} * B'_c$ is bounded in $K_M$. Hence $(\check{B} * B'_c)^o$ is a member of 0-neighborhood base in $\tau_s$. On the other hand we notice that $V(B, U) = (\check{B} * B'_c)^o$, thus $V(B, U)$ is a member of 0-neighborhood base in $\tau_s$.

Finally we show that $\tau_s$ is weaker than $\tau_c$. Let $W(B) = (K'_M)_r \cap B^o$, where $B$ is a bounded subset of $K_M$, be a member of 0-neighborhood base in $\tau_s$. It follows immediatly that $W(B) = V(\check{B}, \{\delta\}^o)$ is a member of 0-neighborhood base in $\tau_c$. This completes the proof of the theorem.

**Acknowledgement.** I would like to thank Professor Stevan Pilipović for providing me with references [**6**] and with a proof of Proposition 1.

REFERENCES

[1] S. Abdullah, *On distributions of rapid growth*, Portugaliae Math. **47** (1990), 191-204.

[2] P. Dierolf, *Some locally convex spaces of regular distributions*, Studia Math. **77** (1984), 393-421.

[3] L. Hormander, *Linear Partial Differntial Operators*, Grundlehren der mathematischen Wissenschaften 116, Springer-Verlag, New York, 1976.

[4] J. Horvath, *Topological Vector Spaces and Distribitions*, Vol. I, Addison-Wesley, Reading, Massachusets, 1966.

[5] S. Pilopović, *On regular $K'\{M_P\}$-distributions*, Ann. Polon. Math. **48** (1988), 31-39.

[6] S. Pilipović, *Remarks on absolutely regular and regular $K'\{M_p\}$-distributions*, Ann. Polon. Math., to appear

[7] Z. Szmydt, *Characterization of regular tempered distributions*, Ann. Polon. Math. **41** (1983), 255-258.

Department of Mathematics
Jordan University of Science & Technology.
Irbid, Jordan

(Received 27 08 1993)