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Non Orthogonal Cutting Problem
The Case of Trapezoidal Pieces

Rachid Ouafi and Abdelkader Khelladi

Faculty of Mathematics, Houari Boumedienne University of Sciences and Technology (USTHB), BP 32 El Alia 16111, Algiers, Algeria
e-mail: rouafi@usthb.dz

Abstract

We study the problem of cutting a rectangular material entity into smaller sub-entities of trapezoidal forms with minimum waste of the material. We introduce an orthogonal build to provide the optimal horizontal and vertical homogenous strips. In this paper we develop a general heuristic search based upon orthogonal build. By solving two one-dimensional knapsack problems, we combine the horizontal and vertical homogenous strips to give a non orthogonal cutting pattern.

Keywords: *combinatorial optimization, cutting problem, heuristic.*

1 Introduction

The cutting problem is NP-complete and has many industrial and commercial applications. Its traditional formulation in the literature is done via the two-dimensional knapsack problem [10]. This problem consists of cutting a number of stock entities of given dimensions into smaller sub-entities with minimum waste or maximum profit. A cutting pattern is represented by a sequence of possible cuts in the stock entities. All the parts which differ from sub-entities are regarded as waste. Generally, we distinguish five versions of the problem;

1. The unconstrained unweighed version: each sub-entity appears with no limits in cutting pattern and the weight of each type of sub-entity is represented by its area. The goal is to minimize the waste or the unused area inside the stock entities.

2. The unconstrained weighted version: this version has the same characteristics as the unconstrained unweighed version, but only the vector of

weights differs. Each sub-entity has a weight assigned independently to its area . Here, the goal is to maximize the weight or the sum of the useful values of the produced sub-entities.

3. The constrained unweighed version: it represents a generalization of the first version, which considers that all the sub-entities can be produced without violating some fixed bounds on the number of occurrences of each sub-entity in the solution.

4. The constrained weighted version: it is a generalization of the second version which additionally uses upper bounds on all sub-entities.

5. The staged version: this problem includes a constraint on the total number of cuts, i.e. the sum of the vertical and horizontal cuts does not exceed a constant $k > 2$[10].

An additional constraint is imposed according to the material used for the cuts considered. We distinguish three types of cuts:

- Guillotine cut: on a rectangular plate, cutting is carried out in only one section while going on a side of the rectangular plate to its opposite.

- Non guillotine cut: in general, this cut generates a solution better than a solution produced by cuts of the guillotine type. Indeed, it consists in using the same process as in guillotine cut, but it can be carried out by marking stops, alternating vertical cut and horizontal cut.

- Non orthogonal cut: the parts can be swivelled and relocated (rotations on the parts are allowed). Generally, this type of cut is typical with the laser cut: a swivelling arm, which moves in all the directions at a variable speed, carrying out cuttings. Sometimes, one is confronted with the problem of optimizing the journey time to be carried out by cutting.

The cutting problem has been intensively studied during the last few years. Many authors were interested in the study of the problem, by supposing that the imposed contraint on the cutting is of the type "guillotine" (see for example, Gilmore and Gomory [10], Herz [13], Hifi [14], Adamowicz and Albano [1], Dyson and Gregory [9], Christofides and Whitlock [6]). In 1985, Beasley [4] interested in the study of the problem by considering that the cutting constraint is of the type "non guillotine". Afterwards, other researchers studied this problem (for example, Daniels and Ghandforoush [7], Hadjiconstantinou and Christofides [11], Arenales and Morabito [3]). Other authors were also interested in the study of the cutting problem by imposing a nonorthogonal cut. Among those which appeared in the literature, one can cite Heassler [12] (by the use of the nonlinear programming), Biro and Boros [5], Rinnoy Kan

[15], Dowsland and Dowsland [8].

We study the unconstrained cutting problem of sub-entities with trapezoidal form on a rectangular plate. This problem will be denoted TCP (Trapezoidal Cutting Problem). It is an alternative of the problem of non orthogonal cutting. The TCP has many applications in manufacturing processes of various industries: pipe line design (petrochemistry), the design of airfoil (aeronautical) or cuts of the components of textile products. This paper is organized as follows: in Section2, we give a detailed description of the problem and in particular the concept of function of fall used as a criterion for optimality. In Section 3, we describe build shapes of the homogeneous strips (composed of only one type of piece). In Section 4, we develop an approximate method for the TCP which is based on a constructive procedure allowing to obtain the best non orthogonal cutting pattern from the combination of the horizontal and vertical optimal homogeneous strips.

2 Presentation of the TCP

The TCP can be simply formulated as follows: maximum cutting of trapezoids on a rectangular support, so as to minimize the total of waste. An instance of the TCP is represented by a rectangular support R of dimension (L, H) where L and H are length and width respectively. The small trapezoidal pieces are represented by the set $S = \{t_1, ...t_n\}$, in which each piece t_i has associated weight $c_i = s_i$ representing the area of the the piece i, $i = 1, ..., n$. The TCP consists in cutting the initial rectangular plate R in small pieces t_i without any limitation on the number of produced pieces, so as to minimize the waste value on the stock entity defined by the function:

$$C(R, t_1, ...t_n, X) = L.H - \sum_{i=1}^{n} s_i x_i \qquad (1)$$

Where: $X = (x_1, ..., x_n)$ indicates a cutting pattern, x_i is the number of occurrences of the piece t_i on the entity R.

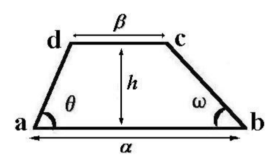
2.1 Basic assumptions

The research about the best placement of the pieces to be cut is closely related to the form and orientation chosen for those pieces. For all trapezoids, we have to specify the allowed orientations, since some orientations are typically impossible for some practical considerations. We suppose that the rotations and the translations allowed on the trapezoids are those which generate horizontally stable trapezoids and therefore nontitled. Thus, the pieces can only be swivelled by 180° (in the two directions). It is clear that for this purpose, on the one hand it is supposed that the support is homogeneous on its two faces and those cuts are of the non orthogonal type. In addition, it is supposed that all dimensions are non negative integer.

2.2 Characteristics of the cuts

A trapezoid $t = (a, b, c, d)$ of irregular shape is completely specified by the parameters $(\alpha, \beta, h, \theta, \omega)$ where:
$\alpha =$ the length of the lower base ab
$\beta =$ the length of the upper base dc
$h =$ the height of t
$\theta = \widehat{dab}$
$\omega = \widehat{cba}$, where : θ and $\omega \in \left]0, \frac{\pi}{2}\right[$.
This supposes obviously that $\beta \prec \alpha$.

Figure 1: Characteristics of the pieces



2.3 Parameterization of the trapezoids

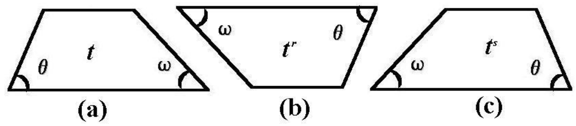
From the above considerations (section 2.1) on the allowed orientations and the potential positions of the trapezoids, it results some particular forms for the same piece. In the following definition, we introduce the notion of duplicated forms of $t = (\alpha, \beta, h, \theta, \omega)$, such as their parameterization.

Definition 2.1 *The transposed of the trapezoid* $t = (\alpha, \beta, h, \theta, \omega)$ *is the trapezoid*
$t^r = (\beta, \alpha, h, \pi - \omega, \pi - \theta)$ *obtained from* t *by a rotation of* 180° *clockwise and counter clockwise.(see fig2)*
The symmetrical of the trapezoid t *is the trapezoid* $t^s = (\alpha, \beta, h, \omega, \theta)$.

All the shapes of duplicated trapezoids have the same area:

$$s(t) = s(t^r) = s(t^s) = \frac{1}{2}(\alpha + \beta).h \tag{2}$$

Figure 2: The duplicated forms of trapezoid t



2.4 Trapezoids regrouping

The expression of the area of the trapezoid t is obtained from the area of the pair of trapezoids $p = (t, t^r)$, where p represents a horizontal regrouping of the two contiguous trapezoids t and t^r which generates a parallelogram of dimension $(\alpha + \beta, h)$ and whose angles are $(\theta, \pi - \theta)$. We adopt this constructive aspect based on the concept of contiguous regrouping of the identical pieces in the construction of the homogeneous strips of trapezoids. We give the following definition for the construction of the various blocks forming a horizontal homogeneous strip.

Definition 2.2 *A sequence of non orthogonal cuts forms a horizontal homogeneous construction associated to piece t if the combination of the two trapezoids t and t^r generates a parallelogram $p = (t, t^r)$ of dimension $(\alpha$ $+\beta, h)$.*

3 Strip Models

Definition 3.1 *A horizontal homogeneous strip is a strip containing one type of participating piece. A homogeneous strip of order k associated to piece t is the rectangular module of minimal area containing a horizontal homogeneous construction made up of k pieces equivalent to piece t .*

Proposition 3.1 *Given a trapezoid $t = (\alpha, \beta, h, \theta, \omega)$. Let:*

$$t^r = (\beta, \alpha, h, \pi - \omega, \pi - \theta) \text{ and } t^s = (\alpha, \beta, h, \omega, \theta)$$

be the duplicated forms obtained by rotation and symmetry of the piece t respectively , then:

1.

$(\mathrm{t}^r)^r = (t)$, $(t^s)^s = (t)$ (3)

2.

$((t^r)^s)^r = (t^s)$, $((t^s)^r)^s = (t^r)$ (4)

3.

$(t^r)^s = (t^s)^r$ (5)

4. If $p = (t, t^r)$ and $p^s = (t^s, t^{s^r})$, then:

s(p)=s(p^s) (6)

Proof. 1. We have : $t^r = (\beta, \alpha, h, \pi - \omega, \pi - \theta)$, thus;

$(t^r)^s = (\beta, \alpha, h, \pi - \theta, \pi - \omega)$ (7)

In other ways, $t^s = (\alpha, \beta, h, \omega, \theta)$, as a results,

$(t^s)^r = (\beta, \alpha, h, \pi - \theta, \pi - \omega)$ (8)

and so: $(t^r)^s = (t^s)^r$.

2. It is sufficient to note that the length of the base of the parallelogram p generated by the regrouping of the pair of contiguous trapezoids (t, t^r) is $\alpha + \beta$, its height h, and their angles are $(\theta, \pi - \theta)$. In other ways, $p^s = (t^s, t^{s^r})$ is of dimension $(\alpha + \beta, h)$, through their angles are $(\omega, \pi - \omega)$.

From this result, we note that the strips $R_{t,k}$, $R_{t^r,k}$ and $R_{t^s,k}$ are all made up of k contiguous pieces of identical area. However, the waste on these strips is not equivalent. In what follows, we show a result that allows characterization of the optimal homogeneous strip.

The regrouping of contiguous trapezoids can be carried out in many ways (according to the direction of provision of the considered piece). We distinguish three types of homogeneous strips denoted $R_{t,k}$, $R_{t^r,k}$ and $R_{t^s,k}$ and associated to $t = (\alpha, \beta, h, \theta, \omega)$ its equivalent forms $t^r = (\beta, \alpha, h, \pi - \omega, \pi - \theta)$, and $t^s = (\alpha, \beta, h, \omega, \theta)$ respectively. these are obtained by the allowed rotations on the piece t. We show in the following result that among all the possibilities of regrouping of the trapezoidal pieces in the generation of the homogeneous strips which are all equivalent in term of component pieces, there is an optimal configuration in terms of wastage in a strip.

Proposition 3.2 *Let us consider the strips $R_{t,k}$, $R_{t^r,k}$ and $R_{t^s,k}$ .Wastes recorded on these strips being respectively $C(R_{t,k})$, $C(R_{t^r,k})$ and $C(R_{t^s,k})$. One has :*

1.

$$\mathrm{C(R}_{t,k}) = \begin{cases} h^2 \tan(\frac{\pi}{2} - \theta) & \text{if} \quad k = 2n \\ \frac{h^2}{2} \tan(\frac{\pi}{2} - \theta) + \frac{h^2}{2} \tan(\frac{\pi}{2} - \omega) & \text{if} \quad k = 2n + 1 \end{cases} \tag{9}$$

2.

$$C(R_{t^r,k}) = C(R_{t^s,k}) \tag{10}$$

Proof. 1. (l, h) are the dimensions of $R_{t,k}$, The waste on $R_{t,k}$ is written in the following form :

$$C(R_{t,k}) = lh - \frac{k}{2}(\alpha + \beta)h = ch \tag{11}$$

- if $k = 2n$,

$$c = 2\frac{h}{2} \tan(\frac{\pi}{2} - \theta) = h \tan(\frac{\pi}{2} - \theta) \tag{12}$$

thus,

$$C(R_{t,k}) = h^2 \tan(\frac{\pi}{2} - \theta) \tag{13}$$

- if $k = 2n + 1$,

$$c = c^{'} + c^{''}$$

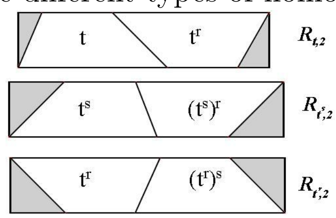
with :

$$\begin{aligned} c^{'} &= \frac{h}{2} \tan(\frac{\pi}{2} - \theta) \\ c^{''} &= \frac{h}{2} \tan(\frac{\pi}{2} - \omega) \end{aligned} \tag{14}$$

from where :

$$C(R_{t,k}) = \frac{h^2}{2}\left(\tan(\frac{\pi}{2} - \theta) + \tan(\frac{\pi}{2} - \omega)\right) \tag{15}$$

Figure 3: The different types of homogeneous strips



2. Strips $R_{t^r,k}$ and $R_{t^s,k}$ are equivalent in terms of the pieces they are made of and which are all equivalent to piece t. Indeed, let $(a_1, a_2, ..., a_k)$ and $(c_1, c_2, ..., c_k)$ be the pieces returning in $R_{t^r,k}$ and $R_{t^s,k}$ respectively. For all $i = 1, ..., k$, we have :

$$a_i = \begin{cases} t^r & \text{if} \quad \text{i is odd} \\ (t^r)^r & \text{if} \quad i \text{ is even} \end{cases} \tag{16}$$

and

$$c_i = \begin{cases} t^s & \text{if} \quad \text{i is odd} \\ (t^s)^r & \text{if} \quad i \text{ is even} \end{cases} \tag{17}$$

The recorded wastes for the strips $R_{t^r,k}$ and $R_{t^s,k}$ are obtained from the configuration of the pieces $a_1 = t^r = (\beta, \alpha, h, \pi - \omega, \pi - \theta)$ and $a_k = t = (\alpha, \beta, h, \theta, \omega)$ for $R_{t^r,k}$ as well as the pieces $c_1 = t^s = (\alpha, \beta, h, \omega, \theta)$ and $c_k = (t^s)^r = (\beta, \alpha, h, \pi - \theta, \pi - \omega)$ for $R_{t^s,k}$. It results from it under the terms of the result from 1)

$$\mathrm{C}(\mathrm{R}_{t^r,k}) = C(R_{t^s,k}) = \begin{cases} h^2 \tan(\frac{\pi}{2} - \omega) & \text{if } k \text{ is even} \\ \frac{h^2}{2}\tan(\frac{\pi}{2} - \omega) + \frac{h^2}{2}\tan(\frac{\pi}{2} - \theta) & \text{if } k \text{ is odd} \end{cases} \tag{18}$$

3.1 Characterization of the optimal homogeneous strip

Proposition 3.1.1 *Let* $(b_1, b_2, ..., b_k)$ *and* $(c_1, c_2, ..., c_k)$ *be the pieces composed respectively* $R_{t,k}$ *and* $R_{t^s,k}$ *, such as for* $i = 1, ..., k$*:*

$$b_i = \begin{cases} t & \text{if} \quad \text{i is odd} \\ (t^r) & \text{if} \quad \text{i is even} \end{cases} \tag{19}$$

and

$$c_i = \begin{cases} t^s & \text{if} \quad \text{i is odd} \\ (t^s)^r & \text{if} \quad \text{i is even} \end{cases} \tag{20}$$

Then,

$$C(R_{t,k}) \leq C(R_{t^r,k}) \Longleftrightarrow \theta \geq \omega \tag{21}$$

Proof. We can easily express from the previous result the waste of the strips $R_{t,k}$ and $R_{t^r,k}$ as follows :

- If $k = 2n$,

$$\begin{aligned} C(R_{t,k}) &= h^2 \tan(\frac{\pi}{2} - \theta) \\ C(R_{t^s,k}) &= h^2 \tan(\frac{\pi}{2} - \omega) \end{aligned} \tag{22}$$

- If $k = 2n + 1$,

$$C(R_{t,k}) = C(R_{t^s,k}) = \frac{h^2}{2}\left(\tan(\frac{\pi}{2} - \theta) + \tan(\frac{\pi}{2} - \omega)\right) \tag{23}$$

  As tangent is an increasing function, our result is an immediate consequence of the last equality.

Consequently, in all what follows, any trapezoidal piece t will be characterized by $t = (\alpha, \beta, h, \theta, \omega)$, with $\theta \geq \omega$.

4 Resolution Algorithm

We propose an algorithm to solve the TCP denoted HTC, and based on a constructive procedure allowing to obtain the horizontal and vertical homogeneous strips. The resolution of two unidimensional knapsack problems enables us to build two guillotine cutting patterns. The first is a horizontal cutting pattern obtained from a combination of the horizontal homogeneous strips of various heights. The second pattern is a vertical cutting pattern obtained by the combination of the vertical homogeneous strips of various lengths. The approximate solution being the value of the best cutting patterns among the two patterns.

4.1 Principle of the algorithm

Let us consider an instance of the TCP defined by : (R, S, c), where: $R = (L, H)$ is the initial rectangle plate, and L and H its length and width respectively. $S = (t_1, t_2, ..., t_n)$ is the set of the pieces to be cut. Each piece i is characterized by $t_i = (\alpha_i, \beta_i, h_i, \theta_i, \omega_i)$. $c = (c_1, c_2, ..., c_n)$ is the vector weight, such that:

$$c_i = s(t_i) = (\alpha_i + \beta_i)\frac{h_i}{2} \; for \; i = 1, ..., n$$

The steps of the algorithm HTC are summarized as follows:

1. Generation of the horizontal homogeneous strips.
   Let $R_{i,a_i}(L)$, $i = 1, .., n$, denote the horizontal homogeneous strips of length L, obtained by the horizontal regrouping of the a_i pieces t_i and having value
   $$\lambda_i = c_i a_i \tag{24}$$

2. Generation of the vertical homogeneous strips.
   Let $R_{i,b_i}(H)$, $i = 1, .., n$, denote the vertical homogeneous strips of height H, obtained by the vertical regrouping. Each strip is made up of b_i pieces, t_i, and of value
   $$\xi_i = c_i b_i \tag{25}$$

3. Horizontal cutting pattern. Order the elements of S, such that:
   $$\lambda_1 \leq \lambda_2 \leq ... \leq \lambda_r$$
   where r is the number of possible values $R_{i,a_i}(L)$. Solve the following knapsack problem:
   $$\begin{aligned} F_{hor} &= \max \sum_{j=1}^{r} \lambda_j x_j \qquad (26) \\ sc &: \sum_{j=1}^{r} h_j x_j \leq H \\ x_j &\in N, j = 1, ...r \end{aligned}$$
   Where F_{hor} is the value of the horizontal cutting pattern $_{hor} = (x_j)_{j=1,...,r}$, with x_j being the number of occurrences of the strip $R_{j,a_j}(L)$ in M_{hor}.

4. Vertical cutting pattern. Order the elements of S as follows:
   $$\xi_1 \leq \xi_2 \leq ... \leq \xi_{r'}$$
   where r' is the number of possible values $R_{i,b_i}(H)$, and solve the following knapsack problem:
   $$F_{ver} = \max \sum_{j=1}^{r'} \xi_j y_j \tag{27}$$

$$\begin{aligned} sc &: \sum_{j=1}^{r'} h_j y_j \leq L \\ y_j &\in N, j = 1, ...r' \end{aligned}$$

where F_{ver} is the value of the vertical cutting pattern $_{ver} = (y_j)_{j=1,\ldots,r'}$, with y_j being the number of occurrences of the strip $R_{j,b_j}(H)$ in M_{ver}. the solution value is : $M^* = \max(F_{hor}, F_{ver})$

Theorem 4.1 *The HTC admits an approximation ratio satisfying:*

$$\frac{A(I)}{Opt(I)} \geq \frac{1}{3} \tag{28}$$

where $A(I)$ (resp $Opt(I)$) is the sub-optimal (resp optimal) value for the instance I.

Proof. The heuristic HTC realizes the best homogenous cutting pattern associated to t_i for $1 \leq i \leq n$. Therefore it satisfies the inequality

$$A(I) \geq \left|\frac{L}{(\alpha_i + \beta_i)/2}\right| \left|\frac{H}{h_i}\right| c_i$$

Where $c_i = (\alpha_i + \beta_i)\frac{h_i}{2}$. we set $\delta = \left|\frac{L}{(\alpha_i+\beta_i)/2}\right|$ and $\delta' = \left|\frac{H}{h_i}\right|$. In addition the optimal solution value verifies

$$Opt(I) \leq LH$$

That enables us to have

$$\frac{Opt(I)}{A(I)} \leq \frac{L.H}{\delta.\delta'.(\alpha_i + \beta_i).\frac{h_i}{2}}$$

In other way, we have

$$\left|\frac{L}{(\alpha_i + \beta_i)/2}\right| . \left|\frac{H}{h_i}\right| \leq \frac{L.H}{(\alpha_i + \beta_i).\frac{h_i}{2}} \leq \left(\left|\frac{L}{(\alpha_i + \beta_i)/2}\right| + 1\right) . \left(\left|\frac{H}{h_i}\right| + 1\right)$$

Thus

$$\delta.\delta' \leq (\delta + 1) . \left(\delta' + 1\right)$$

And consequently

$$\frac{Opt(I)}{A(I)} \leq \frac{(\delta + 1) . \left(\delta' + 1\right)}{\delta.\delta'}$$

Finally, for $\delta \geq 1$ and $\delta^{'} \geq 2$ (or $\delta^{'} \geq 1$ and $\delta \geq 2$) we obtain

$$\frac{A(I)}{Opt(I)} \geq \frac{1}{3}$$

4.2 Illustration of the algorithm with an example

Let us consider the instance (R, S, c), with $R = (9, 7)$ and $S = (t_1, t_2, t_3)$, where:

$$t_1 = (3, 2, 1),\ t_2 = (2, 1, 2) \text{ and } t_3 = (3, 1, 3)$$
$$c_1 = 5/2,\ c_2 = 3,\ c_3 = 6$$

- The horizontal homogenous strips are $R_{1,3}(9)$, $R_{2,5}(9)$, $R_{3,4}(9)$ and have respective values:

$$\lambda_1 = 15/2,\ \lambda_2 = 15,\ \text{and } \lambda_3 = 24$$

The solution of the knapsack problem:

$$\begin{aligned} F_{hor} = \max 15/2x_1 + 15x_2 + 24x_3 & \\ s.c \quad &: \quad x_1 + 2x_2 + 3x_3 \leq 7 \\ x_1, x_2, x_3 \quad &\in \quad N \end{aligned}$$

is $M_{hor} = (1, 0, 2)$ of value $F_{hor} = 55.5$

- The vertical homogenous strips are $R_{1,2}(7)$, $R_{2,4}(7)$, $R_{3,3}(7)$ and have respective values:

$$\xi_1 = 5, \xi_2 = 12, and\ \xi_3 = 18$$

The solution of the knapsack problem:

$$\begin{aligned} F_{ver} = \max 5y_1 + 12y_2 + 18y_3 & \\ s.c \quad &: \quad y_1 + 2.y_2 + 3.y_3 \leq 9 \\ y_1, y_2, y_3 \quad &\in \quad N \end{aligned}$$

is $M_{ver} = (0, 0, 3)$ of value $F_{hor} = 54$

The solution is:

$$M_{hor} = (1, 0, 2)$$

Which correspond to the horizontal cutting pattern composed by two strips of the type $R_{1,3}(9)$ and strip $R_{3,4}(9)$.

4.3 Numerical examples

We consider two groups of 60 randomly generated instances. The first group, with sizes L and H taken in the interval [250, 500] and the number of pieces to be cut is taken in the interval [20, 50]. The second, the parameters L and H are ranged in the interval [500, 750], whereas the number of pieces to be cut are ranged in the interval [50, 80]. The dimensions of the pieces (α_i, β_i, h_i) are taken uniformly in the interval]0, L[,]0, α_i[and]0, H[respectively, and the number of pieces to be cut is also taken uniformly in the specified interval. the average of the total surface used is 78.45 % in the first test and 85.63% in the second.
Results of some examples are summarized in Table 1 and 2, which contain the number of pieces to be cut N, the dimension of the initial rectangle plate $R = (L, H)$, the waste % S, the solution Z^* and the computational time required $Time$(s).

Table 1: Results of the first test

N	R=(L,H)	% S	Z^*	$Time$(s)
21	(398,310)	24.42	93249	0.980
22	(298,296)	16.49	73660	0.060
23	(309,292)	2.59	87888	1.260
24	(488,300)	32.83	98334	0.050
36	(469,315)	35.97	94588	0.050
44	(415,340)	29.71	99186	0.110
45	(339,256)	7.43	80333	0.060
46	(378,255)	7.38	89278	0.113
48	(253,252)	2.90	61908	0.052
49	(393,278)	9.58	98792.5	0.061

Table 2: Results of the second test

N	R=(L,H)	% S	Z*	*Time*(s)
50	(847,632)	17.80	439980	0.110
51	(937,715)	6.07	629286	2.530
53	(863,801)	2.85	671540	2.600
56	(524,506)	5.12	251544	4.070
58	(971,811)	2.79	765445	1.380
62	(867,767)	5.25	630018	0.160
68	(837,817)	7.92	629614	3.180
69	(807,660)	2.56	518959	0.160
73	(836,681)	6.74	530903	0.220
78	(847,632)	18.80	499985	0.110

5 Conclusion

We have presented the general context of the cutting problem of trapezoidal pieces on a rectangular plate and studied the forms of rotations allowed in the provision of the pieces to be cut out. In addition, we established the properties and the characteristics of the optimal homogeneous strips. Finally, we developed a new approach for the resolution of the trapezoidal cutting problem. This approach is based on a constructive procedure allowing to obtain nonorthogonal cutting pattern on the initial entity by means of horizontal and vertical constructions which generate the optimal homogeneous strips. The selection of the best homogeneous strips leads to the construction of the best non orthogonal cutting pattern generated from the combinations of the elements of all the horizontal and vertical homogeneous strips. We showed that the algorithm admits a constant approximation ratio.

6 Open problems

There are a number of interesting open problems linked to the problem presented in this paper, we can mentioned how to provide a good heuristic for the general trapezoidal cutting problem (with many rectangular stock entities) by using sequential and parallel implementations. In addition any other heuristic for solving the (un)weighted (un)constrained (non)staged trapezoidal cutting problem can be presented as a further research.

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