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CERTAIN BOUNDED FUNCTIONS OF COMPLEX ORDER

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Abstract. In this paper we obtain sharp coefficient bounds for functions analytic in the unit disc U and belonging to the class $R(b, M)$, $b \neq 0$ is a complex number. Also, we maximize $|a_3 - \mu a_2^2|$ over the class $R(b, M)$ and obtain distortion theorem for functions in this class.

1. Introduction

Let A denote the class of functions

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \tag{1.1}$$

which are analytic in the unit disc U. Also denote by S the subclass of A, consisting of all univalent functions in U. Let Ω denote the class of bounded analytic functions w in U satisfying the conditions $w(0) = 0$ and $|w(z)| \leq |z|$ for $z \in U$. For $f \in A$, we say that f belongs to the class $F(b, M)$ ($b \neq 0$ complex, $M > \frac{1}{2}$), of bounded starlike functions of complex order, if and only if $\frac{f(z)}{z} \neq 0$ in U and for fixed M,

$$\left| \frac{b - 1 + \frac{zf'(z)}{f(z)}}{b} - M \right| < M, \quad z \in U. \tag{1.2}$$

The class $F(b, M)$ was studied by Nasr and Aouf [13].

We note that:

(i) $F(b, \infty) = S(b)$, where $S(b)$ is the class of starlike functions of complex order, introduced and studied by Nasr and Aouf [14];

(ii) $F(\cos\lambda e^{-i\lambda}, M) = F_{\lambda,M}$ ($|\lambda| < \frac{\pi}{2}, M > \frac{1}{2}$), where $F_{\lambda,M}$ is the class of bounded spiral-like functions, studied by Kulshrestha [9];

(iii) $F((1 - \alpha)\cos\lambda e^{-i\lambda}, M) = F_M(\lambda, \alpha)$ ($|\lambda| < \frac{\pi}{2}, 0 \leq \alpha < 1, M > \frac{1}{2}$), where $F_M(\lambda, \alpha)$ is the class of bounded spiral-like functions of order α, studied by Aouf [3, 4].

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In [1] Halim studied the class $R(b)$ defined as follows:

A function $f \in A$ belongs to the class $R(b)$, if and only if, for $z \in U$

$$\operatorname{Re}\left\{1+\frac{1}{b}(f'(z)-1)\right\}>0, \quad z \in U, \tag{1.3}$$

where b is a non-zero complex number. We note that $R(1)=R$ (see MacGregor [10]). Halim [1] proved that if $\operatorname{Re}\{b\} \geq |b|^2$, then $f \in R(b)$ is univalent.

In the present paper, we consider the class $R(b,M)$ of functions $f \in A$, satisfying the condition:

$$\left|\frac{b-1+f'(z)}{b}-M\right|<M \quad (M>\frac{1}{2};\ z \in U), \tag{1.4}$$

where $b \neq 0$, complex. We note that $R(b,\infty)=R(b)$ and $R(1-\alpha,\infty)=R_\alpha$ $(0 \leq \alpha<1)$ (Ahuja [2]).

Taking different values of b and M, the class $R(b,M)$ reduces to the following subclasses of R:

(1) $R(1-\alpha, \frac{1}{2(1-\beta)})=R_1(\alpha,\beta)$ (Mogra [12])
$=\left\{f \in A: \left|\frac{f'(z)-1}{2\beta(f'(z)-\alpha)-(f'(z)-1)}\right|<1, 0 \leq \alpha<1, 0<\beta \leq 1, z \in U\right\}$;

(2) $R((1-\alpha)\cos\lambda e^{-i\lambda}, \frac{1}{2(1-\beta)})=R_1^\lambda(\alpha,\beta)$ (Ahuja [2])
$=\left\{f \in A: \left|\frac{f'(z)-1}{2\beta(f'(z)-1+(1-\alpha)\cos\lambda e^{-i\lambda})-(f'(z)-1)}\right|<1, 0 \leq \alpha<1, 0<\beta \leq 1, z \in U\right\}$;

(3) $R((1-\alpha)\cos\lambda e^{-i\lambda}, \infty)=R_\alpha^\lambda$ (Ahuja [2])
$=\left\{f \in A: \operatorname{Re} e^{i\lambda} f'(z)>\alpha\sin\lambda,\ 0 \leq \alpha<1,\ |\lambda|<\frac{\pi}{2}, z \in U\right\}$;

(4) $R(\cos\lambda e^{-i\lambda}, \frac{1}{\cos\lambda})=R^{*\lambda}$(Ahuja [2])
$=\left\{f \in A: \left|e^{i\lambda} f'(z)-(1+i\sin\lambda)\right|<1, |\lambda|<\frac{\pi}{2}, z \in U\right\}$;

(5) $R(\cos\lambda e^{-i\lambda}, \frac{1}{2\rho})=R^{*\lambda}(\rho)$ (Ahuja [2])
$=\left\{f \in A: \left|\frac{e^{i\lambda} f'(z)-i\sin\lambda}{\cos\lambda}-\frac{1}{2\rho}\right|<\frac{1}{2\rho},\ |\lambda|<\frac{\pi}{2}, 0 \leq \rho<1, z \in U\right\}$;

(6) $R(\cos\lambda e^{-i\lambda}, M)=R_M^{*\lambda}$ (Ahuja [2])
$=\left\{f \in A: \left|\frac{e^{i\lambda} f'(z)-i\sin\lambda}{\cos\lambda}-M\right|<M, |\lambda|<\frac{\pi}{2}, M>\frac{1}{2}, z \in U\right\}$;

(7) $R((1-\alpha)\cos\lambda e^{-i\lambda}, M)=R_{M,\alpha}^{*\lambda}$ (Aouf and Owa [5])
$=\left\{f(z) \in A: \left|\frac{e^{i\lambda} f'(z)-\alpha\cos\lambda-i\sin\lambda}{(1-\alpha)\cos\lambda}-M\right|<M, 0 \leq \alpha<1, |\lambda|<\frac{\pi}{2}, M>\frac{1}{2}, z \in U\right\}$;

(8) $R(\frac{2\beta(1-\alpha)}{1+\beta}, \frac{1}{1-\beta})=R(\alpha,\beta)$ (Juneja and Mogra [7])
$=\left\{f \in A: \left|\frac{f'(z)-1}{f'(z)+1-2\alpha}\right|<\beta, 0 \leq \alpha<1, 0<\beta \leq 1, z \in U\right\}$;

(9) $R(\frac{2\beta(1-\alpha)\cos\lambda e^{-i\lambda}}{1+\beta}, \frac{1}{1-\beta})=R_{\alpha,\beta}^\lambda$ (Makowka [11])
$=\left\{f \in A: \left|\frac{f'(z)-1}{f'(z)-1+2(1-\alpha)\cos\lambda e^{-i\lambda}}\right|<\beta, 0 \leq \alpha<1, |\lambda|<\frac{\pi}{2}, 0<\beta \leq 1, z \in U\right\}$;

(10) $R(\frac{2\beta}{1+\beta}, \frac{1}{1-\beta})=R(\beta)$ (Padmanabhan [16] and Caplinger and Causey [6])
$=\left\{f \in A: \left|\frac{f'(z)-1}{f'(z)+1}\right|<\beta, 0<\beta \leq 1, z \in U\right\}$.

We further, observe that, by the special choice of M our class $R(b,M)$ gives rise the following new subclasses of R:

(1) $R\left(b, \frac{1}{2(1-\beta)}\right) = R(b,\beta)$
$= \left\{f \in A : \left|\frac{f'(z)-1}{2\beta[f'(z)-1+b]-[f'(z)-1]}\right| < 1, b \neq 0, \text{ complex}, 0 < \beta \leq 1, z \in U\right\}$;

(2) $R\left((1-\alpha)\cos\lambda e^{-i\lambda}, \frac{1}{2\rho}\right) = R^{*\lambda}(\rho,\alpha)$
$= \left\{f \in A : \left|\frac{e^{i\lambda}f'(z)-\alpha\cos\lambda-i\sin\lambda}{(1-\alpha)\cos\lambda} - \frac{1}{2\rho}\right| < \frac{1}{2\rho}, |\lambda| < \frac{\pi}{2}, 0 \leq \alpha < 1, 0 \leq \rho < 1; z \in U\right\}$.

We can easily show that $f \in R(b,M)$ if and only if there exists a function $w \in \Omega$ such that [9]

$$1 + \frac{1}{b}(f'(z)-1) = \frac{1+w(z)}{1-mw(z)}, \quad m = 1 - \frac{1}{M}. \tag{1.5}$$

Thus, from (1.5) it follows that $f \in R(b,M)$ if and only for $z \in U$

$$f'(z) = \frac{1+[(1+m)b-m]w(z)}{1-mw(z)}, \quad w(z) \in \Omega, \ \ m = 1 - \frac{1}{M}. \tag{1.6}$$

2. Coefficient estimates

Theorem 1. *Let the function f defined by* (1.1) *be in the class $R(b,M)$, $M > \frac{1}{2}$. Then*

$$|a_n| \leq \frac{(1+m)\,|b|}{n} \quad (n \geq 2, m = 1 - \frac{1}{M}). \tag{2.1}$$

The estimates are sharp.

Proof. Since $f \in R(b,M)$, we have

$$f'(z) = \frac{1+[(1+m)b-m]w(z)}{1-mw(z)} \quad (w \in \Omega, m = 1 - \frac{1}{M}). \tag{2.2}$$

By simplification, (2.2) yields

$$[(1+m)b + m(f'(z)-1)]w(z) = f'(z) - 1,$$

that is

$$[(1+m)b + m\sum_{n=2}^{\infty} na_n z^{n-1}][\sum_{n=1}^{\infty} t_n z^n] = \sum_{n=2}^{\infty} na_n z^{n-1}. \tag{2.3}$$

Equating corresponding coefficients on both sides of (2.3), we find that the coefficient a_n on the right hand side of (2.3) depends only on $a_2, a_3, \dots, a_{n-1}$, on the left hand side of (2.3). Hence for $n \geq 2$, it follows from (2.3) that

$$[(1+m)b + m\sum_{n=2}^{k-1} na_n z^{n-1}]w(z) = \sum_{n=2}^{k} na_n z^{n-1} + \sum_{n=k+1}^{\infty} d_n z^{n-1},$$

where $\sum_{n=k+1}^{\infty} d_n z^{n-1}$ converges in U. Then, since $|w(z)| < 1$, we get

$$\left|(1+m)b + m\sum_{n=2}^{k-1} n a_n z^{n-1}\right| \geq \left|\sum_{n=2}^{k} n a_n z^{n-1} + \sum_{n=k+1}^{\infty} d_n z^{n-1}\right|. \qquad (2.4)$$

Writing $z = re^{i\theta}, r < 1$, squaring both sides of (2.4), and then integrating we obtain

$$(1+m)^2 |b|^2 + m^2 \sum_{n=2}^{k-1} n^2 |a_n|^2 r^{2(n-1)} \geq \sum_{n=2}^{k} n^2 |a_n|^2 r^{2(n-1)} + \sum_{n=k+1}^{\infty} |d_n|^2 r^{2(n-1)}.$$

Taking the limit as r approaches to 1, we have

$$n^2 |a_n|^2 \leq (1+m)^2 |b|^2 - (1-m^2) \sum_{n=2}^{k-1} n^2 |a_n|^2 . \qquad (2.5)$$

Since $m \geq 1$, it follows that

$$|a_n| \leq (\frac{1+m}{n}) |b| \qquad (n \geq 2). \qquad (2.6)$$

The sharpness of the result follows for the function

$$f(z) = \int_0^z \left[1 + \frac{(1+m)bt^{n-1}}{1-mt^{n-1}}\right] dt \quad (n \geq 2, m = 1 - \frac{1}{M}, M > \frac{1}{2}). \quad \blacksquare \qquad (2.7)$$

Putting $m = 1$ ($M = \infty$) in Theorem 1, we get the following result obtained by Halim [1].

Corollary 1. *Let the function f defined by (1.1) be in the class $R(b, \infty) = R(b)$. Then*

$$|a_n| \leq \frac{2|b|}{n} \quad (n \geq 2).$$

The result is sharp for the function

$$f(z) = \int_0^z [1 + \frac{2bt^{n-1}}{1-t^{n-1}}] \, dt \quad (n \geq 2, z \in U).$$

Putting $b = (1-\alpha)\cos\lambda e^{-i\lambda}$, $0 \leq \alpha < 1, |\lambda| < \frac{\pi}{2}$ and $m = 1 - \frac{1}{M}$ ($M > \frac{1}{2}$) in Theorem 1, we get the following result obtained by Aouf and Owa [5].

Corollary 2. *Let the function f defined by (1.1) be in the class $R((1-\alpha)\cos\lambda e^{-i\lambda}, M) = R^{*\lambda}_{M,\alpha}$ ($|\lambda| < \frac{\pi}{2}, 0 \leq \alpha < 1, M > \frac{1}{2}$). Then*

$$|a_n| \leq (\frac{2M-1}{M})\frac{(1-\alpha)\cos\lambda}{n} \quad (n \geq 2)$$

and the result is sharp.

3. Maximization of $\left|a_3 - \mu a_2^2\right|$

We shall need the following lemmas in our investigation.

Lemma 1. [15]. *Let the function w defined by*

$$w(z) = \sum_{k=1}^{\infty} c_k z^k, \tag{3.1}$$

be in the class Ω. Then $|c_1| \le 1$ and $|c_2| \le 1 - |c_1|^2$.

Lemma 2. [8]. *Let the function w defined by (3.1) be in the class Ω. Then*

$$\left|c_2 - \mu c_1^2\right| \le \max\{1, |\mu|\}, \tag{3.2}$$

for any complex number μ. Equality in (3.2) may be attained with the functions $w(z) = z^2$ and $w(z) = z$ for $|\mu| < 1$ and $|\mu| \ge 1$, respectively.

Theorem 2. *Let the function f defined by (1.1) be in the class $R(b, M)$. Then*
(a) *for any real number μ we have*

$$\left|a_3 - \mu a_2^2\right| \le \frac{(1+m)\,|b|}{12}\,|4m - 3\mu(1+m)b|\,; \tag{3.3}$$

(b) *for any complex number μ we have*

$$\left|a_3 - \mu a_2^2\right| \le \frac{(1+m)\,|b|}{3}\ \max\{1, \frac{|4m - 3\mu(1+m)b|}{4}\}. \tag{3.4}$$

The result is sharp for each μ either real or complex.

Proof. Since $f \in R(b, M)$, we have from (2.2) that

$$f'(z) = \frac{1 + [(1+m)b - m]w(z)}{1 - m\ w(z)} \quad (m = 1 - \frac{1}{M}), \tag{3.5}$$

where $w(z) = \sum_{k=1}^{\infty} c_k z^k \in \Omega$. From (3.5), we have

$$w(z) = \frac{f'(z) - 1}{m(f'(z) - 1) + (1+m)b} = \frac{\sum_{n=2}^{\infty} n a_n z^{n-1}}{(1+m)b}\left[1 - \frac{m}{(1+m)b}\sum_{n=2}^{\infty} n a_n z^{n-1} - \cdots\right] \tag{3.6}$$

and then comparing the coefficients of z and z^2 on both sides of (3.6), we have $c_1 = \frac{2a_2}{(1+m)b}$ and $c_2 = \frac{3a_3}{(1+m)b} - mc_1^2$.

Thus $a_2 = \frac{(1+m)bc_1}{2}$ and $a_3 = \frac{(1+m)b}{3}\left[c_2 + mc_1^2\right]$. Hence

$$a_3 - \mu a_2^2 = \frac{(1+m)b}{3}\left[c_2 - \frac{3\mu(1+m)b - 4m}{4}c_1^2\right]$$

and therefore

$$\left|a_3-\mu a_2^2\right| = \frac{(1+m)\,|b|}{3}\left|c_2-\frac{3\mu(1+m)b-4m}{4}c_1^2\right|. \tag{3.7}$$

(a) When μ is real, (3.7) becomes

$$\left|a_3-\mu a_2^2\right| \le \frac{(1+m)\,|b|}{12}\left[4\,|c_2|+|4m-3\mu(1+m)b|\,|c_1|^2\right]. \tag{3.8}$$

Now, applying Lemma 1 for $|c_2|$ in (3.8), we have

$$\left|a_3-\mu a_2^2\right| \le \frac{(1+m)\,|b|}{12}\left[4+\{|4m-3\mu(1+m)b|-4\}\,|c_1|^2\right]. \tag{3.9}$$

Again, using Lemma 1 for $|c_1|$ in (3.9), we obtain

$$\left|a_3-\mu a_2^2\right| \le \frac{(1+m)\,|b|}{12}\,|4m-3\mu(1+m)b|\,.$$

The equality in (3.3) is attained for the function

$$f'(z)=\frac{[m-(1-m)b]}{m}+\frac{(1+m)b}{m}\;\frac{1}{1-mz}. \tag{3.10}$$

(b) When μ is a complex number, applying Lemma 2 in (3.7), we get

$$\left|a_3-\mu a_2^2\right| \le \frac{(1+m)\,|b|}{3}\max\left\{1,\frac{|4m-3\mu(1+m)b|}{4}\right\}, \tag{3.11}$$

which is (3.4) of Theorem 2.

When $\dfrac{|4m-3\mu(1+m)b|}{4}\ge 1$, we choose the function

$$f(z)=\frac{[m-(1+m)b]}{m}\;z-\frac{(1+m)b}{m^2}\log(1-mz) \tag{3.12}$$

and when $\dfrac{|4m-3\mu(1+m)b|}{4}<1$, we have the function

$$f(z)=\frac{[m-(1+m)b]}{m}\;z+\frac{(1+m)b}{m}\int_0^z\frac{dt}{1-mt^2}, \tag{3.13}$$

for attaining the equality in (3.4). Thus the result is sharp. ■

Putting $b=(1-\alpha)\cos\lambda e^{-i\lambda}, 0\le\alpha<1$ and $|\lambda|<\frac{\pi}{2}$ in Theorem 2, we get the following corollary.

Corollary 3. *Let the function f defined by (1.1) be in the class $R((1-\alpha)\cos\lambda e^{-i\lambda},M)=R_{M,\alpha}^{*\lambda}$. Then*

(a) *for any real μ, we have*

$$\left|a_3-\mu a_2^2\right| \le \frac{(1+m)(1-\alpha)\cos\lambda}{12}\left|4me^{i\lambda}-3\mu(1+m)(1-\alpha)\cos\lambda\right|, \tag{3.14}$$

(b) *for any complex number μ, we have*

$$\left|a_3-\mu a_2^2\right| \le \frac{(1+m)(1-\alpha)\cos\lambda}{3}\max\left\{1,\frac{\left|4me^{i\lambda}-3\mu(1+m)(1-\alpha)\cos\lambda\right|}{4}\right\}. \tag{3.15}$$

The result is sharp for each μ either real or complex.

4. Distortion theorem

THEOREM 3. *Let the function f defined by* (1.1) *be in the class* $R(b, M)$. *Then for* $|z| < r < 1$ *we have*

$$\operatorname{Re} f'(z) \geqslant \frac{1-(1+m)\,|b|\,r+m[(1+m)\operatorname{Re}\{b\}-m]r^2}{1-m^2r^2} \quad (z \in U) \tag{4.1}$$

and

$$\operatorname{Re} f'(z) \leq \frac{1+(1+m)\,|b|\,r+m[(1+m)\operatorname{Re}\{b\}-m]r^2}{1-m^2r^2} \quad (z \in U). \tag{4.2}$$

The result is sharp.

Proof. Since $f \in R(b, M)$, we observe that the condition (1.6) doubled with an application of Schwarz's lemma [15], implies $|f'(z) - \zeta| < R$, where

$$\zeta = \frac{1+m[(1+m)b-m]r^2}{1-m^2r^2}, \quad \text{and} \quad R = \frac{(1+m)\,|b|\,r}{1-m^2r^2}.$$

Hence we have (4.1) and (4.2). By considering the function f defined by

$$f(z) = \frac{[m-(1+m)b]}{m} z - \frac{(1+m)b}{m^2e^{i\gamma}} \log(1 - mze^{i\gamma}),$$

where

$$e^{i\gamma} = \frac{|b| + mzb}{b + mz\,|b|},$$

we find that the bounds in (4.1) and (4.2) are sharp at $z = \pm r$, respectively. ■

Putting $b = (1-\alpha)\cos\lambda e^{-i\lambda}$ $(0 \leq \alpha < 1$ and $|\lambda| < \frac{\pi}{2})$ in Theorem 3, we get

COROLLARY 4. *Let the function f defined by* (1.1) *be in the class* $R((1-\alpha)\cos\lambda e^{-i\lambda}, M) = R^{*\lambda}_{M,\alpha}$. *Then for* $|z| = r < 1$ *we have*

$$\operatorname{Re} f'(z) \geqslant \frac{1-(1+m)(1-\alpha)\cos\lambda.r+m[(1+m)(1-\alpha)\cos^2\lambda-m]r^2}{1-m^2r^2} \tag{4.3}$$

and

$$\operatorname{Re} f'(z) \leq \frac{1+(1+m)(1-\alpha)\cos\lambda.r+m[(1+m)(1-\alpha)\cos^2\lambda-m]r^2}{1-m^2r^2}. \tag{4.4}$$

The equalities in (4.3) *and* (4.4) *are attained, respectively at* $z = \pm r$, *for the function f defined by*

$$f(z) = \frac{[m-(1+m)(1-\alpha)\cos\lambda e^{-i\lambda}]}{m} z - \frac{(1+m)(1-\alpha)\cos\lambda}{m^2e^{i(\gamma+\lambda)}} \log(1 - mze^{i\gamma}),$$

where

$$e^{i\gamma} = \frac{e^{i\lambda} + mz}{1 + mze^{i\lambda}}.$$

The bounds in (4.3) *and* (4.4) *are sharp at* $z = \pm r$, *respectively.*

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