ABSTRACT. In this paper we introduce the notion of unit-regular ideals for unital rings, which is a natural generalization of unitregular rings. It is shown that every square matrix over unitregular ideals admits a diagonal reduction. We also prove that a regular ideal of a unital ring is unit-regular if and only if pseudosimilarity via the ideal is similarity.