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Adaptive Algorithm for Noisy Autoregressive Signals*

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This paper presents a new type of improved least-squares (ILS) algorithm for adaptive parameter estimation of autoregressive (AR) signals from noisy observations. Unlike the previous ILS based methods, the developed algorithm can give consistent parameter estimates in a very direct manner that it does not involve dealing with an augmented noisy AR model. The new algorithm is demonstrated to outperform the previous ILS based methods in terms of its improved numerical efficiency.

Keywords: Adaptive algorithms; Autoregressive signals; Parameter estimation; Noisy signals; Adaptive filtering

1. INTRODUCTION

The autoregressive (AR) modeling techniques have been widely used in many signal processing applications, such as speech analysis, spectral estimation and noise cancellation [3, 4, 7]. The normal least-squares (LS) method is one of the frequently used AR modeling techniques due to its simplicity and ease of implementation. However, when the signal to be modeled is observed in noise, which is a very common phenomenon in practical applications, the normal LS

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estimates of the AR parameters are biased and thus can produce misleading results.

Several methods that can be used to consistently estimate AR signals from noisy measurements are available, such as the modified Yule-Walker (MYW) equations method [3], the maximum likelihood (ML) method [8], the recursive prediction error (RPE) method [5], the modified LS (MLS) method [6], and the improved least-squares (ILS) methods [9, 10]. Among them, the ILS methods are an efficient algorithm for unbiased identification of noisy AR signals. The distinctive feature of the ILS methods is that the estimate of the measurement noise variance is derived in a simple yet effective manner. With this estimated noise variance, the noise-induced bias in the normal LS estimator can then be removed to yield consistent AR parameter estimates.

In a wide range of signal processing applications, the AR parameters may vary with time. The use of off-line identification methods in these situations is obviously inappropriate. Instead, an adaptive identification algorithm is need so as to track the time-varying AR parameters. Needless to say, the numerical efficiency is highly important for an adaptive algorithm to be practical or competitive. In this paper adaptive identification of AR signals where the signal observations contain white noise is studied. The MYW method, though very simple from the numerical point of view, may not be numerically stable during recursive estimation. The ML method is not only rather computationally demanding but also confined to off-line identification. While the RPE method is suited for adaptive estimation, its employment of the Gauss–Newton algorithm in minimization causes some extensive computations. The main drawback with the MLS method is that a series of computations of several intermediate variables are needed in order to estimate the noise variances, which can be rather time-consuming. On the contrary, the fact that the ILS methods (*i.e.*, the prefiltering based ILS (PILS) method presented in [9] and the non-prefiltering based ILS (NPILS) method presented in [10]) are based on linear regression and require a small amount of computations at each time step makes them well suited for adaptive estimation.

In this paper, a new form of direct adaptive ILS (DAILS) algorithm is proposed for on-line estimation of AR signals corrupted by additive white noise. The proposed algorithm is characterized by its direct AR parameter estimation structure, with its numerical efficiency being greatly improved. That is, the underlying noisy AR(p) model, where p is the model order, is identified in a direct manner so that there is no need to deal with an augmented noisy AR($p+1$) model as presented in [9] and [10]. The procedure of this DAILS algorithm is composed of the following steps. First, a normal LS estimate of the parameters of the underlying AR(p) model, which is bound to be biased due to the presence of observation noise, is computed. Second, the variance of the white observation noise, which determines the noise-induced bias in the normal LS estimate, is directly estimated in a novel way that is based on use of the variables only associated with the underlying noisy AR(p) model. Third, the consistent estimate of the AR parameters is obtained *via* the well-known bias correction principle. It is shown that this new DAILS algorithm not only retains the merits of the PILS method and the NPILS method, but also possesses some important computational advantages over the latter. Simulations are provided to support the theoretical predictions.

2. NOISY AUTOREGRESSIVE MODEL

Assume that the pth-order AR random process $x(t)$ is defined as

$$x(t) = a_1 x(t-1) + \cdots + a_p x(t-p) + v(t) \tag{1}$$

where $v(t)$ is driving white noise having variance σ_v^2, and

$$\mathbf{a}^\top = [a_1 \cdots a_p] \tag{2}$$

is the AR parameter vector.

The noisy measurement of the AR signal is described by

$$y(t) = x(t) + w(t) \tag{3}$$

where $w(t)$ is white noise having variance σ_w^2, which denotes measurement errors.

The three assumptions are imposed on the noisy AR(p) model (1) and (3):

A1. $x(t)$ is stationary;
A2. The order p is given;
A3. $v(t)$ and $w(t)$ are independent of each other.

The problem of noisy AR modeling is concerned with estimating the AR parameter vector $\mathbf{a}$ as well as the noise variances σ_v^2 and σ_w^2 from N noisy measurements $\{y(1), \ldots, y(N)\}$.

Let

$$\mathbf{y}_t^{\mathrm{T}} = [y(t-1) \cdots y(t-p)] \tag{4}$$

$$\mathbf{w}_t^{\mathrm{T}} = [w(t-1) \cdots w(t-p)]. \tag{5}$$

Combining (1) and (3) together and using (2), (4) and (5) gives rise to a linear regression expression of the noisy AR model:

$$y(t) = \mathbf{y}_t^{\mathrm{T}} \mathbf{a} + \varepsilon(t) \tag{6}$$

where

$$\varepsilon(t) = v(t) + w(t) - \mathbf{w}_t^{\mathrm{T}} \mathbf{a}. \tag{7}$$

As shown in [2], the normal LS estimate of $\mathbf{a}$ is defined as the minimizer of the LS criterion $J(\mathbf{a}) = E[\varepsilon(t)^2]$, and is given by

$$\mathbf{a}_{LS} = \mathbf{R}_y^{-1} \mathbf{r}_y \tag{8}$$

where $\mathbf{R}_y$ and $\mathbf{r}_y$ are described respectively by

$$\mathbf{R}_y = E[\mathbf{y}_t \mathbf{y}_t^{\mathrm{T}}] = \begin{bmatrix} r_0 & r_1 & \cdots & r_{p-1} \\ r_1 & r_0 & \ddots & \vdots \\ \vdots & \ddots & \ddots & r_1 \\ r_{p-1} & \cdots & r_1 & r_0 \end{bmatrix}, \quad \mathbf{r}_y = E[\mathbf{y}_t y(t)] = \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_p \end{bmatrix} \tag{9}$$

with their entries r_k ($k=0,1,\ldots,p$) being the autocovariances defined as

$$r_k = E[y(t)y(t-k)]. \tag{10}$$

Following the white noise and independence assumptions on $v(t)$ and $w(t)$ and introducing

$$\mathbf{x}_t^{\top} = [x(t-1)\cdots x(t-p)] \tag{11}$$

we get from (7) that

$$E[\mathbf{x}_t\varepsilon(t)] = E[\mathbf{x}_t v(t)] + E[\mathbf{x}_t w(t)] - E[\mathbf{x}_t\mathbf{w}_t^{\top}]\mathbf{a} = \mathbf{0} \tag{12}$$

$$E[\mathbf{w}_t\varepsilon(t)] = E[\mathbf{w}_t v(t)] + E[\mathbf{w}_t w(t)] - E[\mathbf{w}_t\mathbf{w}_t^{\top}]\mathbf{a} = -(\sigma_w^2\mathbf{I}_p)\mathbf{a} = -\sigma_w^2\mathbf{a} \tag{13}$$

where $\mathbf{I}_p$ is an identity matrix of order p. Since

$$\mathbf{y}_t = \mathbf{x}_t + \mathbf{w}_t \tag{14}$$

it follows from (12), (13) and (14) that

$$E[\mathbf{y}_t\varepsilon(t)] = E[\mathbf{x}_t\varepsilon(t)] + E[\mathbf{w}_t\varepsilon(t)] = -\sigma_w^2\mathbf{a} \tag{15}$$

which, using (6) and (9), further yields

$$\mathbf{r}_y = E[\mathbf{y}_t y(t)] = E[\mathbf{y}_t\mathbf{y}_t^{\top}]\mathbf{a} + E[\mathbf{y}_t\varepsilon(t)] = \mathbf{R}_y\mathbf{a} - \sigma_w^2\mathbf{a}. \tag{16}$$

Substitution of (16) into (8) leads to

$$\mathbf{a}_{LS} = \mathbf{R}_y^{-1}(\mathbf{R}_y\mathbf{a} - \sigma_w^2\mathbf{a}) = \mathbf{a} - \sigma_w^2\mathbf{R}_y^{-1}\mathbf{a} = \mathbf{a} + \Delta\mathbf{a} \tag{17}$$

where

$$\Delta\mathbf{a} = -\sigma_w^2\mathbf{R}_y^{-1}\mathbf{a}. \tag{18}$$

It is seen from (17) that the presence of measurement noise will certainly lead to bias. In the meantime (17) explicitly indicates in what way the asymptotic bias is related to the measurement noise variance σ_w^2.

The elementary idea of the ILS type methods is to apply the bias correction scheme

$$\mathbf{a}_{ILS}(t) = \mathbf{a}_{LS} + \sigma_w^2 \mathbf{R}_y^{-1} \mathbf{a}_{ILS}(t-1) \tag{19}$$

to obtain an unbiased estimate of $\mathbf{a}$. Note that practical application of (19) requires the knowledge of the measurement noise variance σ_w^2. An efficient technique for estimating σ_w^2 will be presented in the next section.

3. ADAPTIVE ESTIMATION ALGORITHM

Following the derivation given in [10], we first consider identifying an augmented noisy AR($p+1$) model that is equivalent to the underlying noisy AR(p) model. That is, (1) is now replaced by

$$x(t) = a_1 x(t-1) + \cdots + a_p x(t-p) + \bar{a}_{p+1} x(t-p-1) + v(t) \tag{20}$$

where $\bar{a}_{p+1} = 0$, so that (20) and (1) constitute the augmented noisy AR($p+1$) model.

Define

$$\bar{\mathbf{a}}^{\mathrm{T}} = [\mathbf{a}^{\mathrm{T}} \bar{a}_{p+1}] = [\mathbf{a}^{\mathrm{T}} 0]. \tag{21}$$

Then the normal LS estimate of the augmented parameter vector $\bar{\mathbf{a}}$ is given by

$$\bar{\mathbf{a}}_{LS} = \overline{\mathbf{R}}_y^{-1} \bar{\mathbf{r}}_y \tag{22}$$

where

$$\overline{\mathbf{R}}_y = E[\bar{\mathbf{y}}_t \bar{\mathbf{y}}_t^{\mathrm{T}}], \quad \bar{\mathbf{r}}_y = E[\bar{\mathbf{y}}_t y(t)] \tag{23}$$

$$\bar{\mathbf{y}}_t^{\mathrm{T}} = [\mathbf{y}_t^{\mathrm{T}} y(t-p-1)]. \tag{24}$$

Using the knowledge of $\bar{a}_{p+1} = 0$ about the augmented noisy AR($p+1$) model (20) and (1), it is shown in [10] that the measurement noise variance σ_w^2 can be estimated by means of

$$\sigma_w^2 = \frac{-\boldsymbol{\mu}^{\mathrm{T}} \bar{\mathbf{a}}_{LS}}{\boldsymbol{\mu}^{\mathrm{T}} \overline{\mathbf{R}}_y^{-1} \bar{\mathbf{a}}} \tag{25}$$

where $\boldsymbol{\mu}^{\mathrm{T}} = [0 \cdots 0\ 1] \in \mathbb{R}^{p+1}$.

It is seen from (25) that estimation of σ_w^2 actually involves identification of the augmented noisy AR($p+1$) model due to the use of $\bar{\mathbf{a}}_{LS}$ as well as $\bar{\mathbf{R}}_y^{-1}$. Obviously, additional calculations are incurred. Thus, the estimation scheme given in [10] is far from being numerically efficient.

To achieve efficient implementation of the scheme (25) while without actually identifying the augmented noisy AR($p+1$) model, the particular form of the regression vector $\bar{\mathbf{y}}_t$ given in (24) is exploited to partition $\bar{\mathbf{R}}_y$ and $\bar{\mathbf{r}}_y$ as

$$\bar{\mathbf{R}}_y = \begin{bmatrix} \mathbf{R}_y & \mathbf{T}\mathbf{r}_y \\ \mathbf{r}_y^\top \mathbf{T} & r_0 \end{bmatrix}, \quad \bar{\mathbf{r}}_y = \begin{bmatrix} \mathbf{r}_y \\ r_{p+1} \end{bmatrix} \tag{26}$$

where $r_{p+1} = E[y(t)y(t-p-1)]$, $\mathbf{R}_y$ and $\mathbf{r}_y$ are the same as defined in (9), and $\mathbf{T}$ is the exchange matrix defined by

$$\mathbf{T} = \begin{bmatrix} 0 & 0 & \dots & 0 & 1 \\ 0 & 0 & \dots & 1 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 1 & \dots & 0 & 0 \\ 1 & 0 & \dots & 0 & 0 \end{bmatrix} \in \mathbb{R}^{p \times p}. \tag{27}$$

Then applying the matrix inversion formula [2] to $\bar{\mathbf{R}}_y^{-1}$ gives

$$\bar{\mathbf{R}}_y^{-1} = \begin{bmatrix} \mathbf{R}_y^{-1} + \mathbf{R}_y^{-1}\mathbf{T}\mathbf{r}_y d^{-1}\mathbf{r}_y^\top \mathbf{T}\mathbf{R}_y^{-1} & -\mathbf{R}_y^{-1}\mathbf{T}\mathbf{r}_y d^{-1} \\ -d^{-1}\mathbf{r}_y^\top \mathbf{T}\mathbf{R}_y^{-1} & d^{-1} \end{bmatrix} \tag{28}$$

where $d = r_0 - \mathbf{r}_y^\top \mathbf{T}\mathbf{R}_y^{-1}\mathbf{T}\mathbf{r}_y$. It follows immediately from (21), (22), (26) and (28) that

$$\boldsymbol{\mu}^\top \bar{\mathbf{a}}_{LS} = -d^{-1}\mathbf{r}_y^\top \mathbf{T}\mathbf{R}_y^{-1}\mathbf{r}_y + d^{-1} r_{p+1} \tag{29}$$

$$\boldsymbol{\mu}^\top \bar{\mathbf{R}}_y^{-1}\bar{\mathbf{a}} = -d^{-1}\mathbf{r}_y^\top \mathbf{T}\mathbf{R}_y^{-1}\mathbf{a}. \tag{30}$$

Substituting (29) and (30) into (25) yields

$$\sigma_w^2 = \frac{r_{p+1} - \mathbf{r}_y^\top \mathbf{T}\mathbf{a}_{LS}}{\mathbf{r}_y^\top \mathbf{T}\mathbf{R}_y^{-1}\mathbf{a}}. \tag{31}$$

With (31), we may be able to work solely with the underlying noisy AR(p) model.

Therefore, the above derivation may be summarized as the following DAILS algorithm for adaptive estimation of noisy AR signals.

3.1. The DAILS Algorithm

Step 0 Initialization:

(i) Set $\hat{\mathbf{a}}_{LS}(0)=\mathbf{0}$ and $\mathbf{P}_0=\gamma\mathbf{I}_p$, where γ is a large positive number.
(ii) Set $\hat{\mathbf{r}}_y(0)=\mathbf{0}$ and $\hat{r}_{p+1}(0)=0$.
(iii) Let $\hat{\mathbf{a}}_{ILS}(0)=\hat{\mathbf{a}}_{LS}(0)$.

Step 1 Use the recursive LS procedure to calculate $\hat{\mathbf{a}}_{LS}(t)$:

$$\hat{\mathbf{a}}_{LS}(t)=\hat{\mathbf{a}}_{LS}(t-1)+\mathbf{P}_t\mathbf{y}_t(y(t)-\mathbf{y}_t^{\top}\hat{\mathbf{a}}_{LS}(t-1)) \tag{32}$$

$$\mathbf{P}_t=\mathbf{P}_{t-1}-\frac{\mathbf{P}_{t-1}\mathbf{y}_t\mathbf{y}_t^{\top}\mathbf{P}_{t-1}}{1+\mathbf{y}_t^{\top}\mathbf{P}_{t-1}\mathbf{y}_t} \tag{33}$$

where $(t\mathbf{P}_t)\approx\mathbf{R}_y^{-1}$ for very large t.

Step 2 Evaluate covariance estimates

$$\hat{\mathbf{r}}_y(t)=\hat{\mathbf{r}}_y(t-1)+\frac{1}{t}(\mathbf{y}_t y(t)-\hat{\mathbf{r}}_y(t-1)) \tag{34}$$

$$\hat{r}_{p+1}(t)=\hat{r}_{p+1}(t-1)+\frac{1}{t}(y(t)y(t-p-1)-\hat{r}_{p+1}(t-1)). \tag{35}$$

Step 3 Compute the measurement noise variance estimate $\hat{\sigma}_w^2(t)$:

$$\hat{\sigma}_w^2(t)=\frac{\hat{r}_{p+1}(t)-\hat{\mathbf{r}}_y^{\top}(t)\mathbf{T}\hat{\mathbf{a}}_{LS}(t)}{\hat{\mathbf{r}}_y^{\top}(t)\mathbf{T}(t\mathbf{P}_t)\hat{\mathbf{a}}_{ILS}(t-1)}. \tag{36}$$

Step 4 Find the AR parameter estimate $\hat{\mathbf{a}}_{ILS}(t)$ *via*

$$\hat{\mathbf{a}}_{ILS}(t)=\hat{\mathbf{a}}_{LS}(t)+\hat{\sigma}_w^2(t)(t\mathbf{P}_t)\hat{\mathbf{a}}_{ILS}(t-1). \tag{37}$$

Step 5 If the chosen stop criterion is satisfied, output $\hat{\mathbf{a}}_{ILS}(t)$, $\hat{\sigma}_w^2(t)$ and stop; otherwise, set $t=t+1$ and go to Step 1.

In signal processing tasks such as smoothing and deconvolution, an estimate of the driving noise variance σ_v^2 is required. To this end, we examine the LS errors

$$y(t) - \mathbf{y}_t^\top \mathbf{a}_{LS} = (\mathbf{y}_t^\top \mathbf{a} + \varepsilon(t)) - \mathbf{y}_t^\top \mathbf{a}_{LS} = \mathbf{y}_t^\top (\mathbf{a} - \mathbf{a}_{LS}) + \varepsilon(t) \tag{38}$$

where (6) is used. By means of (8) and (9), it is easy to verify that

$$E[\mathbf{y}_t(y(t) - \mathbf{y}_t^\top \mathbf{a}_{LS})] = E[\mathbf{y}_t y(t)] - E[\mathbf{y}_t \mathbf{y}_t^\top]\mathbf{a}_{LS} = \mathbf{0}. \tag{39}$$

Following the white noise and independence assumptions on $v(t)$ and $w(t)$, we obtain from (7) that

$$\begin{aligned} E[\varepsilon(t)^2] &= E[(v(t) + w(t) - \mathbf{w}_t^\top \mathbf{a})^2] \\ &= E[v(t)^2] + E[w(t)^2] + \mathbf{a}^\top E[\mathbf{w}_t \mathbf{w}_t^\top]\mathbf{a} \\ &= \sigma_v^2 + \sigma_w^2 + \mathbf{a}^\top (\sigma_w^2 \mathbf{I}_p)\mathbf{a} \\ &= \sigma_v^2 + \sigma_w^2 + \sigma_w^2 \mathbf{a}^\top \mathbf{a}. \end{aligned} \tag{40}$$

Using (15), (38), (39) and (40), we may derive the average LS errors as follows:

$$\begin{aligned} J(\mathbf{a}_{LS}) &= E[(y(t) - \mathbf{y}_t^\top \mathbf{a}_{LS})^2] \\ &= E[(\mathbf{y}_t^\top (\mathbf{a} - \mathbf{a}_{LS}) + \varepsilon(t))(y(t) - \mathbf{y}_t^\top \mathbf{a}_{LS})] \\ &= (\mathbf{a} - \mathbf{a}_{LS})^\top E[\mathbf{y}_t(y(t) - \mathbf{y}_t^\top \mathbf{a}_{LS})] + E[\varepsilon(t)(y(t) - \mathbf{y}_t^\top \mathbf{a}_{LS})] \\ &= \mathbf{0} + E[\varepsilon(t)(\mathbf{y}_t^\top (\mathbf{a} - \mathbf{a}_{LS}) + \varepsilon(t))] \\ &= (\mathbf{a} - \mathbf{a}_{LS})^\top E[\mathbf{y}_t \varepsilon(t)] + E[\varepsilon(t)^2] \\ &= (\mathbf{a} - \mathbf{a}_{LS})^\top (-\sigma_w^2 \mathbf{a}) + (\sigma_v^2 + \sigma_w^2 + \sigma_w^2 \mathbf{a}^\top \mathbf{a}) \\ &= \sigma_v^2 + \sigma_w^2 (1 + \mathbf{a}_{LS}^\top \mathbf{a}). \end{aligned} \tag{41}$$

Once the AR parameter vector $\mathbf{a}$ and the measurement noise variance σ_w^2 have been estimated, the driving noise variance σ_v^2 may be estimated by virtue of (41). Specifically, at time step t, the driving noise variance estimate $\hat{\sigma}_v^2(t)$ is computed *via*

$$\hat{\sigma}_v^2(t) = \frac{1}{t}\hat{F}_t - \hat{\sigma}_w^2(t)[1 + \hat{\mathbf{a}}_{LS}(t)^\top \hat{\mathbf{a}}_{ILS}(t-1)] \tag{42}$$

where $(1/t)\hat{F}_t \approx J(\mathbf{a}_{LS})$ for very large t, and $\hat{F}_t$ satisfies the recursive relation [1]:

$$\hat{F}_t = \hat{F}_{t-1} + \frac{(y(t) - \mathbf{y}_t^\top \hat{\mathbf{a}}_{LS}(t-1))^2}{1 + \mathbf{y}_t^\top \mathbf{P}_{t-1}\mathbf{y}_t}. \tag{43}$$

The above two Eqs. (42) and (43) may be incorporated into the DAILS algorithm to perform on-line estimation of the driving noise variance σ_v^2.

Now some main algorithmic differences between the NPILS algorithm presented in [10] and the DAILS algorithm proposed herein are highlighted. The previous NPILS algorithm is based on the introduced augmented noisy AR($p+1$) model since it needs to calculate the variables associated the augmented model, such as, $\bar{\mathbf{R}}_y$, $\bar{\mathbf{r}}_y$ and $\bar{\mathbf{a}}_{LS}$. In contrast, the current DAILS algorithm inherently works with the underlying noisy AR(p) model. Since all the redundant calculations related to the previous NPILS algorithm are removed, we can achieve efficient implementation of the ILS type algorithms. As will be seen in the numerical results given in the next section, the DAILS algorithm can make a significant saving in the computational load over the implementation of the previous NPILS algorithm while maintaining the parameter estimation consistency.

Moreover, the advantages of the ILS type methods over the MYW, the ML, the RPE and the MLS methods, which are mentioned in Section 1, are retained by the proposed DAILS algorithm. In other words, with its great suitability for adaptive estimation, the DAILS algorithm either requires lower computational cost or maintains better numerical stability than those methods.

4. EXPERIMENTAL RESULTS

The simulated example is concerned with identifying an AR(2) model

$$x(t) - 1.1x(t-1) + 0.8x(t-2) = v(t) \tag{44}$$

where $\sigma_v^2 = 1.0$. The variance of the measurement noise $w(t)$ is assumed to be $\sigma_w^2 = 0.7$, so the signal-to-noise ratio (SNR) is

$$\text{SNR} = 10\log_{10}\frac{E[x^2(t)]}{\sigma_w^2} = 8\,\text{dB}. \tag{45}$$

The proposed DAILS algorithm is used to perform adaptive estimation, along with the normal LS method, the PILS method coupled with the prefilter

$$F(q^{-1}) = \frac{1}{1 - 0.4q^{-1}} \tag{46}$$

and the NPILS method. All the four algorithms adopt the following initial values:

$$\hat{\mathbf{a}}_{LS}(0) = \mathbf{0}, \quad \mathbf{P}_0 = 10^4\mathbf{I}_2. \tag{47}$$

The performance criteria used for comparison are the relative error (RE), the normalized root mean square error (RMSE), and the number of flops (count of floating point operations). The RE and the RMSE are defined as

$$\mathrm{RE} = 20\log_{10}\frac{\|\mathbf{m}(\hat{\mathbf{a}}) - \mathbf{a}\|}{\|\mathbf{a}\|}\ (\mathrm{dB}) \tag{48}$$

$$\mathrm{RMSE} = 20\log_{10}\sqrt{\frac{1}{M}\sum_{m=1}^{M}\frac{\|\hat{\mathbf{a}}_m - \mathbf{a}\|^2}{\|\mathbf{a}\|^2}}\ (\mathrm{dB}) \tag{49}$$

where

$$\mathbf{m}(\hat{\mathbf{a}}) = \frac{1}{M}\sum_{m=1}^{M}\hat{\mathbf{a}}_m \tag{50}$$

and $\hat{\mathbf{a}}_m$ stands for the estimate of $\mathbf{a}$ in the mth run over a total of M independent runs. Based on an ensemble average of $M = 200$ independent trials and a sample size of $N = 1000$, the values of the RE and the RMSE are plotted in Figures 1 and 2, while the numerical costs involved per recursion are listed in Table I.

Figures 1 and 2 clearly show that the normal LS estimates are seriously biased, whereas the three ILS based methods all yield good estimation results. In particular, it is observed from Figure 2 that the DAILS estimates are almost the same as the NPILS estimates in terms of accuracy. On the other hand, it is seen from Table I that by comparison with the NPILS method, an about 28% reduction in computations per time step is achieved by use of the DAILS method

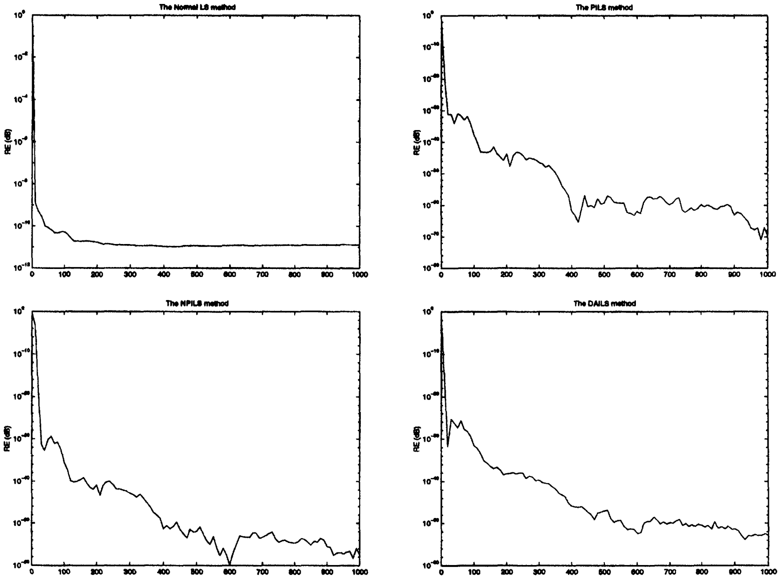

FIGURE 1 RE of parameter estimates.

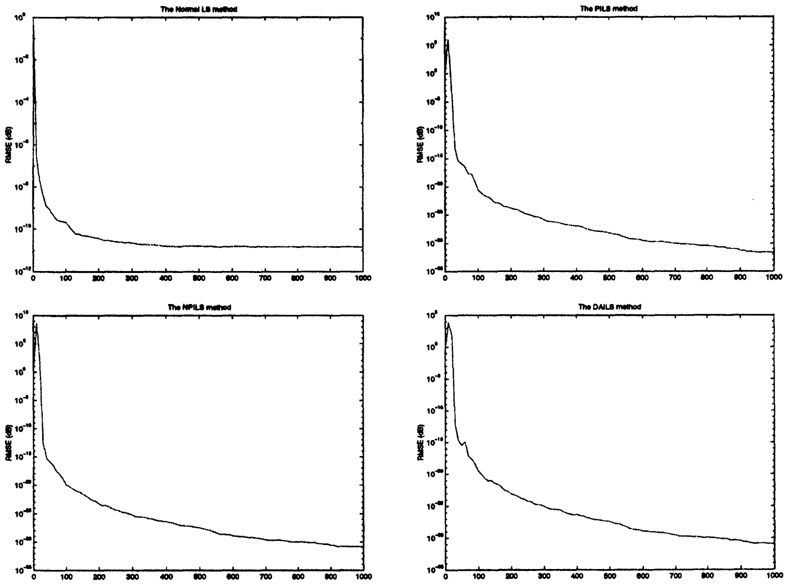

FIGURE 2 RMSE of parameter estimates.

TABLE I Numerical costs (1000 data points, 200 independent trials, SNR = 8 dB)

Method	*LS*	*PILS*	*NPILS*	*DAILS*
flops # per recursion	52	269	157	113

due to no handling of the augmented noisy AR model. Moreover, the numerical cost of the DAILS method at each time step accounts for only about 42% that of the PILS method. All this illustrates that the computational advantage of the DAILS method over the NPILS method as well as the PILS method is very prominent. Thus the DAILS algorithm is equipped with the greatly improved numerical efficiency.

5. CONCLUSIONS

An adaptive ILS based algorithm for on-line identification of noisy AR signals has been described. The proposed DAILS algorithm is consistently convergent. Since the underlying noisy AR model is identified directly, the DAILS algorithm is more numerically efficient than the previous PILS and NPILS methods. The good performances of the DAILS algorithm have been illustrated by the experimental results.

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Intermodal transport refers to the movement of goods in a single loading unit which uses successive various modes of transport (road, rail, water) without handling the goods during mode transfers. Intermodal transport has become an important policy issue, mainly because it is considered to be one of the means to lower the congestion caused by single-mode road transport and to be more environmentally friendly than the single-mode road transport. Both considerations have been followed by an increase in attention toward intermodal freight transportation research.

Various intermodal freight transport decision problems are in demand of mathematical models of supporting them. As the intermodal transport system is more complex than a single-mode system, this fact offers interesting and challenging opportunities to modelers in applied mathematics. This special issue aims to fill in some gaps in the research agenda of decision-making in intermodal transport.

The mathematical models may be of the optimization type or of the evaluation type to gain an insight in intermodal operations. The mathematical models aim to support decisions on the strategic, tactical, and operational levels. The decision-makers belong to the various players in the intermodal transport world, namely, drayage operators, terminal operators, network operators, or intermodal operators.

Topics of relevance to this type of decision-making both in time horizon as in terms of operators are:

- Intermodal terminal design
- Infrastructure network configuration
- Location of terminals
- Cooperation between drayage companies
- Allocation of shippers/receivers to a terminal
- Pricing strategies
- Capacity levels of equipment and labour
- Operational routines and lay-out structure
- Redistribution of load units, railcars, barges, and so forth
- Scheduling of trips or jobs
- Allocation of capacity to jobs
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