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A note on p.q.-Baer modules

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Abstract. A module M_R is called *right principally quasi-Baer* (or simply *right p.q.-Baer*) if the right annihilator of a principal submodule of R is generated by an idempotent. Let R be a ring. Let α be an endomorphism of R and M_R be a α-compatible module and $T = R[[x;\alpha]]$. It is shown that $M[[x]]_T$ is right p.q.-Baer if and only if M_R is right p.q.-Baer and the right annihilator of any countably-generated submodule of M is generated by an idempotent. As a corollary we obtain a generalization of a result of Liu, 2002.

Contents

1. Introduction

Throughout the paper R always denotes an associative ring with unity and M_R will stand for a right R-module. Recall from [15] that R is a *Baer* ring if the right annihilator of every nonempty subset of R is generated by an idempotent. In [15] Kaplansky introduced Baer rings to abstract various properties of von Neumann algebras and complete $*$-regular rings. The class of Baer rings includes the von Neumann algebras. In [10] Clark defines a ring to be *quasi-Baer* if the left annihilator of every ideal is generated, as a left ideal, by an idempotent. Then he used the quasi-Baer concept to characterize when a finite-dimensional algebra with unity over an algebraically closed field is isomorphic to a twisted matrix units semigroup algebra. Every prime ring is a quasi-Baer ring. Another generalization

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of Baer rings are the p.p.-rings. A ring R is called *right* (*resp. left*) *p.p.* if the right (resp. left) annihilator of an element of R is generated by an idempotent. Birkenmeier et al. in [5] introduced the concept of principally quasi-Baer rings. A ring R is called *right principally quasi-Baer* (or simply *right p.q.-Baer*) if the right annihilator of a principal right ideal of R is generated by an idempotent.

In 1974, Armendariz considered the behavior of a polynomial ring over a Baer ring by obtaining the following result: Let R be a *reduced* ring (i.e., R has no nonzero nilpotent elements). Then $R[x]$ is a Baer ring if and only if R is a Baer ring ([3], Theorem B). Armendariz provided an example to show that the reduced condition is not superfluous. In [6] Birkenmeier et al. showed that the quasi-Baer condition is preserved by many polynomial extensions. Also, Birkenmeier et al. [5] showed that a ring R is right p.q.-Baer if and only if $R[x]$ is right p.q.-Baer. Recall from [7], that an idempotent $e \in R$ is left semicentral in R if $ere = er$ for all $r \in R$. Equivalently, $e^2 = e \in R$ is left semicentral if eR is an ideal of R. Since the right annihilator of a right ideal is a ideal, we see that the right annihilator of a principal right ideal is generated by a left semicentral idempotent in a right p.q.-Baer ring. In [21], Z. Liu showed that if all left semicentral idempotents of a ring R are central, then $R[[x]]$ is right p.q.-Baer if and only if R is right p.q.-Baer and any countable family of idempotents in R has a generalized join in the set of idempotents of R.

From now on, we always denote the skew power series ring by $T := R[[x;\alpha]]$, where $\alpha : R \to R$ is an endomorphism. The skew power series ring T is then the ring consisting of all power series of the form $\sum_{i=0}^{\infty} a_i x^i$ ($a_i \in R$), which are multiplied using the distributive law and the Ore commutation rule $xa = \alpha(a)x$, for all $a \in R$.

Given a right R-module M_R, we can make $M[[x]]$ into a right T-module by allowing power series from T to act on power series in $M[[x]]$ in the obvious way, and applying the above "twist" whenever necessary. The verification that this defines a valid T-module structure on $M[[x]]$ is almost identical to the verification that T is a ring, and it is straightforward.

For a nonempty subset X of M, put $\operatorname{ann}_R(X) = \{a \in R \mid Xa = 0\}$. In [20], Lee–Zhou introduced Baer, quasi-Baer and p.p.-modules as follows:

(1) M_R is called *Baer* if, for any subset X of M, $\operatorname{ann}_R(X) = eR$ where $e^2 = e \in R$.
(2) M_R is called *quasi-Baer* if, for any submodule $X \subseteq M$, $\operatorname{ann}_R(X) = eR$ where $e^2 = e \in R$.
(3) M_R is called *p.p.* if, for any element $m \in M$, $\operatorname{ann}_R(m) = eR$ where $e^2 = e \in R$.

Clearly, a ring R is Baer (resp. p.p. or quasi-Baer) if and only if R_R is Baer (resp. p.p. or quasi-Baer) module. If R is a Baer (resp. p.p. or quasi-Baer) ring, then for any right ideal I of R, I_R is Baer (resp. p.p. or quasi-Baer) module.

A module M_R is called *principally quasi-Baer* (or simply p.q.-Baer) if, for any $m \in M$, $\mathrm{ann}_R(mR) = eR$ where $e^2 = e \in R$. It is clear that R is a right p.q.-Baer ring if and only if R_R is a p.q.-Baer module. Every submodule of a p.q.-Baer module is p.q.-Baer and every Baer module is quasi-Baer.

We use $I(R)$, $S_\ell(R)$ and $C(R)$ to denote the set of idempotents, the set of left semicentral idempotents, and the center of R, respectively.

In this note we show that, if M_R is α-compatible module, the $M[[x]]_T$ is p.q.-Baer if and only if M_R is p.q.-Baer and the right annihilator of any countably-generated submodule of M_R is generated by an idempotent. As a corollary, we show that if R is α-compatible and $S_\ell(R) \subseteq C(R)$, then $R[[x;\alpha]]$ is p.q.-Baer if and only if R is p.q.-Baer and any countable family of idempotents in R has a generalized join in $I(R)$. This result is a generalization of [21].

2. Principally quasi-Baer modules

According to Kim et al. [16], a ring R is called *power-serieswise Armendariz* if whenever $f(x)g(x) = 0$ where $f(x) = \sum_{i=0}^{\infty} a_i x^i$, $g(x) = \sum_{j=0}^{\infty} b_j x^j \in R[[x]]$, we have $a_i b_j = 0$ for all i, j. Let $\alpha \in \mathrm{End}(R)$ and M be an R-module. According to Lee and Zhou [20], a module M_R is called *α-Armendariz of power series type* if the following cinditions are satisfied:
(1) For $m \in M$ and $a \in R$, $ma = 0$ if and only if $m\alpha(a) = 0$.
(2) For any $m(x) = \sum_{i=0}^{\infty} m_i x^i \in M[[x]]$ and $f(x) = \sum_{i=0}^{\infty} a_i x^i \in R[[x;\alpha]]$, $m(x)f(x) = 0$ implies $m_i \alpha^i(a_j) = 0$ for all i, j.

Definition 2.1. Let M_R be an R-module and α be an endomorphism of R. We say M_R is *power-serieswise α-quasi-Armendariz* if whenever $m(x) = \sum_{i=0}^{\infty} m_i x^i \in M[[x]]$ and $f(x) = \sum_{j=0}^{\infty} b_j x^j \in R[[x;\alpha]]$ satisfy

$$m(x)R[[x;\alpha]]f(x) = 0,$$

we have $m_i x^i R b_j x^j = 0$ for all i, j.

Definition 2.2 (Annin, [2])**.** Given a module M_R, an endomorphism $\alpha : R \to R$, we say that M_R is *α-compatible* if for each $m \in M$, $r \in R$, we have $mr = 0 \Leftrightarrow m\alpha(r) = 0$.

Theorem 2.3. *Let M_R be an α-compatible module and $T = R[[x;\alpha]]$.*

(1) *If $M[[x]]_T$ is p.q.-Baer, then M_R is p.q.-Baer.*
(2) *If M_R is p.q.-Baer, then M is power-serieswise α-quasi-Armendariz.*

Proof. (1) Let $m \in M$. Since $M[[x]]_T$ is p.q.-Baer, there exists idempotent $e(x) = e_0 + e_1 x + \cdots \in T$, such that $\mathrm{ann}_T(mT) = e(x)T$. Since $mRe(x) = 0$, so $mRe_0 = 0$. Thus $e_0 R \subseteq \mathrm{ann}_R(mR)$. Let $b \in \mathrm{ann}_R(mR)$. Then $b \in \mathrm{ann}_T(mT)$, since M is α-compatible. Thus $b = e(x)b$ and $b = e_0 b \in e_0 R$. Therefore $\mathrm{ann}_R(mR) = e_0 R$ and M_R is p.q.-Baer.

(2) Assume that $(\sum_{i=0}^{\infty} m_i x^i)T(\sum_{j=0}^{\infty} b_j x^j) = 0$ with $m_i \in M$, $b_j \in R$. Let c be an arbitrary element of R. Then we have the following equation:

$$\sum_{k=0}^{\infty}\left(\sum_{i+j=k} m_i x^i c b_j x^j\right) = \sum_{k=0}^{\infty}\left(\sum_{i+j=k} m_i \alpha^i(cb_j)\right)x^k = 0,$$

and hence

$$\sum_{i+j=k} m_i \alpha^i(cb_j) = 0 \text{ for all } \ k \geq 0. \tag{2.1}$$

We show that $m_i x^i R b_j x^j = 0$ for all i, j. We proceed by induction on $i + j$. From Equation (2.1), we obtain $m_0 R b_0 = 0$. This proves the case $i + j = 0$. Now suppose that $m_i x^i R b_j x^j = 0$ for $i + j \leq n - 1$. Hence $b_j \in \operatorname{ann}_R(m_i R)$ for $j = 0, \ldots, n-1$ and $i = 0, \ldots, n-1-j$. Now $\operatorname{ann}_R(m_i R) = e_i R$ for some idempotent $e_i \in R$. Thus, $e_i b_j = b_j$ for $j = 0, \ldots, n-1$ and $i = 0, \ldots, n-1-j$. If we put $f_j = e_0 \ldots e_{n-1-j}$ for $j = 0, \ldots, n-1$, then $f_j b_j = b_j$ and $f_j \in \operatorname{ann}_R(m_0 R) \cap \cdots \cap \operatorname{ann}_R(m_{n-1-j} R)$. For $k = n$ replacing c by cf_0 in (2.1) and using α-compatibility of M, we obtain $m_0 c b_n = m_0 c f_0 b_n = 0$. Hence $m_0 R b_n = 0$. Continuing this process (replacing c by cf_j in (2.1), for $j = 1, \ldots, n-1$ and using α-compatibility of M), we obtain $m_i R b_j = 0$ and so $m_i x^i R b_j x^j = 0$ for $i + j = n$. Therefore M_R is power-serieswise α-quasi-Armendariz. □

Lemma 2.4. *Let M_R be an α-compatible module and M_R be a p.q.-Baer module. Let $\operatorname{ann}_T(m(x)T) = e(x)T$ for some idempotent $e(x) = e_0 + e_1 x + \cdots \in T$. Then $\operatorname{ann}_T(m(x)T) = e_0 T$ and e_0 is an idempotent of R.*

Proof. Let $m(x) = m_0 + m_1 x + \cdots$. By Theorem 2.3, M is power-serieswise α-quasi-Armendariz. Since $m(x)Te(x) = 0$, so $m_i R e_0 = 0$, for each $i \geq 0$. Hence $e_0 \in \operatorname{ann}_T(m(x)T)$, and $e_0 T \subseteq e(x)T$. Now let $f(x) = a_0 + a_1 x + \cdots \in \operatorname{ann}_T(m(x)T)$. Then $m_i R b_j = 0$, for all i, j, since M is power-serieswise α-quasi-Armendariz. Thus $b_j \in \operatorname{ann}_T(m(x)T) = e(x)T$, since M_R is α-compatible. Hence $b_j = e(x)b_j$ and $b_j = e_0 b_j$ for each j. Therefore $f(x) = e_0 f(x) \in e_0 T$. □

Theorem 2.5. *Let M be an α-compatible module and $T = R[[x;\alpha]]$. Then $M[[x]]_T$ is p.q.-Baer if and only if M_R is p.q.-Baer and the right annihilator of any countably-generated submodule of M is generated by an idempotent.*

Proof. If $M[[x]]_T$ is p.q.-Baer, then by Theorem 2.3, M_R is p.q.-Baer. Let $X = \{a_0, a_1, \ldots\}$ be a countable subset of M and $\langle X \rangle$ be the right submodule of M generated by X. Let $m(x) = a_0 + a_1 x + \cdots$. $M[[x]]_T$ is p.q.-Baer, so by Lemma 2.4, there exists an idempotent $e \in R$ such that $\operatorname{ann}_T(m(x)T) = eT$. Clearly, $eR \subseteq \operatorname{ann}_R(\langle X \rangle)$. Let $b \in \operatorname{ann}_R(\langle X \rangle)$. Then $a_i R b = 0$ for each i. Hence $m(x)Tb = 0$, since M_R is α-compatible. Thus $b = eb \in eR$. Consequently, $\operatorname{ann}_R(\langle X \rangle) = eR$.

Now assume M_R is p.q.-Baer and the right annihilator of any countably-generated submodule of M is generated by an idempotent. Let $m(x) = \sum_{i=0}^{\infty} m_i x^i \in M[[x]]$. Let N be the submodule of M generated by the coefficients $\{m_0, m_1, \dots\}$. Then $\mathrm{ann}_R(N) = eR$ for some idempotent $e \in R$. Since $m_i Re = 0$ for each i and M_R is α-compatible, so by α-compatibility of M_R, $m(x)Te = 0$ and that $eT \subseteq \mathrm{ann}_T(m(x)T)$. Now let $f(x) = \sum_{j=0}^{\infty} a_i x^i \in \mathrm{ann}_T(m(x)T)$. Then $m_i R a_j = 0$, for each i, j, since M is power-serieswise α-quasi-Armendariz. Then $a_j \in eR$, for each j, and $a_j = ea_j$. Therefore $f(x) = ef(x) \in eT$. Consequently, $M[[x]]_T$ is p.q.-Baer. □

Corollary 2.6. *Let M be a right R-module. Then $M[[x]]_{R[[x]]}$ is right p.q.-Baer if and only if M_R is right p.q.-Baer and for any countably-generated submodule N of M, $\mathrm{ann}_R(N) = eR$ for an idempotent $e \in R$.*

Remark 2.7. In [20], it was proved that, if M_R is α-Armendariz of power series type, then $M[[x]]_T$ is p.p. if and only if for any countable subset X of M, $\mathrm{ann}_R(X) = eR$ where $e^2 = e \in R$. By Zalesskii and Neroslavskii [9], there is a simple Noetherian ring R which is not a domain and in which 0 and 1 are the only idempotents. Thus R_R is p.q.-Baer ring which is not right p.p. Therefore our Corollary 2.6, is not implied from [20].

There is a p.q.-Baer module M_R such that $M[[x]]_{R[[x]]}$ is not p.q.-Baer.

Example 2.8. Let M_1 be a right p.q.-Baer R_1-module. Let

$$M = \left\{ (m_n) \in \prod_{n=1}^{\infty} M_n \;\middle|\; m_n \text{ is eventually constant} \right\},$$

where $M_n = M_1$ for $n > 1$ and let

$$R = \left\{ (a_n) \in \prod_{n=1}^{\infty} R_n \;\middle|\; a_n \text{ is eventually constant} \right\},$$

where $R_n = R_1$ for $n > 1$. Clearly M is a right R module. Clearly M is right p.q.-Baer. Let m be a nonzero element of M_1. Let $m_1 = (m, 0, 0, \dots)$, $m_2 = (m, 0, m, 0, 0, \dots)$, $m_3 = (m, 0, m, 0, m, 0, 0, \dots)$, ... Let $\langle X \rangle$ be the submodule of M generated by $X = \{m_1, m_2, \dots\}$. One can show that $\mathrm{ann}_R(\langle X \rangle)$ is not generated by any idempotent, hence by Theorem 2.5, $M[[x]]_{R[[x]]}$ is not right p.q.-Baer.

Definition 2.9 (Z. Liu [21])**.** Let $\{e_0, e_1, \dots\}$ be a countable family of idempotents of R. We say $\{e_0, e_1, \dots\}$ has a generalized join in $I(R)$ if there exists an idempotent $e \in I(R)$ such that:

(1) $e_i R(1 - e) = 0$.
(2) If $f \in I(R)$ is such that $e_i R(1 - f) = 0$, then $eR(1 - f) = 0$.

Lemma 2.10. *Let R be a ring and $S_\ell(R) \subseteq C(R)$. Then the following are equivalent:*

(1) *R is right p.q.-Baer and any countable family of idempotents in R has a generalized join in $I(R)$.*
(2) *R is right p.q.-Baer and the right annihilator of any countably-generated right ideal of R is generated by an idempotent.*

Proof. (1)⇒(2) Let $X = \{a_i\}_{i\in I}$ be a countable subset of R and $\langle X\rangle$ be the right ideal of R generated by X. Then for each $a_i \in X$, $\text{ann}_R(a_iR) = e_iR$ for some idempotent $e_i \in R$. Let h be a generalized join of the set $\{1-e_i \mid i \in I\}$. Then $(1-e_i)R(1-h) = 0$. Hence $r(1-h) = e_ir(1-h)$ for all $r \in R$. Since $e_i \in S_\ell(R) \subseteq C(R)$, $a_ir(1-h) = a_ie_ir(1-h) = 0$ for all i and each $r \in R$. Hence $(1-h) \in \text{ann}_R(\langle X\rangle)$ and $(1-h)R \subseteq \text{ann}_R(\langle X\rangle)$. Suppose that $b \in \text{ann}_R(\langle X\rangle)$. Hence $b = e_ib$ for each i. Since $e_i \in S_\ell(R) \subseteq C(R)$, $bR(1-e_i) = 0$ for each i. Since R is right p.q.-Baer, so $\text{ann}_R(bR) = fR$, where f is a left semicentral idempotent of R. Thus $(1-e_i) \in \text{ann}_R(bR) = fR$, so $(1-e_i) = f(1-e_i)$ for each i. Hence from $(1-e_i) \in C(R)$, we have $(1-e_i)R(1-f) = 0$. Since h is a generalized join of the set $\{1-e_i \mid i \in I\}$, $hR(1-f) = 0$. Hence $b = b - bf = (1-f)b = (1-h)(1-f)b \in (1-h)R$. Therefore $\text{ann}_R(\langle X\rangle) = (1-h)R$.

(2)⇒(1) Suppose that $\{e_i \mid i = 0, 1, \dots\}$ is a countable family of idempotents of R. Let J be the right ideal of R generated by $\{e_i \mid i = 0, 1, \dots\}$. Then $\text{ann}_R(J) = eR$ for some left semicentral idempotent e. Let $h = 1 - e$. Then $e_ir(1-h) = 0$ for each $r \in R$. Suppose that f is an idempotent of R such $e_iR(1-f) = 0$ for each i. Then $r(1-f) \in \text{ann}_R(J)$ for each $r \in R$. Thus $r(1-f) = er(1-f)$ and $hr(1-f) = (1-e)r(1-f) = 0$. Hence h is a generalized join of the set $\{e_i \mid i = 0, 1, \dots\}$. □

Theorem 2.11. *Let R be a ring with $S_\ell(R) \subseteq C(R)$ and α be an endomorphism of R. Let R_R be an α-compatible module. Then the following are equivalent:*

(1) *$R[[x;\alpha]]$ is right p.q.-Baer.*
(2) *R is right p.q.-Baer and any countable family of idempotents of R has a generalized join in $I(R)$.*

Proof. This follows from Theorem 2.5 and Lemma 2.10. □

Corollary 2.12 (Z. Liu [21, Theorem 3])**.** *Let R be a ring with $S_\ell(R) \subseteq C(R)$. Then the following conditions are equivalent:*

(1) *$S = R[[x]]$ is right p.q.-Baer.*
(2) *R is right p.q.-Baer and any countable family of idempotents in R has a generalized join in $I(R)$.*

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