

On Some α -Convex Functions

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Abstract

In this paper, we define a general class of α -convex functions, denoted by $ML_{\beta,\alpha}(q)$, with respect to a convex domain D ($q(z) \in \mathcal{H}_u(U)$, $q(0) = 1$, $q(U) = D$) contained in the right half plane by using the linear operator D_λ^β defined by

$$D_\lambda^\beta : A \rightarrow A,$$

$$D_\lambda^\beta f(z) = z + \sum_{j=2}^{\infty} (1 + (j-1)\lambda)^\beta a_j z^j,$$

where $\beta, \lambda \in \mathbb{R}$, $\beta \geq 0$, $\lambda \geq 0$ and $f(z) = z + \sum_{j=2}^{\infty} a_j z^j$. Regarding the class $ML_{\beta,\alpha}(q)$, we give a inclusion theorem and a transforming theorem, from which we may obtain many particular results.

Keywords: α -convex functions, generalized Libera integral operator, Briot-Bouquet differential subordination, modified Sălăgean operator.

1 Introduction

Let $\mathcal{H}(U)$ be the set of functions which are regular in the unit disc U , $A = \{f \in \mathcal{H}(U) : f(0) = f'(0) - 1 = 0\}$, $\mathcal{H}_u(U) = \{f \in \mathcal{H}(U) : f \text{ is univalent in } U\}$ and $S = \{f \in A : f \text{ is univalent in } U\}$.

Let D^n be the Sălăgean differential operator (see [14]) defined as:

$$\begin{aligned} D^n : A &\rightarrow A, \quad n \in \mathbb{N} \text{ and } D^0 f(z) = f(z) \\ D^1 f(z) &= Df(z) = zf'(z), \quad D^n f(z) = D(D^{n-1}f(z)). \end{aligned}$$

Remark

If $f \in A$, $f(z) = z + \sum_{j=2}^{\infty} a_j z^j$, then $D^n f(z) = z + \sum_{j=2}^{\infty} j^n a_j z^j$ is also in A .

Let $n \in \mathbb{N}$ and $\lambda \geq 0$. Let denote with D_λ^n the Al-Oboudi operator (see [7]) defined by

$$\begin{aligned} D_\lambda^n : A &\rightarrow A, \\ D_\lambda^0 f(z) &= f(z), \quad D_\lambda^1 f(z) = (1 - \lambda)f(z) + \lambda z f'(z) = D_\lambda f(z), \\ D_\lambda^n f(z) &= D_\lambda (D_\lambda^{n-1} f(z)). \end{aligned}$$

We observe that D_λ^n is a linear operator and for $f(z) = z + \sum_{j=2}^{\infty} a_j z^j$ we have

$$D_\lambda^n f(z) = z + \sum_{j=2}^{\infty} (1 + (j-1)\lambda)^n a_j z^j.$$

The aim of this paper is to define a general class of α -convex functions with respect to a convex domain D , contained in the right half plane, by using a modified Sălăgean operator, which is also a modified Al-Oboudi operator, and to obtain some properties of this class.

2 Preliminary results

We recall here the definitions of the well - known classes of starlike functions, convex functions and α -convex functions (see [8])

$$S^* = \left\{ f \in A : \operatorname{Re} \frac{z f'(z)}{f(z)} > 0, z \in U \right\},$$

$$S^c = CV = K = \left\{ f \in A; \operatorname{Re} \left(1 + \frac{z f''(z)}{f'(z)} \right) > 0, z \in U \right\},$$

$$M_\alpha = \{ f \in A; \operatorname{Re} J(\alpha, f; z) > 0, z \in U, \alpha \in \mathbb{R} \},$$

where

$$J(\alpha, f; z) = (1 - \alpha) \frac{z f'(z)}{f(z)} + \alpha \left(1 + \frac{z f''(z)}{f'(z)} \right)$$

We observe that $M_0 = S^*$ and $M_1 = S^c$ where S^* and S^c are the class of starlike functions, respectively the class of convex functions.

Remark By using the subordination relation, we may define the class M_α thus

if $f(z) = z + a_2 z^2 + \dots$, $z \in U$, then $f \in M_\alpha$ if and only if $J(\alpha, f; z) \prec \frac{1+z}{1-z}$, $z \in U$, where by " \prec " we denote the subordination relation.

Let consider the generalized Libera integral operator $L_a : A \rightarrow A$ defined as:

$$f(z) = L_a F(z) = \frac{1+a}{z^a} \int_0^z F(t) \cdot t^{a-1} dt, \quad a \in \mathbb{C}, \quad \text{Re } a \geq 0. \quad (1)$$

In the case $a = 1$ this operator was introduced by Libera [7] and it was studied by many authors in different general cases. In this general form ($a \in \mathbb{C}$, $\text{Re } a \geq 0$) was used first time by Pascu in [13].

The next theorem is result of the so called "admissible functions method" introduced by Miller and Mocanu (see [10], [11], [12]).

Theorem Let q be convex in U and $\text{Re}[\beta q(z) + \gamma] > 0$, $z \in U$. If $p \in H(U)$ with $p(0) = q(0)$ and p satisfied the Briot-Bouquet differential subordination

$$p(z) + \frac{z p'(z)}{\beta p(z) + \gamma} \prec q(z), \quad \text{then } p(z) \prec q(z).$$

Definition 2.1 [4] Let $\beta, \lambda \in \mathbb{R}$, $\beta \geq 0$, $\lambda \geq 0$ and $f(z) = z + \sum_{j=2}^{\infty} a_j z^j$. We denote by D_λ^β the linear operator defined by

$$D_\lambda^\beta : A \rightarrow A,$$

$$D_\lambda^\beta f(z) = z + \sum_{j=2}^{\infty} (1 + (j-1)\lambda)^\beta a_j z^j.$$

Definition 2.2 [4] Let $q(z) \in \mathcal{H}_u(U)$, with $q(0) = 1$ and $q(U) = D$, where D is a convex domain contained in the right half plane, $\beta, \lambda \in \mathbb{R}$, $\beta \geq 0$ and $\lambda \geq 0$. We say that a function $f(z) \in A$ is in the class $SL_\beta^*(q)$ if

$$\frac{D_\lambda^{\beta+1} f(z)}{D_\lambda^\beta f(z)} \prec q(z), \quad z \in U.$$

Theorem 2.1 [4] Let $\beta, \lambda \in \mathbb{R}$, $\beta \geq 0$ and $\lambda \geq 1$. If $F(z) \in SL_\beta^*(q)$ then $f(z) = L_a F(z) \in SL_\beta^*(q)$, where L_a is the integral operator defined by (1).

Definition 2.3 [5] Let $q(z) \in \mathcal{H}_u(U)$, with $q(0) = 1$ and $q(U) = D$, where D is a convex domain contained in the right half plane, $\beta, \lambda \in \mathbb{R}$, $\beta \geq 0$ and $\lambda \geq 0$. We say that a function $f(z) \in A$ is in the class $SL_\beta^c(q)$ if $\frac{D_\lambda^{\beta+2} f(z)}{D_\lambda^{\beta+1} f(z)} \prec q(z)$, $z \in U$.

Theorem 2.2 [5] Let $\beta, \lambda \in \mathbb{R}$, $\beta \geq 0$ and $\lambda \geq 1$. If $F(z) \in SL_\beta^c(q)$ then $f(z) = L_\alpha F(z) \in SL_\beta^c(q)$, where L_α is the integral operator defined by (1).

3 Main results

Definition 3.1 Let $q(z) \in \mathcal{H}_u(U)$, with $q(0) = 1$, $q(U) = D$, where D is a convex domain contained in the right half plane, $\beta \geq 0$, $\lambda \geq 0$ and $\alpha \in [0, 1]$. We say that a function $f(z) \in A$ is in the class $ML_{\beta, \alpha}(q)$ if

$$J_{\beta, \lambda}(\alpha, f; z) = (1 - \alpha) \frac{D_\lambda^{\beta+1} f(z)}{D_\lambda^\beta f(z)} + \alpha \frac{D_\lambda^{\beta+2} f(z)}{D_\lambda^{\beta+1} f(z)} \prec q(z), \quad z \in U$$

Remark 3.1 Geometric interpretation: $f(z) \in ML_{\beta, \alpha}(q)$ if and only if $J_{\beta, \lambda}(\alpha, f; z)$ take all values in the convex domain D contained in the right half-plane.

Remark 3.2 We have $ML_{\beta, 0}(q) = SL_\beta^*(q)$ and $ML_{\beta, 1}(q) = SL_\beta^c(q)$.

Remark 3.3 It is easy to observe that if we choose different function $q(z)$ we obtain variously classes of α -convex functions, such as (for example), for $\lambda = 1$ and $\beta = 0$, the class of α -convex functions, the class of α -uniform convex functions with respect to a convex domain (see [3]), and, for $\lambda = 1$ and $\beta = n \in \mathbb{N}$, the class $UD_{n, \alpha}(b, \gamma)$, $b \geq 0$, $\gamma \in [-1, 1)$, $b + \gamma \geq 0$ (see [2]), the class of α - n -uniformly convex functions with respect to a convex domain (see [3]).

Remark 3.4 For $q_1(z) \prec q_2(z)$ we have $ML_{\beta, \alpha}(q_1) \subset ML_{\beta, \alpha}(q_2)$. From the above we obtain $ML_{\beta, \alpha}(q) \subset ML_{\beta, \alpha}\left(\frac{1+z}{1-z}\right)$.

Remark 3.5 It is easy to observe that for every $\beta \geq 0$, $\alpha \in [0, 1]$ and $\lambda \geq 0$ we have $id(z) \in ML_{\beta, \alpha}(q)$, where $id(z) = z$, $z \in U$.

Theorem 3.1 Let $q(z) \in \mathcal{H}_u(U)$, with $q(0) = 1$, $q(U) = D$, where D is a convex domain contained in the right half plane, $\beta \geq 0$ and $\lambda \geq 0$. For all $\alpha, \alpha' \in [0, 1]$, with $\alpha < \alpha'$, we have $ML_{\beta, \alpha'}(q) \subset ML_{\beta, \alpha}(q)$.

Proof. From $f(z) \in ML_{\beta, \alpha'}(q)$ we have

$$J_{\beta, \lambda}(\alpha', f; z) = (1 - \alpha') \frac{D_{\lambda}^{\beta+1} f(z)}{D_{\lambda}^{\beta} f(z)} + \alpha' \frac{D_{\lambda}^{\beta+2} f(z)}{D_{\lambda}^{\beta+1} f(z)} \prec q(z), \quad (2)$$

where $q(z)$ is univalent in U with $q(0) = 1$ and maps the unit disc U into the convex domain D contained in the right half-plane.

Define the function

$$p(z) = \frac{D_{\lambda}^{\beta+1} f(z)}{D_{\lambda}^{\beta} f(z)} = 1 + p_1 z + \dots$$

for $f(z) \in A$ with

$$f(z) = z + \sum_{j=2}^{\infty} a_j z^j.$$

Note that

$$\begin{aligned} z \left(D_{\lambda}^{\beta+1} f(z) \right)' &= z + \sum_{j=2}^{\infty} j (1 + (j-1)\lambda)^{\beta+1} a_j z^j = z + \sum_{j=2}^{\infty} ((j-1) + 1) (1 + (j-1)\lambda)^{\beta+1} a_j z^j \\ &= z + \sum_{j=2}^{\infty} (1 + (j-1)\lambda)^{\beta+1} a_j z^j + \sum_{j=2}^{\infty} (j-1) (1 + (j-1)\lambda)^{\beta+1} a_j z^j \\ &= z + D_{\lambda}^{\beta+1} f(z) - z + \sum_{j=2}^{\infty} (j-1) (1 + (j-1)\lambda)^{\beta+1} a_j z^j \\ &= D_{\lambda}^{\beta+1} f(z) + \frac{1}{\lambda} \sum_{j=2}^{\infty} ((j-1)\lambda) (1 + (j-1)\lambda)^{\beta+1} a_j z^j \\ &= D_{\lambda}^{\beta+1} f(z) + \frac{1}{\lambda} \sum_{j=2}^{\infty} (1 + (j-1)\lambda - 1) (1 + (j-1)\lambda)^{\beta+1} a_j z^j \\ &= D_{\lambda}^{\beta+1} f(z) - \frac{1}{\lambda} \sum_{j=2}^{\infty} (1 + (j-1)\lambda)^{\beta+1} a_j z^j + \frac{1}{\lambda} \sum_{j=2}^{\infty} (1 + (j-1)\lambda)^{\beta+2} a_j z^j \\ &= D_{\lambda}^{\beta+1} f(z) - \frac{1}{\lambda} \left(D_{\lambda}^{\beta+1} f(z) - z \right) + \frac{1}{\lambda} \left(D_{\lambda}^{\beta+2} f(z) - z \right) \\ &= D_{\lambda}^{\beta+1} f(z) - \frac{1}{\lambda} D_{\lambda}^{\beta+1} f(z) + \frac{z}{\lambda} + \frac{1}{\lambda} D_{\lambda}^{\beta+2} f(z) - \frac{z}{\lambda} \\ &= \frac{\lambda-1}{\lambda} D_{\lambda}^{\beta+1} f(z) + \frac{1}{\lambda} D_{\lambda}^{\beta+2} f(z) \\ &= \frac{1}{\lambda} \left((\lambda-1) D_{\lambda}^{\beta+1} f(z) + D_{\lambda}^{\beta+2} f(z) \right). \end{aligned}$$

Similarly we have

$$z \left(D_\lambda^\beta f(z) \right)' = z + \sum_{j=2}^{\infty} j (1 + (j-1)\lambda)^\beta a_j z^j = \frac{1}{\lambda} \left((\lambda-1) D_\lambda^\beta f(z) + D_\lambda^{\beta+1} f(z) \right).$$

Thus we see that

$$\begin{aligned} p(z) + \alpha' \lambda \frac{z p'(z)}{p(z)} &= \frac{D_\lambda^\beta f(z)}{D_\lambda^{\beta+1} f(z)} \\ + \alpha' \lambda \left(\frac{(\lambda-1) D_\lambda^{\beta+1} f(z) + D_\lambda^{\beta+2} f(z)}{\lambda D_\lambda^{\beta+1} f(z)} - \frac{(\lambda-1) D_\lambda^\beta f(z) + D_\lambda^{\beta+1} f(z)}{\lambda D_\lambda^\beta f(z)} \right) \\ &= (1 - \alpha') \frac{D_\lambda^{\beta+1} f(z)}{D_\lambda^\beta f(z)} + \alpha' \frac{D_\lambda^{\beta+2} f(z)}{D_\lambda^{\beta+1} f(z)} \\ &= J_{\beta, \lambda}(\alpha', f; z). \end{aligned}$$

From (2) we have

$$p(z) + \frac{z p'(z)}{\frac{1}{\alpha' \lambda} \cdot p(z)} \prec q(z),$$

with $p(0) = q(0)$, $Re q(z) > 0$, $z \in U$, $\alpha' > 0$ and $\lambda \geq 0$. In this conditions from Theorem 2 we obtain $p(z) \prec q(z)$ or $p(z)$ take all values in D .

If we consider the function $g : [0, \alpha'] \rightarrow \mathbb{C}$,

$$g(u) = p(z) + u \cdot \frac{\lambda z p'(z)}{p(z)},$$

with $g(0) = p(z) \in D$ and $g(\alpha') = J_{\beta, \lambda}(\alpha', f; z) \in D$, it easy to see that

$$g(\alpha) = p(z) + \alpha \cdot \frac{\lambda z p'(z)}{p(z)} \in D, \quad 0 \leq \alpha < \alpha'.$$

Thus we have

$$J_{\beta, \lambda}(\alpha, f; z) \prec q(z)$$

or

$$f(z) \in ML_{\beta, \alpha}(q).$$

From the above theorem we have

Corollary 3.1 *For every $\beta \geq 0$, $\lambda \geq 0$ and $\alpha \in [0, 1]$, we have*

$$ML_{\beta, \alpha}(q) \subset ML_{\beta, 0}(q) = SL_\beta^*(q)$$

Theorem 3.2 Let $q(z) \in \mathcal{H}_u(U)$, with $q(0) = 1$, $q(U) = D$, where D is a convex domain contained in the right half plane, $\beta \geq 0$, $\alpha \in [0, 1]$ and $\lambda \geq 1$. If $F(z) \in ML_{\beta, \alpha}(q)$ then $f(z) = L_a F(z) \in SL_{\beta}^*(q)$, where L_a is the integral operator defined by (1).

Proof. From (1) we have

$$(1 + a)F(z) = af(z) + zf'(z).$$

Note that

$$\begin{aligned} (1 + a)D_{\lambda}^{\beta+1}F(z) &= aD_{\lambda}^{\beta+1}f(z) + z \left(D_{\lambda}^{\beta+1}f(z) \right)' \\ &= aD_{\lambda}^{\beta+1}f(z) + \frac{1}{\lambda} \left((\lambda - 1)D_{\lambda}^{\beta+1}f(z) + D_{\lambda}^{\beta+2}f(z) \right), \end{aligned}$$

or

$$\lambda(1 + a)D_{\lambda}^{\beta+1}F(z) = ((a + 1)\lambda - 1)D_{\lambda}^{\beta+1}f(z) + D_{\lambda}^{\beta+2}f(z)$$

and

$$\lambda(1 + a)D_{\lambda}^{\beta}F(z) = ((a + 1)\lambda - 1)D_{\lambda}^{\beta}f(z) + D_{\lambda}^{\beta+1}f(z).$$

With the following definition for $p(z)$,

$$\frac{D_{\lambda}^{\beta+1}f(z)}{D_{\lambda}^{\beta}f(z)} = p(z), \quad p(0) = 1,$$

we obtain

$$zp'(z)D_{\lambda}^{\beta}f(z) + zp(z) \left(D_{\lambda}^{\beta}f(z) \right)' = z \left(D_{\lambda}^{\beta+1}f(z) \right)'.$$

This implies that

$$\lambda zp(z)'D_{\lambda}^{\beta}f(z) + (\lambda - 1)p(z)D_{\lambda}^{\beta}f(z) + p(z)D_{\lambda}^{\beta+1}f(z) = (\lambda - 1)D_{\lambda}^{\beta+1}f(z) + D_{\lambda}^{\beta+2}f(z).$$

Therefore, we have that

$$\lambda zp(z)' \frac{D_{\lambda}^{\beta}f(z)}{D_{\lambda}^{\beta+1}f(z)} + (\lambda - 1)p(z) \frac{D_{\lambda}^{\beta}f(z)}{D_{\lambda}^{\beta+1}f(z)} + p(z) = (\lambda - 1) + \frac{D_{\lambda}^{\beta+2}f(z)}{D_{\lambda}^{\beta+1}f(z)},$$

that is, that

$$\frac{D_{\lambda}^{\beta+2}f(z)}{D_{\lambda}^{\beta+1}f(z)} = \frac{1}{p(z)} (p(z)^2 + \lambda zp'(z)).$$

Therefore, we obtain

$$\frac{D_{\lambda}^{\beta+1}F(z)}{D_{\lambda}^{\beta}F(z)} = \frac{p(z)^2 + \lambda zp'(z) + ((a + 1)\lambda - 1)p(z)}{p(z) + ((a + 1)\lambda - 1)} = p(z) + \lambda \frac{zp'(z)}{p(z) + ((a + 1)\lambda - 1)},$$

where $a \in \mathbb{C}$, $Re a \geq 0$, $\beta \geq 0$ and $\lambda \geq 1$.

If we denote $\frac{D_\lambda^{\beta+1}F(z)}{D_\lambda^\beta F(z)} = h(z)$, with $h(0) = 1$, we have from $F(z) \in ML_{\beta,\alpha}(q)$ (see the proof of the above Theorem):

$$J_{\beta,\lambda}(\alpha, F; z) = h(z) + \alpha \cdot \lambda \cdot \frac{zh'(z)}{h(z)} \prec q(z)$$

Using the hypothesis, from Theorem 2, we obtain

$$h(z) \prec q(z)$$

or

$$p(z) + \lambda \frac{zp'(z)}{p(z) + ((a+1)\lambda - 1)} \prec q(z),$$

where $a \in \mathbb{C}$, $Re a \geq 0$ and $\lambda \geq 1$.

By using the Theorem 2 and the hypothesis we have

$$p(z) \prec q(z)$$

or

$$\frac{D_\lambda^{\beta+1}f(z)}{D_\lambda^\beta f(z)} \prec q(z),$$

where $\beta \geq 0$ and $\lambda \geq 1$.

This means $f(z) = L_a F(z) \in SL_\beta^*(q)$.

Remark 3.6 *It is easy to observe that if we choose different values for the parameters β and λ , and different functions $q(z)$, in the present results, we obtain variously previously results (for example see [2], [3]).*

4 Open problem

In [1] the author define a generic class of analytical functions from which it is easy to obtain a great number of subclasses of starlike, convex and close to convex functions, and so to study properties for this generic class and simply translate them to the corresponding subclasses. We recall here the main ideas of this paper:

We have already see in the *Preliminary results* section the definitions for the classes $SL_\beta^*(q)$ and $SL_\beta^c(q)$. More, in [6] the authors consider the following general class of close to convex functions:

Definition 4.1 Let $q(z) \in \mathcal{H}_u(U)$, with $q(0) = 1$ and $q(U) = D$, where D is a convex domain contained in the right half plane, $\beta, \lambda \in \mathbb{R}$, $\beta \geq 0$ and $\lambda \geq 0$. We say that a function $f(z) \in A$ is in the class $CCL_\beta(q)$, with respect to the function $g(z) \in SL_\beta^*(q)$, if $\frac{D_\lambda^{\beta+1} f(z)}{D_\lambda^\beta g(z)} \prec q(z)$, $z \in U$.

We observe that the conditions from the definitions of the classes $SL_\beta^*(q)$, $SL_\beta^c(q)$, $CCL_\beta(q)$ can be written

$$p(z) \prec q(z)$$

where

$$p(z) = \frac{D_\lambda^{\beta+1} f(z)}{D_\lambda^\beta f(z)} \text{ or } \frac{D_\lambda^{\beta+2} f(z)}{D_\lambda^{\beta+1} f(z)} \text{ or } \frac{D_\lambda^{\beta+1} f(z)}{D_\lambda^\beta g(z)}. \quad (3)$$

From the above notations we have $p(0) = 1$.

This mean that instead of all three classes, we may define a generic class of analytic functions by

$$CLASS(p, q, \beta, \lambda) = \{f \in A : p \in \mathcal{H}(U) \text{ having the form (3) satisfy } p(z) \prec q(z)\} .$$

Remark 4.1 In the case of the third form of $p(z)$ from (3) it is sufficiently to us to consider that g is also in the class $CLASS(p, q, \beta, \lambda)$.

The open problem is to define a generic class of analytical functions such that the class $ML_{\beta, \alpha}(q)$ is contained inside and is possible to obtain, in a similarly way with the results from [1], where results for all the four general classes are included.

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