

ABSTRACT. We extend the classical Gauss–Bonnet theorem for the Euclidean, elliptic, hyperbolic, and Lorentzian planes to the other three Cayley–Klein geometries of dimension two, all three of which are absolute-time spacetimes, providing one proof for all nine geometries. Suppose that M is a polygon in any one of the nine geometries. Let Γ , the boundary of M , have length element ds , discontinuities θ_i , and signed geodesic curvature κ_g , where M and Γ are oriented according to Stokes’ theorem. Let K denote the constant Gaussian curvature of the geometry with area form dA . Then

$$\int_{\Gamma} \kappa_g ds + \sum_i \theta_i + \int \int_M K dA = 2\pi$$

for the nonspacetimes and

$$\int_{\Gamma} \kappa_g ds + \sum_i \theta_i + \int \int_M K dA = 0$$

for the spacetimes, where we assume that Γ is timelike.