

ABSTRACT. We obtain sufficient and necessary conditions for the Choquet–Deny theorem to hold in the class of compactly generated totally disconnected locally compact groups of polynomial growth, and in a larger class of totally disconnected generalized \overline{FC} -groups. The following conditions turn out to be equivalent when G is a metrizable compactly generated totally disconnected locally compact group of polynomial growth:

- (1) The Choquet–Deny theorem holds for G .
- (2) The group of inner automorphisms of G acts distally on G .
- (3) Every inner automorphism of G is distal.
- (4) The contraction subgroup of every inner automorphism of G is trivial.
- (5) G is a SIN group.

We also show that for every probability measure μ on a totally disconnected compactly generated locally compact second countable group of polynomial growth, the Poisson boundary is a homogeneous space of G , and that it is a compact homogeneous space when the support of μ generates G .