

# Errata: Primes in Classes of the Iterated Totient Function

Tony D. Noe

In September 2008, Prof. Lenny Jones and his student Joshua Harrington found a gap in the proof of the author's [2] Theorem 4, which states that

*If the smallest number in a class is odd prime  $p$ , then the prime factors of  $p - 1$  are 2 and primes that are the smallest numbers in their class.*

After trying unsuccessfully to correct the proof, the author found a counterexample to the theorem in class 41, whose smallest number is the prime 2336497860617. The prime factorization of  $2336497860617 - 1$  is  $2^3 \cdot 292062232577$ . But 292062232577 is not the smallest number in its class.

The failure of Theorem 4 affects Theorem 5 and some of the computational results in section 5. From additional calculations [3] we know that

- Tables 4 and 5 have four errors: the least numbers in classes 41, 57, 63, and 65 are 2336497860617, 153127060291256329, 9800131858640405057, 39199929291092525057, respectively. Due to the failure of Theorem 4, the symbolic formulas for these four numbers cannot be written in terms of the other  $c_n$  values.
- For  $n \leq 1000$ , there are 288 classes in which the least number  $c_n$  is a provable prime.
- As reported in the paper, only the first 22 powers of the Fermat prime  $F_5$  are the least number in their class.
- Figure 1 is essentially unchanged because the corrected values of  $c_n$  are only slightly smaller than the original values.

The computational part of the paper implicitly uses the following theorem, which is similar to Catlin's [1] Theorem 1, but applies to every class.

**Theorem 6.** *Let  $m$  be odd and the smallest composite number of its class. Then the prime factors of  $m$  are primes that are the smallest numbers in their class.*

*Proof.* The proof is essentially the same as Catlin's proof. Let  $q$  be any prime factor of  $m$ . Because  $m$  is odd,  $q$  must be odd. Then  $m = qs$  and  $C(m) = C(q) + C(s)$ , where  $s = m/q$ . If  $q$  is not the least number in its class, then there is a number  $p < q$  with  $C(p) = C(q)$ . The composite number  $ps$  is in the same class as  $m$ , but less than  $m$ , which contradicts the assumption that  $m$  is the smallest composite number in its class. Hence, every prime factor of  $m$  must be the smallest number in its class.  $\square$

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## References

- [1] P. A. Catlin, Concerning the iterated  $\phi$  function, *Amer. Math. Monthly*, **77** (1970), 60–61.
- [2] Tony D. Noe, [Primes in classes of the iterated totient function](#), *J. Integer Sequences*, **11** (2008), Article 08.1.2.
- [3] Tony D. Noe, [Computing numbers in section I of the totient iteration](#).