Classification of Maps by Their Membership on Maximal Clones that Contain Minimum and Complement

Rade Doroslovački, Jovanka Pantović
Faculty of Engineering, University of Novi Sad
Trg D. Obradovića 6, 21000 Novi Sad, Yugoslavia
Gradimir Vojvodić
Institute of Mathematics, University of Novi Sad
Trg D. Obradovića 4, 21000 Novi Sad, Yugoslavia

Abstract

In this paper, five-valued logic functions are classified according to their membership in the maximal clones which contain $min(x, y)$ and $\overline{x} = 4 - x$.

1 Notation and Preliminaries

Denote by $\mathbf{N}$ the set $\{1, 2, \ldots\}$ of positive integers. For $k, n \in \mathbf{N}$, $E_k = \{0, 1, \ldots, k-1\}$, denote by $P_k^{(n)}$ the set of all maps $E_k^n \to E_k$, and $P_k = \bigcup_{n\in\mathbf{N}} P_k^{(n)}$. We say that f is an *i-th projection of arity n* $(1 \leq i \leq n)$ if $f \in P_k^{(n)}$ and f satisfies the identity $f(x_1, \ldots, x_n) \approx x_i$. We say that $f \in P_k^{(n)}$ is *essential* if it depends on at least two variables and it takes all values from E_k. Let π_i^n denote the i-th projection of arity n, and let Π_k denote the set of all projections over E_k. For $n, m \geq 1, f \in P_k^{(n)}$ and $g_1, \ldots, g_n \in P_k^{(m)}$, *the superposition of* f and $g_1, \ldots, g_n$, denoted by $f(g_1, \ldots, g_n)$, is defined by $f(g_1, \ldots, g_n)(a_1, \ldots, a_m) = f(g_1(a_1, \ldots, a_m), \ldots, g_n(a_1, \ldots, a_m))$ for all $(a_1, \ldots, a_m) \in E_k^m$. A set $F \subseteq P_k$ is *a clone of operations on* E_k (or *clone* for short) if $\Pi_k \subseteq F$ and F is closed with respect to superposition. For $F \subseteq P_k$, $\langle F\rangle_{\mathrm{CL}}$ stands for the clone generated by F. We say that clone F is *maximal* if there is no clone G such that $F \subset G \subset P_k$. $F \subseteq P_k$ is *complete* if $\langle F\rangle_{\mathrm{CL}} = P_k$.

Let $\varrho \subseteq E_k^h$ be an h-ary relation and $f \in P_k^{(n)}$. We say that f *preserves* ϱ if for all h-tuples $(a_{11}, \ldots, a_{1h}), \ldots, (a_{n1}, \ldots, a_{nh})$ from ϱ we have $(f(a_{11}, \ldots, a_{n1}), \ldots, f(a_{1h}, \ldots, a_{nh})) \in \varrho$. $\mathrm{Pol}\,\varrho$ is the set of all $f \in P_k$ which preserve ϱ. For $F \subseteq P_k$, $\mathrm{Inv}\,F$ denotes the set of all the relations preserved by each $f \in F$.

Let C be a clone on E_k and $F \subseteq P_k$. F is a *complete relative* to C (or *C-complete*) if $\langle F \cup C\rangle_{\mathrm{CL}} = P_k$.

A relative complete set F in P_k is called a *relative base of P_k with respect to C* if no proper subset of F is relative complete in P_k with respect to C.

The functions from P_k may be classified into nonempty equivalence classes by their membership in the relative maximal clones. Then we can discuss the completeness properties in P_k in terms of these classes instead of individual functions: if a set is complete then replacing a function in the set by any function in the corresponding equivalence class yields another complete set.

Let $M_1, \ldots, M_m$ be relative maximal clones. Define the map $\phi : P_k \to \{0, 1\}^m$ by setting $\phi(f) := a_1 \ldots a_m$ where $a_i = 0$ if $f \in M_i$ and $a_i = 1$ if $f \notin M_i$. We call $\phi(f)$ *the characteristic vector of* f. We put $f\varrho g$ if $f, g \in P_k$ have the same characteristic vector, i.e. if $\phi(f) = \phi(g)$. It means that for all $j \in \{1, 2, \ldots, n\}$, either $\{f, g\} \subset M_j$ or $\{f, g\} \cap M_j = \emptyset$. Clearly, ϱ is an equivalence relation on P_k and so it partitions P_k into pairwise disjoint nonempty sets called *(equivalence) classes.*

To $a_1, \ldots, a_m \in \{0, 1\}^m$ we associate $A = \{i : a_i = 1\}$ and if $A_1, \ldots, A_l$ are the subsets of $\{1, \ldots, m\}$ corresponding to the characteristic vectors, the relative completeness problem is reduced to the listing of subsets of $\{A_1, \ldots, A_l\}$ covering $\{1, \ldots, m\}$.

All maximal clones in P_k containing the functions min and complement and all maximal clones in $P_k, 2 < k < 9$ containing two special unary operations are determined in [3].

2 Results and discussion

Using the statements of paper [3] it follows:

Theorem 2.1 *There are 11 relative maximal clones in* P_5 *:*

$$Q_1 = Pol\begin{pmatrix}012340134\\012341043\end{pmatrix} = Pol(E_5^2 - P_{02} - P_{03} - P_{04} - P_{12} - P_{13} - P_{14} - P_{23} - P_{24})$$
$$Q_2 = Pol\begin{pmatrix}01234121323\\01234213132\end{pmatrix} = Pol(E_5^2 - P_{01} - P_{02} - P_{03} - P_{04} - P_{14} - P_{24} - P_{34})$$
$$Q_3 = Pol(0134)$$
$$Q_4 = Pol(04)$$
$$Q_5 = Pol(123)$$
$$Q_6 = Pol(13)$$
$$Q_7 = Pol(024)$$
$$Q_8 = Pol(2)$$
$$Q_9 = Pol\begin{pmatrix}0123401020312131423243 4\\012341020302131413242 43\end{pmatrix} = Pol(E_5^2 - P_{04})$$
$$Q_{10} = Pol\begin{pmatrix}0123401021213232434\\0123410202131324243\end{pmatrix} = Pol(E_5^2 - P_{04} - P_{03} - P_{14})$$
$$Q_{11} = Pol\begin{pmatrix}01234010212232434\\01234102021324243\end{pmatrix} = Pol(E_5^2 - P_{04} - P_{03} - P_{13} - P_{14})$$

From theorems 1-14 [4] it follows respectively:

Theorem 2.2

1. $Q_5Q_7 \subset Q_8 \ \wedge \ Q_3Q_5 \subset Q_6 \ \wedge \ Q_3Q_7 \subset Q_4$.

2. $\overline{Q}_9Q_{10} \subset Q_8 \ \wedge \ \overline{Q}_9Q_{11} \subset Q_8 \ \wedge \ \overline{Q}_{10}Q_{11} \subset Q_8$.

3. $Q_2Q_6 \subset Q_5 \ \wedge \ Q_2Q_8 \subset Q_5 \ \wedge \ Q_1Q_4 \subset Q_3 \ \wedge \ Q_1Q_6 \subset Q_3$.

4. $Q_1Q_7Q_9 \subset Q_{10} \ \wedge \ Q_1Q_7Q_{10} \subset Q_{11}$.

5. $Q_7\overline{Q}_8\overline{Q}_{10} \subset \overline{Q}_{11} \ \wedge \ Q_7\overline{Q}_8\overline{Q}_{11} \subset \overline{Q}_{10} \ \wedge \ Q_7\overline{Q}_8\overline{Q}_9 \subset \overline{Q}_{10} \ \wedge \ Q_7\overline{Q}_8\overline{Q}_9 \subset \overline{Q}_{11}$.

6. $Q_2\overline{Q}_5\overline{Q}_{11} \subset \overline{Q}_{10} \ \wedge \ Q_2\overline{Q}_5\overline{Q}_9 \subset \overline{Q}_{10} \ \wedge \ Q_2\overline{Q}_5\overline{Q}_9 \subset \overline{Q}_{11}$.

7. $Q_4Q_9\overline{Q}_{10} \subset \overline{Q}_{11}$

8. $Q_3Q_{10} \subset Q_9 \ \wedge \ Q_6Q_{10} \subset Q_9 \ \wedge \ Q_4Q_{10} \subset Q_{11}$.

9. $Q_1Q_2 \subset Q_9$.

10. $Q_2Q_{10} \subset Q_9 \ \wedge \ Q_2Q_{11} \subset Q_9$.

11. $Q_2Q_{11} \subset Q_{10}$.

12. $Q_1Q_2\overline{Q}_5 \subset Q_9 \ \wedge \ Q_1Q_2\overline{Q}_5 \subset Q_{10} \ \wedge \ Q_1Q_2\overline{Q}_5 \subset Q_{11}$.

13. $Q_1\overline{Q}_5\overline{Q}_9 \subset \overline{Q}_{10} \ \wedge \ Q_1\overline{Q}_5\overline{Q}_{11} \subset \overline{Q}_{10}$.

14. $Q_1Q_2\overline{Q}_5 \subset Q_3$.

Remarks It is important to notice that theorem 2.2. is useful for classification of functions in P_5 (according to their membership in all maximal clones), which is still an open problem.

Theorem 2.3 *There are 391 classes of functions in* P_5 *with respect to the clone generated by* $min(x, y)$ *and* $\overline{x} = 4 - x$.

Proof In P_5 we have 2048 possible classes of functions. According to the previous theorem we have found 1657 empty classes of functions. Representatives are efectively constructed for other 391 classes of functions. They are given in the following tables.

	class of funct.	repre.		class of funct.	repre.		class of funct.	repre.		class of funct.	repre.
1	00000000000	#72	68	01011010011	#157	135	10011111011	#4	202	11001000000	#246
2	00000010000	#113	69	01011010110	#158	136	10011111111	#99	203	11001000011	#247
3	00000010011	#114	70	01011010111	#159	137	10100000000	#201	204	11001000110	#248
4	00000011000	#62	71	01011011000	#54	138	10100000011	#202	205	11001000111	#249
5	00000011011	#71	72	01011011011	#60	139	10100000111	#203	206	11001001000	#250
6	00001101000	#1	73	01011011111	#85	140	10100010000	#204	207	11001001011	#58
7	00001111000	#115	74	01011110000	#33	141	10100010011	#205	208	11001001111	#57
8	00010010000	#73	75	01011110010	#44	142	10100010111	#206	209	11001010000	#251
9	00010010001	#116	76	01011110011	#160	143	10100011000	#207	210	11001010011	#252
10	00010010011	#117	77	01011110110	#48	144	10100011011	#208	211	11001010110	#253
11	00010011000	#63	78	01011110111	#161	145	10100011111	#209	212	11001010111	#254
12	00010011001	#108	79	01011111000	#7	146	10100100000	#77	213	11001011000	#255
13	00010011011	#80	80	01011111011	#18	147	10100100011	#210	214	11001011011	#256
14	00011111000	#2	81	01011111111	#22	148	10100100111	#211	215	11001011111	#257
15	00110100000	#79	82	01110100000	#162	149	10100110000	#212	216	11001100000	#258
16	00110110000	#110	83	01110100100	#163	150	10100110011	#213	217	11001100011	#42
17	00110110001	#118	84	01110100110	#164	151	10100110111	#214	218	11001100110	#259
18	00110110011	#119	85	01110100111	#165	152	10100111000	#67	219	11001100111	#36
19	00110111000	#69	86	01110110000	#166	153	10100111011	#70	220	11001101000	#260
20	00110111001	#111	87	01110110001	#167	154	10100111111	#215	221	11001101011	#16
21	00110111011	#84	88	01110110010	#168	155	10101101000	#216	222	11001101111	#10
22	01000000000	#120	89	01110110011	#169	156	10101101011	#217	223	11001110000	#261
23	01000000110	#121	90	01110110100	#170	157	10101101111	#218	224	11001110011	#262
24	01000000111	#122	91	01110110101	#171	158	10101111000	#219	225	11001110110	#263
25	01000010000	#123	92	01110110110	#172	159	10101111011	#220	226	11001110111	#264
26	01000010011	#124	93	01110110111	#173	160	10101111111	#221	227	11001111000	#265
27	01000010110	#124	94	01110111000	#174	161	10110000000	#74	228	11001111011	#29
28	01000010111	#126	95	01110111001	#175	162	10110000001	#222	229	11001111111	#24
29	01000011000	#127	96	01110111011	#176	163	10110000011	#96	230	11010010000	#266
30	01000011011	#128	97	01110111111	#177	164	10110000111	#223	231	11010010001	#267
31	01000011111	#129	98	01111100000	#40	165	10110010000	#109	232	11010010010	#268
32	01001000000	#130	99	01111100110	#178	166	10110010001	#224	233	11010010011	#269
33	01001000110	#131	100	01111100111	#179	167	10110010011	#225	234	11010010110	#270
34	01001000111	#132	101	01111101000	#13	168	10110010111	#226	235	11010010111	#271
35	01001001000	#53	102	01111101111	#52	169	10110011000	#64	236	11010011000	#272
36	01001001111	#61	103	01111110000	#102	170	10110011001	#107	237	11010011001	#273
37	01001010000	#133	104	01111110010	#180	171	10110011011	#81	238	11010011011	#274
38	01001010011	#134	105	01111110011	#181	172	10110011111	#227	239	11010011111	#275
39	01001010110	#135	106	01111110110	#182	173	10110100000	#91	240	11011010000	#276
40	01001010111	#136	107	01111110111	#183	174	10110100001	#228	241	11011010001	#277
41	01001011000	#137	108	01111111000	#27	175	10110100011	#98	242	11011010010	#278
42	01001011011	#138	109	01111111011	#51	176	10110100111	#229	243	11011010011	#279
43	01001011111	#139	110	01111111111	#103	177	10110110000	#78	244	11011010110	#280
44	01001100000	#32	111	10000000000	#184	178	10110110001	#230	245	11011010111	#281
45	01001100110	#45	112	10000000011	#76	179	10110110011	#97	246	11011011000	#282
46	01001100111	#140	113	10000000111	#185	180	10110110111	#231	247	11011011001	#283
47	01001101000	#6	114	10000010000	#186	181	10110111000	#68	248	11011011011	#56
48	01001101111	#19	115	10000010011	#187	182	10110111001	#106	249	11011011111	#87
49	01001110000	#141	116	10000010111	#188	183	10110111011	#83	250	11011110000	#284
50	01001110011	#142	117	10000011000	#189	184	10110111111	#232	251	11011110001	#37
51	01001110110	#143	118	10000011011	#66	185	10111101000	#3	252	11011110010	#285
52	01001110111	#144	119	10000011111	#190	186	10111101011	#233	253	11011110011	#35
53	01001111000	#23	120	10001101000	#191	187	10111101111	#100	254	11011110110	#286
54	01001111011	#49	121	10001101011	#192	188	10111111000	#101	255	11011110111	#46
55	01001111111	#30	122	10001101111	#5	189	10111111011	#112	256	11011111000	#287
56	01010010000	#145	123	10001111000	#193	190	10111111111	#234	257	11011111001	#25
57	01010010001	#146	124	10001111011	#194	191	11000000000	#235	258	11011111011	#9
58	01010010010	#147	125	10001111111	#195	192	11000000011	#236	259	11011111111	#20
59	01010010011	#148	126	10010010000	#196	193	11000000110	#237	260	11100000000	#288
60	01010010110	#149	127	10010010001	#75	194	11000000111	#238	261	11100000011	#289
61	01010010111	#150	128	10010010011	#95	195	11000010000	#239	262	11100000110	#290
62	01010011000	#151	129	10010010111	#197	196	11000010011	#240	263	11100000111	#291
63	01010011001	#152	130	10010011000	#198	197	11000010110	#241	264	11100010000	#292
64	01010011011	#153	131	10010011001	#65	198	11000010111	#242	265	11100010011	#293
65	01010011111	#154	132	10010011011	#82	199	11000011000	#243	266	11100010110	#294
66	01011010000	#155	133	10010011111	#199	200	11000011011	#244	267	11100010111	#295
67	01011010010	#156	134	10011111000	#200	201	11000011111	#245	268	11100011000	#296

	class of funct.	repre.
269	11100011011	#297
270	11100011111	#298
271	11100100000	#299
272	11100100011	#300
273	11100100100	#301
274	11100100110	#302
275	11100100111	#303
276	11100110000	#304
277	11100110011	#305
278	11100110100	#306
279	11100110110	#307
280	11100110111	#308
281	11100111000	#309
282	11100111011	#310
283	11100111111	#311
284	11101000000	#312
285	11101000011	#313
286	11101000110	#314
287	11101000111	#315
288	11101001000	#316
289	11101001011	#317
290	11101001111	#318
291	11101010000	#319
292	11101010011	#320
293	11101010110	#321
294	11101010111	#322
295	11101011000	#323
296	11101011011	#324
297	11101011111	#325
298	11101100000	#38
299	11101100011	#326
300	11101100100	#41
301	11101100110	#327
302	11101100111	#90
303	11101101000	#11
304	11101101011	#88
305	11101101111	#15
306	11101110000	#328
307	11101110011	#329
308	11101110100	#330
309	11101110110	#331
310	11101110111	#332
311	11101111000	#26
312	11101111011	#50
313	11101111111	#28
314	11110000000	#333
315	11110000001	#334
316	11110000010	#335
317	11110000011	#336
318	11110000110	#337
319	11110000111	#338
320	11110010000	#339
321	11110010001	#340
322	11110010010	#341
323	11110010011	#342
324	11110010110	#343
325	11110010111	#344
326	11110011000	#345
327	11110011001	#346
328	11110011011	#347
329	11110011111	#348
330	11110100000	#349
331	11110100001	#350
332	11110100010	#351
333	11110100011	#352
334	11110100100	#353
335	11110100101	#354
336	11110100110	#355
337	11110100111	#356
338	11110110000	#357
339	11110110001	#358
340	11110110010	#359
341	11110110011	#360
342	11110110100	#361
343	11110110101	#362
344	11110110110	#363
345	11110110111	#364
346	11110111000	#365
347	11110111001	#366
348	11110111011	#367
349	11110111111	#368
350	11111000000	#369
351	11111000001	#370
352	11111000010	#371
353	11111000011	#372
354	11111000110	#373
355	11111000111	#374
356	11111001000	#55
357	11111001011	#59
358	11111001111	#86
359	11111010000	#375
360	11111010001	#376
361	11111010010	#377
362	11111010011	#378
363	11111010110	#379
364	11111010111	#380
365	11111011000	#104
366	11111011001	#381
367	11111011011	#105
368	11111011111	#382
369	11111100000	#34
370	11111100001	#383
371	11111100010	#43
372	11111100011	#384
373	11111100100	#93
374	11111100101	#385
375	11111100110	#47
376	11111100111	#386
377	11111101000	#8
378	11111101011	#17
379	11111101111	#21
380	11111110000	#39
381	11111110001	#387
382	11111110010	#388
383	11111110011	#89
384	11111110100	#94
385	11111110101	#389
386	11111110110	#390
387	11111110111	#92
388	11111111000	#12
389	11111111001	#391
390	11111111011	#14
391	11111111111	#31

UNARY REPRESENTATIVES:

#	01234
1	00000
2	00001
3	00002
4	00003
5	00004
6	00010
7	00011
8	00012
9	00013
10	00014
11	00020
12	00021
13	00022
14	00023
15	00024
16	00030
17	00032
18	00033
19	00034
20	00041
21	00042
22	00043
23	00100
24	00104
25	00113
26	00120
27	00122
28	00124
29	00130
30	00134
31	00142
32	00200
33	00201
34	00202
35	00203
36	00204
37	00213
38	00220
39	00221
40	00222
41	00224
42	00230
43	00232
44	00233
45	00234
46	00241
47	00242
48	00243
49	00300
50	00320
51	00322
52	00422
53	01010
54	01011
55	01012
56	01013
57	01014
58	01030
59	01032
60	01033
61	01034
62	01110
63	01111
64	01112
65	01113
66	01114
67	01120
68	01121
69	01122
70	01124
71	01134
72	01210
73	01211
74	01212
75	01213
76	01214
77	01220
78	01221
79	01222
80	01311
81	01312
82	01313
83	01321
84	01322
85	01411
86	01412
87	01413
88	02030
89	02203
90	02204
91	02222
92	02241
93	02242
94	02243
95	03211
96	03212
97	03221
98	03222
99	04441
100	04442
101	10002
102	10222
103	10422
104	11012
105	11032
106	11123
107	11132
108	11133
109	11212
110	11222
111	11322
112	14442

BINARY REPRESENTATIVES :

f(00)f(01)f(02)f(03)f(04) | f(10)f(11)f(12)f(13)f(14) | f(20)f(21)f(22)f(23)f(24) | f(30)f(31)f(32)f(33)f(34) | f(40)f(41)f(42)f(43)f(44)

113	43334	43334	43234	43334	44444
114	01334	11333	22233	33333	43334
115	01110	00000	00000	00000	00000
116	11333	11333	33233	33333	33333
117	10001	11111	33211	11111	11111
118	22133	22133	22222	22222	22222
119	10001	11111	33222	22222	22222
120	44244	43234	22222	43234	44244
121	00200	01210	22222	01234	00244
122	44444	43333	43234	43133	44044
123	01100	11111	11211	11111	01110
131	00000	11111	22222	33433	44444
132	44444	43334	43244	43134	44044
133	44444	43434	44243	43434	44444
134	44444	43334	43244	43134	44144
135	00000	11111	22222	33433	43334
136	44344	43334	43244	43134	44044
137	44444	43434	44344	43434	44444
138	44444	43334	43344	43134	44144
139	44444	43334	43344	43134	44044
140	44444	43344	43244	43134	44044
141	44444	44444	44243	44444	44444

149	01111	11111	22222	43333	34444
150	11111	11110	11234	11111	11111
151	11111	11111	01111	11111	11111
152	11311	11311	11111	11111	11011
153	33333	33333	33311	33334	33333
154	33333	33333	33310	33334	33333
155	44443	43434	44244	43434	44444
156	11111	11011	22222	33333	33333
157	33333	33333	33211	33434	33333
158	11110	11111	22222	43434	44444
159	33333	33333	33200	33334	33333
160	33333	33343	33211	33333	33333
161	33333	33343	33201	33333	33333
162	22222	22222	22222	43222	44222
163	01222	01222	22222	22233	22244
164	22210	22211	22222	01234	00244
165	22222	22222	22210	22234	22244
166	22243	22234	22222	22222	22222
167	22022	22122	11222	33222	33222
168	22211	22211	22222	22234	22244
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171	11133	01133	22222	22233	22243
172	01222	11222	22222	43222	33222
173	22222	22222	33210	22334	22444
174	22011	22111	11111	11111	11111
175	22133	22133	22322	22333	22344
176	22222	22222	22322	22133	22144
177	22222	22222	22333	22133	22044
178	22222	22222	00244	22222	22222
179	22222	22222	01244	22422	22422
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181	22222	22222	11244	22322	22322
182	01122	10122	22222	43333	33333
183	22222	22222	10243	22322	22322
184	01210	11210	22220	11210	01210
185	44444	43330	43230	43330	44444
186	00000	11211	11211	11111	01110
187	41114	11111	11211	11111	41114
188	40004	11111	11211	11111	41114
189	01110	11111	11112	11111	01110
190	40004	11111	11111	11111	41114
191	01210	00000	00000	00000	00000
192	41214	44444	44444	44444	44444
193	01210	10000	10000	10000	00000
194	41114	44444	44444	44444	44444
195	40004	44444	44444	44444	43334
196	11211	01211	02222	01211	11211
197	34443	33333	33233	33333	30004
198	33333	33233	32323	33233	33233
199	33333	33333	23333	33333	04443
200	11111	10001	20001	10001	11111
201	44444	23233	22222	23233	43234
202	41214	43233	42222	43233	42224
203	40004	43233	42222	43233	42224
204	02220	21212	22222	21212	02120
205	41114	23334	23234	23334	44444
206	40004	23333	23233	23333	43334
207	02220	21212	22122	21212	02220
208	41114	23333	23333	23333	43334
209	40004	23333	23333	23333	43334
210	44444	42224	42224	42124	44444
211	40004	42224	42224	42124	44444
212	02120	22222	22222	22222	02220
213	41114	22223	22223	22223	42224
214	40004	22223	22223	22223	42224
215	40004	22223	22323	22223	42224
216	00000	20000	20000	20000	00000
217	42224	14444	24444	14444	44444
218	42224	04444	04444	04444	44444
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242	40200	01110	11214	01110	00000
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245	40200	01110	01110	01114	00000
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247	03030	33033	03230	33333	03030
248	43434	43433	22222	11011	00200
249	00000	01210	00200	01014	00000
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252	03330	33033	03233	33333	03330
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257	00000	01010	00104	01010	00000
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259	00200	00000	22222	44444	44444
260	00200	00000	20002	00000	00200
261	00100	00000	00202	00000	00000
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271	11411	11111	21210	11111	11111
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274	33333	03333	33333	33333	33333
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277	11111	11111	10213	11111	11111
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279	03333	33033	03233	33333	33333
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390	22122	20222	22222	24222	22222
391	22222	20222	22123	22222	22222

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