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NEARLY pT_i-CONTINUOUS MAPPINGS

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Abstract. Some generalizations of T_i-pairwise continuous functions and similar generalizations of pairwise T_i-spaces, for $i = 1, 2, 3$ are introduced and their relationship with the concept of T_i-pairwise continuity is studied.

1. Introduction

In a bitopological space $X = (X, \tau_1, \tau_2)$ a cover $\mathcal{U}$ of X is pairwise open if $\mathcal{U} \subset \tau_1 \cup \tau_2$ and if furthermore $\mathcal{U}$ contains a non-empty member of τ_1 and a non-empty member of τ_2 [1]. A pairwise open cover is called pT_1-open. A pairwise open cover $\mathcal{U}$ of a bitopological space X is said to be pT_2-open if for each $U \in \mathcal{U}$, $\tau_i\text{-int}(X \setminus U) \neq \emptyset$, for $i = 1$ or $i = 2$. A pairwise open cover $\mathcal{W}$ of a bitopological space X is called pT_3-open if for each $W \in \mathcal{W}$ whenever $W \in \tau_i$, there exist τ_j-open sets V_1 and V_2 such that V_1, $V_2 \neq \emptyset$, $V_1 \subset \tau_i\text{-cl}\, V_1 \subset V_2 \subset X \setminus W$, for $i \neq j$ and i, $j = 1, 2$ [3]. A function f from a bitopological space X into a bitopological space Y is called T_i-pairwise continuous if for every pT_i-open cover $\mathcal{V}$ of Y there exists a τ_k-open cover $\mathcal{W}$ of X such that for every $W \in \mathcal{W}$ there is a $V \in \mathcal{V}$ such that $f(W) \subset V$, where $k \in \{1, 2\}$ and $i \in \{1, 2, 3\}$ [3]. Pairwise T_1, T_2 and R_0 axioms will be denoted by PT_1, PT_2 and PR_0 respectively [9,11].

2. Some new bitopological axioms

DEFINITION 2.1. A bitopological space X is mPT_1 if for every pair of distinct points x and y in X the following holds: $\tau_1\text{-cl}\{x\} \cap \tau_2\text{-cl}\{y\} = \emptyset$ or $\tau_2\text{-cl}\{x\} \cap \tau_1\text{-cl}\{y\} = \emptyset$ [5].

A bitopological space X is $MNPT_1$ if for every pair of distinct points x and y in X there exists a τ_1-open set or a τ_2-open set containing x but not y [6].

A bitopological space X is wPT_1 if for each pair of distinct points there is a τ_1-open set containing one of the points but not the other and a τ_2-open set containing the second point but not the first [10].

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Definition 2.2. A bitopological space X is said to be a nearly PT_i-space (briefly nPT_i-space), $i \in \{1,2,3\}$, if for each point $x \in X$ and a τ_k-open neighbourhood V of x, $k \in \{1,2\}$, there exists a pT_i-open cover $\mathcal{U}$ of X such that $St(x, \mathcal{U}) \subset V$.

It is easy to verify that every PT_i-space is nPT_i, but the converse is not true in general, as it follows from

Example 1. Let $X = \{a,b,c\}$, $\tau_1 = \{\emptyset, X, \{a\}\}$ and $\tau_2 = \{X, \emptyset, \{b,c\}\}$. Then X is nPT_1 but not $MNPT_1$. Hence it does not satisfy any of the axioms wPT_1, mPT_1 or PT_1. Also X is nPT_2 but not wPT_2.

Example 2. Let $X = \{a,b,c\}$, $\tau_1 = \{\emptyset, X, \{a,c\}, \{b,a\}, \{a\}\}$ and $\tau_2 = \{\emptyset, X, \{b,c\}\}$. Then X is $MNPT_1$ but not nPT_1.

The following diagram of implications holds and none of these implications is reversible.

$$\begin{array}{ccccc} mPT_1 & \longrightarrow & wPT_1 & \longrightarrow & MNPT_1 \\ \uparrow & & & & \\ PT_1 & \longrightarrow & nPT_1 & & \end{array}$$

Moreover, the axiom nPT_1 is independent from any of $MNPT_1$, wPT_1 and mPT_1.

Proposition 2.3. *Every PR_0 space is nPT_1.*

Proof. Let $x \in X$ and let U be a τ_i-open neighbourhood of x where $i \in \{1,2\}$. Since X is PR_0, then τ_j-cl$\{x\} \subset U$, for $i \neq j$ and $j \in \{1,2\}$. Then $\mathcal{U} = \{U, X \setminus \tau_j\text{-cl}\{x\}\}$ is a pT_1-open cover of the bitopological space X and $St(x, \mathcal{U}) = U$. Therefore X is an nPT_1 space. ∎

Remark 1. The converse of Proposition 2.3 is not true in general, which follows from

Example 3. Let $X = \{a,b,c\}$, $\tau_1 = \{\emptyset, X, \{c\}, \{b,c\}\}$, and $\tau_2 = \{X, \emptyset, \{a\}, \{a,b\}, \{a,c\}\}$. Then X is nPT_1 but not a PR_0 space.

Definition 2.4. A bitopological space X is said to be a $PT(i,k)$ space, $i,k \in \{1,2,3\}$, if for every $x \in X$ and every pT_k-open cover $\mathcal{U}$ of X there exists a pT_i-open cover $\mathcal{V}$ of X and a $U \in \mathcal{U}$ such that $St(x, \mathcal{V}) \subset U$.

It is easy to prove that the following diagram holds:

$$\begin{array}{ccccc} PT(3,3) & \longrightarrow & PT(2,3) & \longrightarrow & PT(1,3) \\ \uparrow & & \uparrow & & \uparrow \\ PT(3,2) & \longrightarrow & PT(2,2) & \longrightarrow & PT(1,2) \\ \uparrow & & \uparrow & & \uparrow \\ PT(3,1) & \longrightarrow & PT(2,1) & \longrightarrow & PT(1,1) \\ \uparrow & & \uparrow & & \uparrow \\ nPT_3 & \longrightarrow & nPT_2 & \longrightarrow & nPT_1 \end{array}$$

The following example shows that any of $PT(i,k)$ axioms does not imply nPT_j, $j \in \{1,2,3\}$.

EXAMPLE 4. Let $X = \{a,b,c\}$, $\tau_1 = \{\emptyset, X, \{a\}, \{a,c\}\}$ and $\tau_2 = \{X, \emptyset, \{b,c\}\}$. Then X is $PT(3,1)$ but not nPT_1.

3. Nearly pT_i-continuous mappings

In the following properties, corollaries, examples and definition, $i \in \{1,2,3\}$ and $k \in \{1,2\}$.

DEFINITION 3.1. A mapping $f\colon (X,\tau_1.\tau_2) \to (Y,\mathcal{U}_1,\mathcal{U}_2)$ is said to be *nearly pT_i-continuous* at a point $x \in X$ if for every pT_i-open cover $\mathcal{U}$ of Y there exists a τ_k-open neighbourhood $V \subset X$ of x such that $f(V) \subset St(f(x),\mathcal{U})$.

A mapping f is nearly pT_i-continuous if it is nearly pT_i-continuous at each point of X. It is evident that every T_i-pairwise continuous mapping is nearly pT_i-continuous, but the converse is not necessarily true in general, as the following example shows.

EXAMPLE 5. Let $X = \{a,b,c,d\}$, $\tau_1 = \{\emptyset, X, \{a,d\}, \{a\}, \{d\}, \{b,c\}, \{a,b,c\}, \{b,c,d\}\}$, $\tau_2 = \{\emptyset, X, \{a\}, \{d\}, \{a,d\}, \{a,c\}, \{a,c,d\}, \{a,b,d\}\}$ and $Y = \{1,2,3\}$, $\mathcal{U}_1 = \{\emptyset, Y, \{1,3\}\}$, $\mathcal{U}_2 = \{\emptyset, Y, \{2,3\}\}$. Define $f\colon X \to Y$ as follows: $f(a) = 2$, $f(b) = 3 = f(c)$ and $f(d) = 1$. Then f is nearly pT_1-continuous but not T_1-pairwise continuous.

PROPOSITION 3.2. *Let a bitopological space $(Y,\mathcal{U}_1,\mathcal{U}_2)$ be nPT_i and let $f\colon X \to Y$ be a nearly pT_i-continuous mapping from a bitopological space (X,τ_1,τ_2). Then f is p-continuous.*

Proof. Let $V \in \mathcal{U}_k$ be a neighbourhood of $f(x) \in Y$. Since Y is nPT_i, there exists a pT_i-open cover $\mathcal{U}$ of Y such that $St(f(x),\mathcal{U}) \subset V$. Since f is nearly pT_i-continuous, there exists a τ_k-open neighbourhood U of x such that $f(U) \subset St(f(x),\mathcal{U}) \subset V$. Thus f is p-continuous. ∎

REMARK 2. A p-continuous mapping $f\colon X \to Y$, where Y is an nPT_i space, need not be nearly pT_i-continuous, which follows from

EXAMPLE 6. Let $X = \{1,2,3\}$, $\tau_1 = \{\emptyset, X, \{1\}, \{1,2\}, \{1,3\}\}$, $\tau_2 = \{\emptyset, X, \{3\}, \{2,3\}\}$ and $Y = \{a,b,c\}$, $\mathcal{U}_1 = \{\emptyset, Y, \{a\}\}$, $\mathcal{U}_2 = \{\emptyset, Y, \{b,c\}\}$. Define the mapping $f\colon X \to Y$ as follows: $f(1) = a$, $f(2) = c$ and $f(3) = b$. Then Y is nPT_1, f is p-continuous but not nearly pT_1-continuous.

COROLLARY 1. *a) Let Y be a PR_0 space and let $f\colon X \to Y$ be a nearly pT_1-continuous mapping. Then f is p-continuous.*

b) Let Y be a PT_i space and let $f\colon X \to Y$ be a nearly pT_i-continuous mapping. Then f is p-continuous.

COROLLARY 2. *a) Let Y be an nPT_i space and let $f\colon X \to Y$ be a T_i-pairwise continuous mapping. Then f is p-continuous.*

b) Let Y be a PT_i space and $f\colon X \to Y$ be a T_i-pairwise continuous mapping. Then f is p-continuous.

Proposition 3.3. *Let Y be a $PT(i,k)$ bitopological space, for $i,k \in \{1,2,3\}$ and let $f\colon X \to Y$ be a nearly pT_i-continuous mapping. Then f is T_k-pairwise continuous.*

Proof. Let X be a bitopological space and let $f\colon X \to Y$ be nearly pT_i-continuous. Let $\mathcal{U}$ be a pT_k-open cover of Y and let $x \in X$. Since Y is $PT(i,k)$, there exists a pT_i-open cover $\mathcal{V}$ of Y and $U \in \mathcal{U}$ such that $St(f(x),\mathcal{V}) \subset U$. Since f is nearly pT_i-continuous, there is a τ_j-open neighbourhood $W \subset X$ of x such that $f(W) \subset St(f(x),\mathcal{V}) \subset U$, for $j \in \{1,2\}$. Thus f is T_k-pairwise continuous. ■

Remark 3. Let $f\colon X \to Y$ be a nearly pT_3-continuous mapping and let X be a p-connected space. Then Y need not be p-connected [8].

The last Remark is true even if (X,τ_1) and (X,τ_2) are connected spaces, which follows from

Example 7. Let $X = \{a,b,c\}$, $\tau_1 = \{\emptyset, X, \{a\}, \{a,b\}\}$, $\tau_2 = \{\emptyset, X, \{b\}\}$ and $Y = \{1,2,3\}$, $\mathcal{U}_1 = \{\emptyset, Y, \{2\}\}$, $\mathcal{U}_2 = \{\emptyset, Y, \{3\}, \{1,3\}\}$. Define $f\colon X \to Y$ by $f(b) = 1$ and $f(a) = f(c) = 3$. Then f is nearly pT_3-continuous, X is p-connected, (X,τ_1) and (X,τ_2) are connected, but Y is not p-connected.

Remark 4. Let $f\colon (X,\tau_1,\tau_2) \to (Y,\mathcal{U}_1,\mathcal{U}_2)$ be a nearly pT_i-continuous mapping onto a $PT(i,3)$ space Y and let (X,τ_k) be connected. Then Y is p-connected. The proof follows from Theorem 4.4 in [3] and Proposition 3.3.

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