WATER-WAVE PROBLEM FOR A VERTICAL SHELL

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To Professor Jindřich Nečas on the occasion of his 70th birthday

Abstract. The uniqueness theorem is proved for the linearized problem describing radiation and scattering of time-harmonic water waves by a vertical shell having an arbitrary horizontal cross-section. The uniqueness holds for all frequencies, and various locations of the shell are possible: surface-piercing, totally immersed and bottom-standing. A version of integral equation technique is outlined for finding a solution.

Keywords: time-harmonic velocity potential, uniqueness theorem, Helmholtz equation, Neumann's eigenvalue problem for Laplacian, integral equation method, weighted Hölder spaces

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1. Introduction

In the present note we consider the linearized water-wave problem describing time-harmonic waves on the free surface in the presence of a vertical cylindrical shell having arbitrary horizontal cross-section. The water layer is assumed to have a constant (possibly infinite) depth. We show that the uniqueness theorem for this problem holds for all frequencies and for any location of the shell. Also, an integral equation of the first kind is applied for solving the problem. This equation arises when the solution is sought in the form of a double layer potential.

Vertical shells can be used in ocean engineering for different purposes. In particular, totally immersed and bottom-standing shells can serve as devices extracting wave energy (see papers by Simon [14] and Thomas [17]) as was proposed by Lighthill who investigated the corresponding two-dimensional model in [7]. Using surface-piercing shells as bottomless harbours was suggested by Garrett [1].

It is well-known that in studies of time-harmonic water waves interacting with obstacles the question of uniqueness in the linearized problem is not yet fully answered despite its importance (see Ursell [19], where this problem is placed first in the list of unfinished problems). A substantial progress in this field has been achieved since 1950, when first uniqueness theorems were proved by John [3] for surface-piercing bodies and by Ursell [18] for a submerged circular cylinder. In the 1970s, Vainberg & Maz'ya [20] obtained some criteria of uniqueness for layers of variable depth, and Maz'ya [8] proved uniqueness for a class of submerged bodies (see also Hulme [2] where illustrations are given to Maz'ya's theorem). Simon & Ursell [16] gave various criteria of uniqueness in the two-dimensional problem involving surface-piercing as well as submerged bodies.

During the last two decades several results concerning the uniqueness of solution, and the existence of trapped modes (non-trivial solutions to the homogeneous boundary value problem leading to non-uniqueness in the non-homogeneous problem) have been obtained for obstacles separating a bounded portion of the free surface from infinity. Thus Kuznetsov [4] considered the two-dimensional problem for a pair of surface-piercing bodies, and under certain geometrical restrictions obtained a single bounded interval of frequencies in which uniqueness holds. When McIver [12] had constructed the first example of trapped mode for this problem, it became clear that restrictions on geometry and/or frequency intervals are unavoidable when proving uniqueness for such obstacles. In three dimensions, Simon & Kuznetsov [15] generalized the result in [4] for a surface-piercing toroidal body. Their theorem also imposes restrictions on geometry, and provides uniqueness only in a finite frequency interval. Examples of axisymmetric toroids trapping different azimuthal modes were constructed by McIver & McIver [13] and Kuznetsov & McIver [6]. In the latter paper, it is also shown that the so-called John's condition (which requires lines extending vertically downwards from every point of the free surface not to intersect the body transversally) provides an infinite sequence of frequency intervals for each mode so that the solution is unique there. On the other hand, we demonstrate that uniqueness holds for *all* frequencies in the case of vertical shells.

Section 2 contains the formulation of the problem and some known auxiliary results. We prove the uniqueness theorem in Section 3. In order to avoid superfluous technical details we restrict ourselves to shells having horizontal edges in Sections 2 and 3. More general geometries for which our uniqueness theorem holds are listed in Section 4. In the last Section 5, we outline a version of the integral equation method for solving the problem.

2. Statement of the water-wave problem and auxiliary assertions

The small-amplitude three-dimensional motion of an inviscid, incompressible fluid under gravity (water, for example) is considered. We assume the motion to be time-periodic (ω denotes the radian frequency), and irrotational. Thus it is described by a velocity potential $\mathrm{Re}\{u(x,y)\,\mathrm{e}^{-\mathrm{i}\omega t}\}$, where $x=(x_1,x_2)$, and (x,y) are Cartesian coordinates with the origin in the mean free surface and the y-axis directed vertically upwards.

Let the fluid occupy a layer $L=\{x\in\mathbb{R}^2,\ -d<y<0\}$ of constant depth $d>0$, outside a shell S which:

(i) is a vertical cylindrical surface, that is, has its generators parallel to the y-axis;

(ii) has a boundary ∂S consisting of two horizontal edges belonging to planes $\{y=-a\}$, $\{y=-b\}$ where $0\leqslant a<b\leqslant d$, and equalities $a=0$, $b=d$ cannot hold simultaneously;

(iii) is assumed (for simplicity) to be smooth, that is, the projection of S onto the x-plane is a simple closed C^2-curve ℓ, dividing $\mathbb{R}^2$ into a simply connected bounded domain F_0 and an infinite domain F_∞.

So $W=L\setminus S$ is the fluid domain; the free surface F coincides with $\{y=0\}$ when $a>0$, and $F=F_0\cup F_\infty$ when $a=0$; the bottom B coincides with $\{y=-d\}$ when $b<d$, and $B=\{x\in F_0\cup F_\infty,\ y=-d\}$ when $b=d$.

In the water-wave problem, u must satisfy the Laplace equation

$$\nabla^2 u \equiv u_{x_1x_1}+u_{x_2x_2}+u_{yy}=0 \quad \text{in} \quad W \tag{1}$$

and the following boundary conditions:

$$u_y-\nu u=0 \quad \text{on} \quad F, \qquad u_y=0 \quad \text{on} \quad B \tag{2}$$

on the free surface and bottom; here $\nu=\omega^2/g$ and g is the acceleration due to gravity. Neumann's condition

$$\partial u/\partial n = f \quad \text{on} \quad \operatorname{int} S = S\setminus\partial S \tag{3}$$

prescribes the normal velocity on the shell, and the unit normal n is directed to infinity on $\operatorname{int} S$. Also, (1)–(3) must be supplemented by two conditions. First, u must belong to the Sobolev space $H^1\big(W^{(a)}\big)$ for any finite $a>0$, where $W^{(a)}=W\cap\big\{|x|<a\big\}$ and $|x|=\big(x_1^2+x_2^2\big)^{1/2}$. Secondly, a radiation condition

$$u_{|x|}-\mathrm{i}k_0u=o\big(|x|^{-1/2}\big) \quad \text{as} \quad |x|\to\infty \tag{4}$$

must hold uniformly in y and the polar angle in the x-plane. Thus (4) requires waves to behave at infinity like outgoing cylindrical waves having a wavenumber k_0 which denotes the unique positive root of $k_0\tanh k_0 d=\nu$.

It is well-known (see, for example, Maz'ya & Rossmann [11]) that if $u \in H^1\big(W^{(a)}\big)$ for any a large enough (so that $S \subset \overline{W^{(a-1)}}$), then u is continuous throughout $\overline{W}$, and near the immersed edge (where $a \neq 0$, $b \neq d$) we have

$$\big|\nabla u(x,y)\big| = O(\varrho^{-1/2}) \quad \text{as} \quad \varrho \to 0. \tag{5}$$

We denote by ϱ the distance of a point $(x,y) \in W$ from ∂S. When $a = 0$ or $b = d$, one can replace (5) by

$$\big|\nabla u(x,y)\big| = O(1) \quad \text{as} \quad \varrho \to 0. \tag{6}$$

The question of uniqueness reduces to the demonstration that u vanishes when it satisfies the homogeneous water-wave problem, that is, when f vanishes in (3). It is known (see, for example, Vainberg & Maz'ya [20]) that in this case the total energy of the corresponding waves is finite:

$$\int_W |\nabla u|^2 \,\mathrm{d}x\,\mathrm{d}y + \nu \int_F |u|^2 \,\mathrm{d}x < \infty.$$

Then, the equipartition of the kinetic and potential energy

$$\int_W |\nabla u|^2 \,\mathrm{d}x\,\mathrm{d}y - \nu \int_F |u|^2 \,\mathrm{d}x = 0 \tag{7}$$

is a consequence of Green's formula.

3. Uniqueness theorem

The aim of the present section is to prove the following result.

Theorem. *Under assumptions* (i)–(iii) *the homogeneous water-wave problem has only a trivial solution.*

Proof. As in [3] (see also [6]) we consider the simple wave component of order zero

$$w(x) = \int_{-d}^{0} u(x,y) \cosh k_0(y+d)\,\mathrm{d}y \tag{8}$$

defined for $x \in F_0 \cup F_\infty$ under assumptions (i)–(iii). It is demonstrated in [3] that

$$\nabla_x^2 w + k_0^2 w = 0, \quad \text{where} \quad \nabla_x w = (w_{x_1}, w_{x_2}). \tag{9}$$

The proof is based on integration by parts twice in

$$\nabla_x^2 w = -\int_{-d}^{0} u_{yy}(x,y)\, \cosh k_0(y+d)\, \mathrm{d}y,$$

which is a consequence of (1). Also, (2) must be applied as well as the definition of k_0. Two other assertions proved in [3] will be used in what follows:
1) (9) and (4) provide that

$$w = 0 \quad \text{in} \quad F_\infty, \tag{10}$$

2) (10) implies

$$\nu \int_{F_\infty} |u|^2 \,\mathrm{d}x \leqslant \frac{1}{2} \int_{W_\infty} |u_y|^2 \,\mathrm{d}x\,\mathrm{d}y, \quad W_\infty = \{x \in F_\infty,\ -d < y < 0\}. \tag{11}$$

Attention is now turned to obtaining a similar inequality between the potential and kinetic energy for the fluid region $W_0 = \{x \in F_0,\ -d < y < 0\}$ bounded above by F_0. First, let us prove that $\partial w/\partial n_0$ is continuous across ℓ (n_0 is the unit normal to ℓ).

We note that by (5) and (6) the integrals defining $\nabla_x w$ converge absolutely and uniformly on either side of ℓ (in F_0 and F_∞), as well as across ℓ. Moreover, $\partial u/\partial n$ is continuous across the vertical cylindrical surface having ℓ as the director (of course, only the part within W is considered, and edges of S should be excluded). On $\operatorname{int} S$, the homogeneous Neumann condition yields that $\partial u/\partial n$ is continuous. Outside S, it is a consequence of the smoothness of solutions to (1). Then, $\partial w/\partial n_0$ is continuous across ℓ.

The last fact and (10) imply that

$$\partial w/\partial n_0 = 0 \quad \text{on} \quad \ell, \tag{12}$$

where w is considered as a function in F_0. This and (9) show that the homogeneous water-wave problem can have a non-trivial solution only if $\nu = k_0 \tanh k_0 d$, where k_0^2 is an eigenvalue of (9) and (12) in F_0. Then for a solution of this problem Green's formula gives

$$\int_{F_0} |\nabla_x w|^2 \,\mathrm{d}x = k_0^2 \int_{F_0} |w|^2 \,\mathrm{d}x, \tag{13}$$

which is the crucial point for deriving an inequality similar to (11).

Integrating by parts in (8) we have for $x \in F_0$:

$$u(x,0)\sinh k_0 d = k_0 w(x) + \int_{-d}^{0} u_y(x,y)\,\sinh k_0(y+d)\,\mathrm{d}y.$$

Squaring this and using Cauchy's inequality with ε (its value is to be chosen later for convenience) and the Schwarz inequality one gets

$$\begin{aligned}\left|u(x,0)\sinh k_0 d\right|^2 \leqslant{}& (1+\varepsilon)k_0^2\left|w(x)\right|^2 \\ &+ \left(1+\varepsilon^{-1}\right)\left(\int_{-d}^{0}\left|u_y(x,y)\right|^2\mathrm{d}y\right)\left(\int_{-d}^{0}\sinh^2 k_0(y+d)\,\mathrm{d}y\right).\end{aligned}$$

Let us calculate the last integral, integrate over F_0, and take into account (13), thus obtaining

$$\nu\int_{F_0}|u|^2\,\mathrm{d}x \leqslant \tfrac{\nu(1+\varepsilon)}{\sinh^2 k_0 d}\int_{F_0}\left|\nabla_x w\right|^2\mathrm{d}x + \tfrac{1+\varepsilon^{-1}}{2}\left(1-\tfrac{\nu d}{\sinh^2 k_0 d}\right)\int_{W_0}|u_y|^2\,\mathrm{d}x\,\mathrm{d}y. \tag{14}$$

On the other hand, (8) gives

$$\nabla_x w(x) = \int_{-d}^{0}\nabla_x u(x,y)\,\cosh k_0(y+d)\,\mathrm{d}y.$$

Using the Schwarz inequality we have

$$\left|\nabla_x w\right|^2 \leqslant \left(\int_{-d}^{0}\left|\nabla_x u(x,y)\right|^2\mathrm{d}y\right)\left(\int_{-d}^{0}\cosh^2 k_0(y+d)\,\mathrm{d}y\right).$$

After calculation of the last integral and integration over F_0 this inequality takes the form

$$\int_{F_0}\left|\nabla_x w\right|^2\mathrm{d}x \leqslant \frac{\sinh^2 k_0 d + \nu d}{2\nu}\int_{W_0}\left|\nabla_x u(x,y)\right|^2\mathrm{d}x\,\mathrm{d}y.$$

This and (14) combine to give

$$\nu\int_{F_0}|u|^2\,\mathrm{d}x \leqslant \tfrac{1+\varepsilon}{2}\left(1+\tfrac{\nu d}{\sinh^2 k_0 d}\right)\int_{W_0}|\nabla_x u|^2\,\mathrm{d}x\,\mathrm{d}y + \tfrac{1+\varepsilon^{-1}}{2}\left(1-\tfrac{\nu d}{\sinh^2 k_0 d}\right)\int_{W_0}|u_y|^2\,\mathrm{d}x\,\mathrm{d}y.$$

The choice

$$\varepsilon = \frac{\sinh^2 k_0 d - \nu d}{\sinh^2 k_0 d + \nu d}\,,$$

which is positive by the definition of k_0, simplifies the last inequality to

$$\nu\int_{F_0}|u|^2\,\mathrm{d}x \leqslant \int_{W_0}|\nabla u|^2\,\mathrm{d}x\,\mathrm{d}y. \tag{15}$$

Finally, combination of (15) with (11) produces

$$\nu \int_F |u|^2 \,\mathrm{d}x \leqslant \int_{W_0} |\nabla u|^2 \,\mathrm{d}x\,\mathrm{d}y + \frac{1}{2} \int_{W_\infty} |u_y|^2 \,\mathrm{d}x\,\mathrm{d}y.$$

Comparing this with (7) one immediately finds that $\nabla u = 0$ in W_∞, and as u is analytic ∇u vanishes throughout W. Then (7) shows that $u = 0$ on F, which substituted into the free surface boundary condition gives that $u_y = 0$ on F. Now, application of the uniqueness theorem for the Cauchy problem for the Laplace equation proves the theorem.

□

4. More geometries providing the uniqueness theorem

The proof in Section 3 provides uniqueness in the three-dimensional water-wave problem for a vertical shell of arbitrary horizontal cross-section in the fluid of finite depth. The uniqueness holds for all positions of the shell: surface-piercing, totally immersed and bottom-standing. For a surface-piercing shell, Theorem 3.1 is the first uniqueness theorem which is valid for all frequencies in the case when the free surface consists of two components in the three-dimensional problem. In two other cases mentioned above, Theorem 3.1 also extends the uniqueness results known up to the present (cf Maz'ya [8] and Vainberg & Maz'ya [20] where uniqueness criteria are given for cases of totally submerged bodies and curved bottom respectively, and Kuznetsov [5] where extensions of these results are obtained using the technique of the auxiliary integral identity proposed in [8]).

Let us discuss other geometries for which the method applied in Section 3 provides the uniqueness theorem. First we note that in the case of infinite depth one has simply to use

$$w(x) = \int_{-\infty}^{0} u(x,y)\,\mathrm{e}^{\nu y}\,\mathrm{d}y$$

instead of (8).

Now we subject to further analysis the uniqueness conditions (i)–(iii). Condition (i) is crucial for defining w throughout $\mathbb{R}^2 \setminus \ell$, but it can be weakened in two directions as follows. Consider a finite number of rigid vertical cylinders extending throughout the depth, and having such smooth horizontal cross-sections that their projections on the x-plane are contained within ℓ. Then F_0 becomes a smooth multiply connected domain, but (13) still holds, and hence, considerations in Section 3 remain valid. Also, a finite number of shells is admissible if projections of their contours on the x-plane are disjoint, and each of them lies outside the others.

Condition (ii) is not necessary, and any edge bounding the shell inside W might be an arbitrary smooth curve because (5) remains true in this case (see Maz'ya & Rossmann [11]). Even a shell extending throughout the depth is allowed, but such a shell must have a hole so that W is a connected fluid domain. Also, condition (iii) can be replaced by a requirement that ℓ is a piecewise smooth curve without cusps, but this involves more technical details.

5. On integral equation method

When the uniqueness theorem holds for the water-wave problem, the solvability theorem in the Sobolev space $H^1\big(W^{(a)}\big)$ with any $a > 0$ can be proved by means of a functional analytic technique developed by Vainberg & Maz'ya [20]. Let us consider how the integral equation method can be applied for finding the solution.

Green's function $G(P,Q)$ (for the sake of brevity, we put $P = (x,y)$ and $Q = (\xi,\eta)$) is well-known for the water-wave problem (see, for example, John [3]). This function satisfies (1) with $-4\pi\delta\big(|P-Q|\big)$ instead of zero on the right-hand side, and (2), (4). Seeking the solution in the form

$$u(P) = \int_S \frac{\partial G}{\partial n_q}(P,q)\,\sigma(q)\,\mathrm{d}S_q, \quad P \in \overline{L}, \tag{16}$$

and applying (3) we get the integro-differential equation of the first kind

$$\frac{\partial}{\partial n_p}\int_S \frac{\partial G}{\partial n_q}(p,q)\,\sigma(q)\,\mathrm{d}S_q = f(p), \quad p \in \operatorname{int} S. \tag{17}$$

Here and below we use p, q for points on S, and P, Q for points elsewhere. The operator in (17) is well-defined because the normal derivative of a double layer potential does exist on a smooth surface (see, for example, Maz'ya [9]). Also, we need the well-known relation between $\sigma(p)$ and the limits $u^{(+)}(p)$ and $u^{(-)}(p)$ of (16) as P tends to $p \in \operatorname{int} S$ from the side directed to infinity and the 'interior' side, respectively:

$$u^{(+)}(p) - u^{(-)}(p) = -\pi^{-1}\sigma(p) \quad p \in \operatorname{int} S. \tag{18}$$

To study how the behaviour of a solution to (17) depends on the properties of f, we introduce several function spaces (cf [9], Ch. 5). By $C^\alpha_\beta(S)$ we denote the space of functions given on S and such that

$$\sup_{p,q}\frac{\big|\varrho_p^\beta f(p) - \varrho_q^\beta f(q)\big|}{|p-q|^\alpha} + \sup\big|f(p)\big| < \infty, \quad \alpha \in (0,1), \quad \beta \in \mathbb{R}^1,$$

where ϱ_p denotes the distance of p from the edge (or edges when S is totally immersed) of ∂S lying inside L. Similarly, $C^{1,\alpha}_\beta(S)$ consists of functions for which

$$\sup_{p,q} \frac{\left|\varrho_p^\beta \nabla_\tau f(p) - \varrho_q^\beta \nabla_\tau f(q)\right|}{|p-q|^\alpha} + \sup\left|f(p)\right| < \infty,$$

where ∇_τ denotes the tangential gradient on S. When $\beta - \alpha < 1$, $C^{1,\alpha}_\beta(S)$ consists of functions which are continuous throughout $\overline{S}$, and we can introduce the space $\mathring{C}^{1,\alpha}_\beta(S)$ including those functions which vanish on the edge (edges) mentioned above.

Let a be so large that $S \subset \overline{W^{(a-1)}}$. We introduce the function space $C^{1,\alpha}_\beta\big(W^{(a)}\big)$ consisting of functions given in $W^{(a)}$ for which

$$\sup_{P,Q\in W^{(a)}} \frac{\left|\varrho_P^\beta \nabla f(P) - \varrho_Q^\beta \nabla f(Q)\right|}{|P-Q|^\alpha} + \sup_{P\in W^{(a)}} \left|f(P)\right| < \infty.$$

We note that functions in $C^{1,\alpha}_\beta\big(W^{(a)}\big)$ have their traces in $C^{1,\alpha}_\beta(S)$.

Using a technique developed in Maz'ya & Plamenevskii [10] and Maz'ya & Rossmann [11] one can demonstrate that if the right-hand side term f in (3) belongs to $C^\alpha_\beta(S)$, $1/2 < \beta - \alpha < 1$, then $u \in H^1(W^{(a)})$ for any $a > 0$, and solving the water-wave problem is in $C^{1,\alpha}_\beta\big(W^{(a)}\big)$ for any finite $a > 0$. This fact and (18) imply that $\sigma \in \mathring{C}^{1,\alpha}_\beta(S)$. Thus we formulate

Proposition 1. *If $f \in C^\alpha_\beta(S)$ and $1/2 < \beta - \alpha < 1$, then σ solving* (17) *belongs to $\mathring{C}^{1,\alpha}_\beta(S)$.*

Another integral equation for the water-wave problem arises from Green's representation for u in W. This equation is as follows (cf Maz'ya [9], subsect. 4.1):

$$-2\pi u(p) + \int_S \frac{\partial G}{\partial n_q}(p,q)\, u(q)\, \mathrm{d}S_q = \int_S G(p,q)\, f(q)\, \mathrm{d}S_q, \quad p \in \operatorname{int} S,$$

and the trace on S of a solution to the water-wave problem is to be found. Using again that $u \in C^{1,\alpha}_\beta\big(W^{(a)}\big)$ when $f \in C^\alpha_\beta(S)$, $1/2 < \beta - \alpha < 1$ we formulate

Proposition 2. *If $f \in C^\alpha_\beta(S)$ and $1/2 < \beta - \alpha < 1$, then $u(p)$ solving the last equation belongs to $C^{1,\alpha}_\beta(S)$.*

Also, we note that the single layer potential in the last equation belongs to $C^{1,\alpha}_\beta(S)$.

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