

COMPLETELY REGULAR AND ORTHODOX CONGRUENCES ON REGULAR SEMIGROUPS

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Abstract. Let S be a regular semigroup and $E(S)$ the set of all idempotents of S . Let $\text{Con } S$ be the congruence lattice of S , and let T, K, U and V be equivalences on $\text{Con } S$ defined by $\rho T \xi \Leftrightarrow \text{tr } \rho = \text{tr } \xi$, $\rho K \xi \Leftrightarrow \ker \rho = \ker \xi$, $\rho U \xi \Leftrightarrow \rho \cap \leq = \xi \cap \leq$ and $V = U \cap K$, where $\text{tr } \rho = \rho \upharpoonright_{E(S)}$, $\ker \rho = E(S)\rho$, and \leq is the natural partial order on $E(S)$. It is known that T, U and V are complete congruences on $\text{Con } S$ and T -, K -, U - and V -classes are intervals $[\rho_T, \rho^T]$, $[\rho_K, \rho^K]$, $[\rho_U, \rho^U]$, and $[\rho_V, \rho^V]$, respectively ([13], [10], [9]). In this paper U -classes for which ρ^U is a semilattice congruence, and V -classes for which ρ^V is an inverse congruence are considered. It turns out that the union of all such U -classes is the lattice $\text{CRCon } S$ of all completely regular congruences on S , and the union of all such V -classes is the lattice $\text{OCon } S$ of all orthodox congruences on S . Also, some complete epimorphisms of the form $\rho \rightarrow \rho^U$ and $\rho \rightarrow \rho^V$ are obtained.

1. Preliminaries. In the following we shall use the terminology and notation of [4] and [11]. Throughout the paper, S stands for a regular semigroup. If $\rho \in \text{Con } S$, and α is an equivalence on S/ρ , then the equivalence $\bar{\alpha}$ on S is defined by $a\bar{\alpha}b \Leftrightarrow (a\rho)\alpha(b\rho)$, ($a, b \in S$). If α is a relation on S then α^* denotes the congruence on S generated by α . If α is an equivalence on S , then α° denotes the greatest congruence on S contained in α , and if $T \subseteq S$, then $T\alpha$ denotes the union of α -classes of all elements of T . If $\rho, \xi \in \text{Con } S$ and $\rho \subseteq \xi$, then the relation ξ/ρ on S/ρ defined by $(a\rho)\xi/\rho(b\rho) \Leftrightarrow a\xi b$ ($a, b \in S$) is a congruence. If $\rho, \xi, \zeta \in \text{Con } S$, $\rho \subseteq \xi$ and $\rho \subseteq \zeta$, then $(\xi \vee \zeta)/\rho = (\xi/\rho) \vee (\zeta/\rho)$ and $(\xi \circ \zeta)/\rho = (\xi/\rho) \circ (\zeta/\rho)$.

Let \mathcal{C} be a class of semigroups, and let $\rho \in \text{Con } S$. Then ρ is a \mathcal{C} -congruence if $S/\rho \in \mathcal{C}$, and ρ is over \mathcal{C} if $(\forall e \in E(S))e\rho \in \mathcal{C}$.

In the paper, $\sigma, \eta, \alpha, \nu, Y, o$ denote the least group, semilattice, rectangular band, Clifford, inverse and orthodox congruences on S , respectively. Also, μ and τ denote the greatest idempotent separating and idempotent pure congruences on S , respectively.

LEMMA 1. *Let \mathcal{C} be a class of semigroups closed for homomorphisms and let $\gamma = \gamma_S$ be the least \mathcal{C} -congruence on S . If $\rho \in \text{Con } S$, then*

- (1) ρ is a \mathcal{C} -congruence on $S \Leftrightarrow \rho \supseteq \gamma$,
- (2) $\gamma_{S/\rho} = (\rho \vee \gamma)/\rho$.

Proof. (1) Clearly, if $S/\rho \in \mathcal{C}$, then $\rho \supseteq \gamma$. Conversely, if $\rho \supseteq \gamma$, then $S/\gamma/(\rho/\gamma) \in \mathcal{C}$, i.e. $S/\rho \in \mathcal{C}$.

(2) From (1) it follows that $(\rho \vee \gamma)/\rho \supseteq \gamma_{S/\rho}$. Since $\rho \subseteq \bar{\gamma}_{S/\rho}$ and $\bar{\gamma}_{S/\rho}$ is a \mathcal{C} -congruence on S , we get $\rho \vee \gamma \subseteq \bar{\gamma}_{S/\rho}$, which yields $(\rho \vee \gamma)/\rho \subseteq \gamma_{S/\rho}$.

For $a \in S$, let $V(a)$ denote the set of all inverses of a in S . Let \mathcal{U} and \mathcal{V} be equivalences on S defined by $a\mathcal{U}b \Leftrightarrow V(a)\mathcal{H} = V(b)\mathcal{H}$, and $a\mathcal{V}b \Leftrightarrow V(a) = V(b)$.

RESULT 1. [9] *If $\rho, \xi \in \text{Con } S$, then*

- (1) $\rho^T \xi \Leftrightarrow \bar{\mathcal{H}}_{S/\rho} = \bar{\mathcal{H}}_{S/\xi}$ and $\rho^T = \bar{\mathcal{H}}_{S/\rho}^\circ$, $\rho^T = (\text{tr } \rho)^*$,
- (2) $\rho^K \xi \Leftrightarrow \bar{\tau}_{S/\rho} = \bar{\tau}_{S/\xi}$ and $\rho^K = \bar{\tau}_{S/\rho}$,
- (3) $\rho^U \xi \Leftrightarrow \bar{\mathcal{U}}_{S/\rho} = \bar{\mathcal{U}}_{S/\xi}$ and $\rho^U = \bar{\mathcal{U}}_{S/\rho}^\circ$, $\rho^U = (\rho \cap \leq)^*$,
- (4) $\rho^V \xi \Leftrightarrow \bar{\mathcal{V}}_{S/\rho} = \bar{\mathcal{V}}_{S/\xi}$ and $\rho^V = \bar{\mathcal{V}}_{S/\rho}^\circ$.

In particular we have $\varepsilon^T = \mathcal{H}^\circ = \mu$, $\varepsilon^K = \tau$, $\varepsilon^U = \mathcal{U}^\circ$, $\omega_U = (\leq)^*$ and $\varepsilon^V = \mathcal{V}^\circ$, where ε is the equality, and ω is the universal relation on S .

COROLLARY 1. *For $\rho \in \text{Con } S$ we have $\rho^T/\rho = \mathcal{H}_{S/\rho}^\circ = \mu_{S/\rho}$, $\rho^K/\rho = \tau_{S/\rho}$, $\rho^U/\rho = \mathcal{U}_{S/\rho}^\circ$, $\rho^V/\rho = \mathcal{V}_{S/\rho}^\circ$.*

RESULT 2. [9] *A congruence ρ on S is over completely simple semigroups if and only if $\rho \subseteq \mathcal{U}$.*

RESULT 3. [8] *Let L be a complete lattice and C be a complete congruence on L . Then for any $x \in L$ the C -class x_C is the interval $[x_C, x^C]$ of L , and for any $A \subseteq L$,*

$$\left(\bigvee_{x \in A} x \right)_C = \bigvee_{x \in A} x_C, \quad \left(\bigwedge_{x \in A} x \right)^C = \bigwedge_{x \in A} x^C.$$

RESULT 4. [7] *Let S be a regular semigroup and $\rho, \xi \in \text{Con } S$. Then*

- (i) *if ξ is idempotent separating, then $\rho \vee \xi = \rho \circ \xi \circ \rho$. In particular $\rho \vee \mu = \rho \circ \mu \circ \rho$;*
- (ii) *if S is completely regular, then $\rho \vee \eta = \rho \circ \eta \circ \rho$;*
- (iii) *if S is orthodox, then $\rho \vee Y = \rho \circ Y \circ \rho$.*

2. Completely regular congruences. In this section we establish certain characterizations of completely regular congruences.

LEMMA 2. *For a regular semigroup S , the following are equivalent:*

- (i) *S is completely regular,*
- (ii) $\eta \subseteq \mathcal{U}$,
- (iii) $\eta = \mathcal{U}$.

Proof. Since any completely regular semigroup is a semilattice of completely simple semigroups, the equivalence (i) \Leftrightarrow (ii) follows immediately from Result 2.

By definition of \mathcal{U} , it follows that $\mathcal{U} \subseteq \mathcal{D}$, and if S is completely regular, then $\mathcal{D} = \eta$. Hence, (ii) \Leftrightarrow (iii).

THEOREM 1. *For $\rho \in \text{Con } S$, the following are equivalent:*

- (i) ρ is a completely regular congruence,
- (ii) ρ^U is a semilattice congruence,
- (iii) $\rho^U = \rho \vee \eta$,
- (iv) $\rho^U(\rho \vee \eta)$.

Proof. (i) \Rightarrow (iii) $\rho \in \text{CRCon } S \Leftrightarrow S/\rho$ is completely regular
 $\Leftrightarrow \mathcal{U}_{S/\rho} = \eta_{S/\rho}$ (by Lemma 2)
 $\Leftrightarrow \bar{\mathcal{U}}_{S/\rho} = \rho \vee \eta$ (by Lemma 1)
 $\Rightarrow \rho^U = \rho \vee \eta$ (by Result 1).

(iii) \Rightarrow (iv) This is evident.

(iv) \Rightarrow (ii) $\rho^U(\rho \vee \eta) \Rightarrow \rho \vee \eta \subseteq \rho^U \Rightarrow \rho^U$ is a semilattice congruence.

(ii) \Rightarrow (i) $\eta \subseteq \rho^U \Rightarrow \rho \vee \eta \subseteq \rho^U$
 $\Rightarrow (\rho \vee \eta)/\rho \subseteq \rho^U/\rho$
 $\Rightarrow \eta_{S/\rho} \subseteq \mathcal{U}_{S/\rho}$ (by Lemma 1 and Corollary 1)
 $\Rightarrow S/\rho$ is completely regular (by Lemma 2).

Let $\text{SCon } S$ denote the lattice of all semilattice congruences on S .

- COROLLARY 2.** (1) $(\forall \rho, \xi \in \text{CRCon } S)(\rho^U \xi \Leftrightarrow \rho \vee \eta = \xi \vee \eta)$,
(2) $\text{CRCon } S = [\eta_U, \omega] = [\eta, \omega]U$ and $\eta^U = [\eta_U, \eta]$,
(3) $\text{SCon } S = [\eta, \omega] = \{\rho \in \text{CRCon } S \mid \rho^U = \rho\}$,
(4) $\mu \subseteq \mathcal{U}^\circ \subseteq \eta$.

PROPOSITION 1. *For $\rho \in \text{CRCon } S$, $\rho \vee \eta = \rho \circ \eta \circ \rho$.*

Proof. Let δ denote the least completely regular congruence on S . By Corollary 2 we have $\delta = \eta_U \subseteq \rho$. According to Lemma 1 we get $\eta_{S/\delta} = (\delta \vee \eta)/\delta = \eta/\delta$. So we have

$$\begin{aligned} (\rho/\delta) \vee (\eta/\delta) &= (\rho/\delta) \circ (\eta/\delta) \circ (\rho/\delta) \quad (\text{by Result 4(ii)}) \\ \Leftrightarrow (\rho \vee \eta)/\delta &= (\rho \circ \eta \circ \rho)/\delta \\ \Leftrightarrow \rho \vee \eta &= \rho \circ \eta \circ \rho. \end{aligned}$$

From (1) of Corollary 2 and the implication (i) \Rightarrow (iii) of Theorem 1 we get Proposition 8.1 of [9].

COROLLARY 3. *For S the following are equivalent:*

- (i) S is completely regular,
- (ii) $(\forall \rho \in \text{Con } S)\rho^U = \rho \vee \eta$,

- (iii) $\eta = U^\circ$,
- (iv) $\varepsilon U \eta$.

The next result is an analogue of Theorem 1 of [1].

THEOREM 2. *For $\rho \in \text{Con } S$, the following are equivalent:*

- (i) ρ is a completely simple congruence,
- (ii) ρ^T is a rectangular band congruence,
- (iii) $\rho^T = \rho \vee \alpha$,
- (iv) $\rho T(\rho \vee \alpha)$,
- (v) $\rho U \omega$.

Proof. (i) \Leftrightarrow (ii) \Leftrightarrow (iii) \Leftrightarrow (iv) follows immediately from [1].

(i) \Leftrightarrow (v) ρ is a completely simple congruence

$$\begin{aligned} &\Leftrightarrow \omega/\rho \text{ is over completely simple semigroups} \\ &\Leftrightarrow \rho U \omega \quad (\text{by Theorem 4.10 of [9]}). \end{aligned}$$

Let $\text{CSCon } S$ ($\text{RBCon } S$) denote the lattice of all completely simple (rectangular band) congruences on S . Then we have

- COROLLARY 4.** (1) $(\forall \rho, \xi \in \text{CSCon } S) \rho T \xi \Leftrightarrow \rho \vee \alpha = \xi \vee \alpha$,
- (2) $\text{CSCon } S = [\alpha_T, \omega] = [\alpha, \omega] T$ and $\alpha T = [\alpha_T, \alpha]$,
- (3) $\text{RBCon } S = [\alpha, \omega] = \{\rho \in \text{CSCon } S \mid \rho^T = \rho\}$,
- (4) $\alpha_T = \omega U = (\text{tr } \alpha)^* = (\leq)^*$.

COROLLARY 5. *For S the following are equivalent:*

- (i) S is completely simple,
- (ii) $(\forall \rho \in \text{Con } S) \rho^T = \rho \vee \alpha$,
- (iii) $\mu = \alpha$,
- (iv) $U^\circ = \omega$,
- (v) $\varepsilon U \omega$.

PROPOSITION 2. *For $\rho \in \text{CSCon } S$, $\rho \vee \alpha = \rho \circ \alpha \circ \rho$.*

Proof. Let ζ denote the least completely simple congruence on S . By Corollary 4 we have $\zeta = \alpha_T \subseteq \rho$. According to Lemma 1 we get $\alpha_{S/\zeta} = (\zeta \vee \alpha)/\zeta = \alpha/\zeta$. Hence, by Corollary 5, $\mu_{S/\zeta} = \alpha/\zeta$. Thus

$$\begin{aligned} &(\rho/\zeta) \vee (\alpha/\zeta) = (\rho/\zeta) \circ (\alpha/\zeta) \circ (\rho/\zeta) \quad (\text{by Result 4(i)}) \\ \Leftrightarrow &(\rho \vee \alpha)/\zeta = (\rho \circ \alpha \circ \rho)/\zeta \\ \Leftrightarrow &\rho \vee \alpha = \rho \circ \alpha \circ \rho. \end{aligned}$$

3. Orthodox congruences. Now we describe orthodox congruences on S in terms of K and V . Let $\rho \in \text{Con } S$. It is easy to see that ρ is an orthodox congruence if and only if $\ker \rho$ is a subsemigroup of S .

LEMMA 3. *For a regular semigroup S , the following are equivalent:*

- (i) S is orthodox,
- (ii) $Y = \mathcal{V}$,
- (iii) $Y \subseteq \mathcal{V}$,
- (iv) $Y \subseteq \tau$.

Proof. (i) \Rightarrow (ii) It is proved in [3] and [14].

(ii) \Rightarrow (iii) This is evident.

(iii) \Rightarrow (iv) $Y \subseteq \mathcal{V} \Rightarrow Y \subseteq \mathcal{V}^\circ = \varepsilon^V \subseteq \varepsilon^K = \tau$.

(iv) \Rightarrow (i) Since Y is orthodox, $\ker Y$ is a subsemigroup of S . Thus $Y \subseteq \tau \Leftrightarrow \ker Y = E(S) \Rightarrow E(S)$ is a subsemigroup of $S \Leftrightarrow S$ is orthodox.

THEOREM 3. *For $\rho \in \text{Con } S$ the following are equivalent:*

- (i) ρ is an orthodox congruence,
- (ii) ρ^V is an inverse congruence,
- (iii) $\rho^V = \rho \vee Y$,
- (iv) $\rho V(\rho \vee Y)$,
- (v) $\rho K(\rho \vee Y)$,
- (vi) ρ^K is an inverse congruence.

Proof. (i) \Rightarrow (iii) ρ is orthodox $\Leftrightarrow \mathcal{V}_{S/\rho} = Y_{S/\rho}$ (by Lemma 3)
 $\Leftrightarrow \bar{\mathcal{V}}_{S/\rho} = \rho \vee Y$ (by Lemma 1)
 $\Rightarrow \rho^V = \rho \vee Y$. (by Result 1).

(iii) \Rightarrow (iv) \Rightarrow (v) \Rightarrow (vi) This is evident.

(vi) \Rightarrow (i) $Y \subseteq \rho^K \Rightarrow \rho \vee Y \subseteq \rho^K$
 $\Rightarrow (\rho \vee Y)/\rho \subseteq \rho^K/\rho$
 $\Leftrightarrow Y_{S/\rho} \subseteq \tau_{S/\rho}$ (by Lemma 1 and Corollary 1)
 $\Leftrightarrow \rho$ is orthodox (by Lemma 3).

(iii) \Rightarrow (ii) \Rightarrow (vi) This is evident.

Let $\text{ICon } S$ denote the set of all inverse congruences on S .

- COROLLARY 6. (1) $(\forall \rho, \xi \in \text{OCon } S) \rho V \xi \Leftrightarrow \rho \vee Y = \xi \vee Y$,
(2) $\text{OCon } S = [o, \omega] = [Y, \omega]V$ and $YV = [o, Y]$,
(3) $\text{ICon } S = [Y, \omega] = \{\rho \in \text{OCon } S \mid \rho^V = \rho\}$,
(4) $\mathcal{V}^\circ \subseteq Y$.

From Corollary 6(1) and the implication (i) \Rightarrow (iii) of Theorem 3 we get Proposition 8.5 of [9], and from (i) \Rightarrow (v) of Theorem 3 we get oKY [6] and Lemma 2.1 of [2].

COROLLARY 7. *For S the following are equivalent:*

- (i) S is orthodox,
- (ii) $(\forall \rho \in \text{Con } S) \rho^V = \rho \vee Y$,

- (iii) $Y = \mathcal{V}^\circ$,
- (iv) $\varepsilon V Y$.

Using Result 4(iii), the proof of the following proposition is similar to the proof of the Proposition 1.

PROPOSITION 3. *For $\rho \in \text{OCon } S$, $\rho \vee Y = \rho \circ Y \circ \rho$.*

In the following we describe orthodox completely regular (i.e. orthogroup) congruences and orthodox completely simple congruences on S . By [5], any orthodox completely simple semigroup is a rectangular group and conversely.

THEOREM 4. *For $\rho \in \text{Con } S$, the following are equivalent:*

- (i) ρ is an orthogroup congruence,
- (ii) ρ^V is a Clifford congruence,
- (iii) $\rho^V = \rho \vee \nu$,
- (iv) $\rho V(\rho \vee \nu)$.

Proof. (i) \Rightarrow (ii) By Theorem 3, ρ^V is an inverse completely regular congruence, i.e. a Clifford congruence.

(ii) \Rightarrow (iii)

$$\begin{aligned} \rho^V \text{ is a Clifford congruence} &\Rightarrow \rho^V = \rho \vee Y && \text{(by Theorem 3)} \\ &\Rightarrow \rho^V \subseteq \rho \vee \nu \subseteq \rho^V && \text{(since } Y \subseteq \nu) \\ &\Rightarrow \rho^V = \rho \vee \nu. \end{aligned}$$

(iii) \Rightarrow (iv) This is evident.

(iv) \Rightarrow (i) By Theorem 3, $\rho V(\rho \vee \nu)$ implies that ρ is orthodox. On the other hand,

$$\begin{aligned} \rho V(\rho \vee \nu) &\Rightarrow \rho U(\rho \vee \nu) && \text{(since } V \subseteq U) \\ &\Rightarrow \rho \text{ is completely regular} && \text{(by Theorem 1)}. \end{aligned}$$

Hence ρ is an orthogroup congruence.

Let $\text{OGCon } S$ ($\text{SGCon } S$) denote the lattice of all orthogroup (Clifford) congruences on S . Then we have

- COROLLARY 8.** (1) $(\forall \rho, \xi \in \text{OGCon } S) \rho V \xi \Leftrightarrow \rho \vee \nu = \xi \vee \nu$.
 (2) $\text{OGCon } S = [\nu_V, \omega] = [\nu, \omega]V$ and $\nu V = [\nu_V, \nu]$.
 (3) $\text{SGCon } S = [\nu, \omega] = \{\rho \in \text{OGCon } S \mid \rho^V = \rho\}$.

COROLLARY 9. *For S the following are equivalent:*

- (i) S is an orthogroup,
- (ii) $(\forall \rho \in \text{Con } S) \rho^V = \rho \vee \nu$,
- (iii) $\mathcal{V}^\circ = \nu$,
- (iv) $\varepsilon V \nu$.

THEOREM 5. *For $\rho \in \text{Con } S$, the following are equivalent:*

- (i) ρ is a rectangular group congruence,
- (ii) ρ^V is a group congruence,

- (iii) $\rho^V = \rho \vee \sigma$,
 (iv) $\rho V(\rho \vee \sigma)$.

Proof. (i) \Rightarrow (ii) By Theorem 3, ρ^V is an inverse completely simple congruence, i.e. ρ^V is a group congruence.

(ii) \Rightarrow (iii) If ρ^V is a group congruence, then $\rho^V = \rho^K = \rho \vee \sigma$, by Theorem 3 of [1].

(iii) \Rightarrow (iv) This is evident.

(iv) \Rightarrow (i) $\rho V(\rho \vee \sigma) \Rightarrow \rho$ is orthodox (by Theorem 3). On the other hand,

$$\begin{aligned} \rho V(\rho \vee \sigma) &\Rightarrow \rho U(\rho \vee \sigma) U \omega && \text{(by Theorem 2)} \\ &\Rightarrow \rho \text{ is completely simple.} && \text{(by Theorem 2).} \end{aligned}$$

Hence, ρ is a rectangular group congruence.

Let $\text{RGCon } S$ ($\text{GCon } S$) denote the lattice of all rectangular group (group) congruences on S . Then we have

- COROLLARY 10. (1) $(\forall \rho, \xi \in \text{RGCon } S) \rho V \xi \Leftrightarrow \rho \vee \sigma = \xi \vee \sigma$.
 (2) $\text{RGCon } S = [\sigma_V, \omega] = [\sigma, \omega] V$ and $\sigma V = [\sigma_V, \sigma]$,
 (3) $\text{GCon } S = [\sigma, \omega] = \{\rho \in \text{RGCon } S \mid \rho^V = \rho\}$.

COROLLARY 11. For S the following are equivalent:

- (i) S is a rectangular group,
 (ii) $(\forall \rho \in \text{Con } S) \rho^V = \rho \vee \sigma$,
 (iii) $\mathcal{V}^\circ = \sigma$,
 (iv) $\varepsilon V \sigma$.

4. Some complete epimorphisms. Using the results of Theorems 1–5 and the Result 2 we get the following

THEOREM 6. Let S be a regular semigroup. The mappings

$$\begin{aligned} \varphi_1 : \text{CRCon } S &\longrightarrow \text{SCon } S && \text{defined by } \varphi_1(\rho) = \rho \vee \eta, \\ \varphi_2 : \text{CSCon } S &\longrightarrow \text{RRBon } S && \text{defined by } \varphi_2(\rho) = \rho \vee \alpha, \\ \varphi_3 : \text{OCon } S &\longrightarrow \text{ICon } S && \text{defined by } \varphi_3(\rho) = \rho \vee Y, \\ \varphi_4 : \text{OGCon } S &\longrightarrow \text{SGCon } S && \text{defined by } \varphi_4(\rho) = \rho \vee \nu, \\ \varphi_5 : \text{RGCon } S &\longrightarrow \text{GCon } S && \text{defined by } \varphi_5(\rho) = \rho \vee \sigma \end{aligned}$$

are complete epimorphisms. The classes of the complete congruence $\bar{\varphi}$ induced by the epimorphism φ are U -classes, if $\varphi = \varphi_1$; T -classes, if $\varphi = \varphi_2$; and V -classes, if $\varphi = \varphi_i$, $i = 3, 4, 5$.

For a completely regular semigroup S , the statement concerning the mapping φ_1 is given in [12] (Theorem 4.3(i)).

- COROLLARY 12. (i) $(\cap_{\rho \in F} \rho) \vee \eta = \cap_{\rho \in F} (\rho \vee \eta)$ ($F \subseteq \text{CRCon } S$),
 (ii) $(\cap_{\rho \in F} \rho) \vee \alpha = \cap_{\rho \in F} (\rho \vee \alpha)$ ($F \subseteq \text{CSCon } S$),
 (iii) $(\cap_{\rho \in F} \rho) \vee Y = \cap_{\rho \in F} (\rho \vee Y)$ ($F \subseteq \text{OCon } S$),
 (iv) $(\cap_{\rho \in F} \rho) \vee \nu = \cap_{\rho \in F} (\rho \vee \nu)$ ($F \subseteq \text{OGCon } S$),

$$(v) \quad (\bigcap_{\rho \in F} \rho) \vee \sigma = \bigcap_{\rho \in F} (\rho \vee \sigma) \quad (F \subseteq \text{RGCon } S).$$

From (i) of this corollary we get Theorem 4.7 of [7], and from (iii) we get Theorem 2.4 of [2].

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(Received 29 09 1992)