

ABSTRACT. Let  $\mathbb{F}_\theta^+$  be a  $k$ -graph on a single vertex. We show that every irreducible atomic  $*$ -representation is the minimal  $*$ -dilation of a group construction representation. It follows that every atomic representation decomposes as a direct sum or integral of such representations. We characterize periodicity of  $\mathbb{F}_\theta^+$  and identify a symmetry subgroup  $H_\theta$  of  $\mathbb{Z}^k$ . If this has rank  $s$ , then  $C^*(\mathbb{F}_\theta^+) \cong C(\mathbb{T}^s) \otimes \mathfrak{A}$  for some simple  $C^*$ -algebra  $\mathfrak{A}$ .