Weighted Multirate q-Markov Cover Identification Using PRBS – An Application to Engine Systems

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The q-Markov COVariance Equivalent Realization (q-Markov Cover) method for identification uses either pulse, white noise or PRBS (Pseudo-Random Binary Signal) as test excitation. This paper extended the q-Markov Cover using PRBS to the weighted multirate case, that is, the sample rate of the PRBS signal is different from the system output one. Then, the multirate PRBS q-Markov Cover is applied to identify a diesel engine model from the fuel command input to the engine speed output. The identified engine model has order of two and approximates the pure fuel system time delay using a first-order transfer function with a non-minimum phase numerator. Finally, the identified engine model was successfully used for designing engine idle speed governor and obtained satisfactory performance in the first try.

Keywords: System identification; Pseudo-random binary signal; Control systems

I. INTRODUCTION

The q-Markov COVariance Equivalent Realization (q-Markov Cover) theory was originally developed for model reduction. The reduced order model obtained using q-Markov Cover preserves the first q Markov and covariance parameters of the original system. The parameterization of all q-Markov Covers for both continuous and discrete systems is addressed in Ref. [1]. The realization of all q-Markov Covers from the input/output data of discrete systems is useful for identification; see Ref. [2]. This algorithm generates all q-Markov

Covers with only q Markov parameters and q covariance parameters. This data set requires both pulse experiments to obtain the Markov parameters and white noise experiments to get the covariance parameters. The deterministic approach using only pulse experimental data and the stochastic approach using only white noise experimental data to generate the q Markov parameters and covariance parameters are described in Ref. [3]. Using the results in Ref. [3], a q-Markov Cover of a linear discrete system can be found by using experimental data from either pulse or white noise input. The q-Markov Cover identification using experimental data from Pseudo-Random Binary Signal (PRBS) (see Refs. [4,5]) is presented in Ref. [6] to replace the white noise version algorithm due to the easy implementation of PRBS. These signal can be generated using simple shift register circuits and it is very practical for engineering applications.

The extension of the single rate q-Markov Cover to the multirate case is motivated by its application to the diesel engine system to obtain a linearized transfer function from commanded fueling (mm^3/stroke) to engine speed (RPM) at 100 Hz sample rate. For a six-cylinder four-stroke diesel engine, the engine firing (or fuel injection) frequency is 30 Hz at 600 RPM. The PRBS fuel excitation sampled at more than 30 Hz will not be executed by the fuel injectors, providing a badly identified model due to poor correlation between the fuel command input and the engine speed output. To improve the system identification for diesel engine applications using PRBS q-Markov Cover, it is necessary to have a q-Markov Cover capable of using the input and output data sampled at different frequencies. This paper provides a q-Markov Cover theory that allows the input PRBS sampled at a frequency slower than the system output sample rate at which the linear model is constructed.

A Matlab Graphic User Interface (GUI) has been written to make the multirate PRBS q-Markov Cover system identification tool easy to use. The system identification GUI displays the input PRBS and output response, along with the model order selection plot. This makes it convenient to study the relationship between identification error and the model order selection.

The paper is organized as follows. In Section II, the PRBS signals and its multirate sampling scheme are discussed. Section III extends the single rate PRBS q-Markov Cover theory to a weighted multirate

version. An application example to diesel engine identification is provided in Section IV, and the last section adds some conclusions.

II. PRBS AND ITS STOCHASTIC PROPERTIES

II.A. PRBS Used for q-Markov Cover

The most commonly used PRBSs are based on maximum length sequences (called m-sequences) for which the length of the PRBS signals is $m = 2^n - 1$, where n is an integer called the order of the PRBS. Let z^{-1} represent a delay operator, and define $\hat{p}(z^{-1})$ to be the polynomials

$$\hat{p}(z^{-1}) = a_n z^{-(n-1)} \oplus \cdots \oplus a_2 z^{-1} \oplus a_1, \qquad \text{(II-1)}$$

where a_i is either $a > 0$ or $-a$, and for any number $a \oplus a$ obeys,

$$a \oplus a = -a = -a \oplus -a, \quad a \oplus -a = a = -a \oplus a. \qquad \text{(II-2)}$$

The following table (Table I) provides the selection of coefficients a_i $(i = 1, 2, \ldots, n)$ that guarantees $\hat{p}(z^{-1})$ to be a maximum sequence.

The coefficients of the maximum polynomials with order between 12 and 34 can be found in Ref. [4]. The PRBS with magnitude a can be generated by the following formula:

$$\hat{u}[(k + 1)T_p] = \hat{p}(z^{-1})\hat{u}(kT_p), \quad k = 0, 1, 2, \ldots, \qquad \text{(II-3)}$$

TABLE I   Coefficients for maximum sequences

Order of polynomial n	Period of the sequence m	Non-negative coefficients a_i
2	3	a_1, a_2
3	7	a_2, a_3
4	15	a_3, a_4
5	31	a_3, a_5
6	63	a_5, a_6
7	127	a_4, a_7
8	255	a_2, a_3, a_4, a_8
9	511	a_5, a_9
10	1023	a_7, a_{10}
11	2047	a_9, a_{11}

where $\hat{u}(0) = a$ and $\hat{u}(-T_p) = \hat{u}(-2T_p) = \cdots = \hat{u}[-(n-1)T_p] = -a$, T_p is the sample rate used to generate the PRBS, and a is the magnitude of the PRBS signal. The inverse PRBS, used for q-Markov Cover, is constructed by

$$u(kT_p) = s(k) \oplus \hat{u}(kT_p), \quad s(k) = \begin{cases} a, & k \text{ even,} \\ -a, & k \text{ odd.} \end{cases} \tag{II-4}$$

It is clear after some analysis that $u(k)$ has a period $2m$ and $u(k) = -u(k+m)$. The mean of the inverse PRBS is

$$m_u = E_{2m}u(kT_p) = \frac{1}{2m} \sum_{i=0}^{2m-1} u(kT_p) = 0, \tag{II-5}$$

and the autocorrelation of $u(k)$ is

$$R_{uu}(\tau T_p) = E_{2m}u[(k+\tau)T_p]u^T(kT_p) = \frac{1}{2m} \sum_{i=0}^{2m-1} u[(k+\tau)T_p]u^T(kT_p)$$

$$= \begin{cases} a^2, & \tau = 0, \\ -a^2, & \tau = m, \\ -\dfrac{a^2}{m}, & \tau \text{ even,} \\ \dfrac{a^2}{m}, & \tau \text{ odd.} \end{cases} \tag{II-6}$$

In the rest of this paper, the term "PRBS" is used to represent the inverse PRBS.

II.B. PRBS Sampled at Higher Rate than its Generation Rate

Consider the situation that the PRBS is generated at the sample rate $1/T_p$ and the signal is sampled at a higher rate $1/T$ than its generation rate. Assume that the ratio of two sample rates is an integer n_r defined as follows:

$$n_r = \frac{T_p}{T}. \tag{II-7}$$

The period number of the PRBS signal sampled at $1/T$ is $2n_r m$, and the mean of the signal is zero computed by the following equation:

$$m_u^{\mathrm{T}} = E_{2n_r m} u(kT) = \frac{1}{2n_r m} \sum_{i=0}^{2n_r m - 1} u(kT) = 0. \qquad \text{(II-8)}$$

However, the autocorrelation is quite different from the situation that the PRBS is sampled at its generation rate. In this case, the auto-correlation is defined by

$$R_{uu}(\tau T) = E_{2n_r m} u[(k+\tau)T] u^{\mathrm{T}}(kT) = \frac{1}{2n_r m} \sum_{i=0}^{2n_r m - 1} u[(k+\tau)T] u^{\mathrm{T}}(kT), \qquad \text{(II-9)}$$

and equals to

$R_{uu}(\tau T)$

$$= \begin{cases} (n_r - \bar{\tau})a^2 + \bar{\tau}\dfrac{a^2}{m}, & 0 \le \tau < n_r \ \text{ or } \ n_r m \le \tau < n_r(m+1), \\[2mm] -\bar{\tau}a^2 - (n_r - \bar{\tau})\dfrac{a^2}{m}, & n_r(m-1) \le \tau < n_r m \ \text{ or } \ n_r(2m-1) \le \tau < 2n_r m, \\[2mm] -(n_r - 2\bar{\tau})\dfrac{a^2}{m}, & \text{otherwise if } \tau \text{ even,} \\[2mm] (n_r - 2\bar{\tau})\dfrac{a^2}{m}, & \text{otherwise if } \tau \text{ odd,} \end{cases}$$

$$\text{(II-10)}$$

where $\bar{\tau} = \tau - \mathrm{fix}(t/n_r)n_r$ and $\mathrm{fix}[\,\cdot\,]$ is a round-off operator which takes integer portion of $[\,\cdot\,]$. Figure 1 is an autocorrelation plot of a 5th order PRBS with unit magnitude sampled three times faster than its generation sample rate. When τ is between -2 and 2, the autocorrelation is greater than one. The absolute value of the rest of the autocorrelation is less than $3/31$ (that is $n_r a^2/m$) when the absolute value of τ is greater than 2 and less than $n_r(m-1) = 90$. Note that when the order of the PRBS increases, $n_r a^2/m$ reduces rapidly. For example, for 10th order PRBS with unit magnitude and $n_r = 3$, $n_r a^2/m = 0.00293$ which is less than 0.3 percent of the autocorrelation when $\tau \in [-2, 2]$. This property is very important for applying PRBS to q-Markov Cover identification.
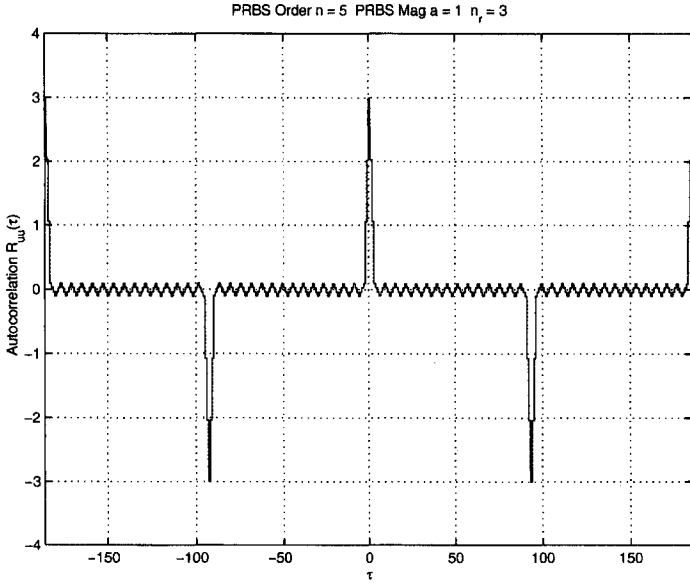
PRBS Order n = 5   PRBS Mag a = 1   n$_r$ = 3

FIGURE 1 Autocorrelation of PRBS sampled faster than its generation rate. (See Color Plate I at the back of the issue.)

III. WEIGHTED MULTIRATE q-MARKOV COVER USING PRBS

Consider the discrete asymptotically stable system

$$
\begin{aligned}
x(k + 1) &= Ax(k) + Dw(k), \\
y(k) &= Cx(k) + Hw(k),
\end{aligned}
\tag{III-1}
$$

where $w \in \mathfrak{R}^{n_w}$, $x \in \mathfrak{R}^{n_x}$ and $y \in \mathfrak{R}^{n_y}$ are the input, state, and output vectors. Suppose that X is the solution of the following Lyapunov equation:

$$
X = AXA^{\mathrm{T}} + DWD^{\mathrm{T}},
\tag{III-2}
$$

where W is the covariance matrix of the white noise input w. The Markov parameters H_i and covariance parameters R_i ($i = 0, 1, 2, \ldots$) of (III-1) are defined by

$$
\begin{aligned}
H_0 &= H, \quad H_i = CA^{i-1}D, \ i = 1, 2, \ldots, \\
R_i &= CA^i XC^{\mathrm{T}} + H_i WH^{\mathrm{T}}, \ i = 0, 1, 2, \ldots
\end{aligned}
\tag{III-3}
$$

III.A. System Responses and its Autocorrelation

Use the same approach as Ref. [6], let $x(k, I)$ denote the state response of (III-1) with $e_I u$ as input, where e_I is the Ith standard base vector in $\mathfrak{R}^{n_w}$ and u is the PRBS sampled n_r times faster than its generation sample rate with a period number m. Since the PRBS is a periodic signal with period number $2n_r m$, the steady state response of the linear system (III-1) will also be periodic with the same period number as the PRBS. For any integer $p > 0$. Let $x(2n_r mp + k)$ represent the state response of (III-1) at time $(2n_r mp + k)T$ and for $0 \leq k < 2n_r m$ define the steady state response with input $e_I u$ as

$$x_s^I(k) = \lim_{p \to \infty} x^I(2n_r mp + k)$$

$$= A^k \sum_{j=0}^{\infty} A^{2n_r mj} \sum_{i=1}^{2n_r m-1} A^{2n_r m-1-i} De_I u(i) + \sum_{i=0}^{k-1} A^{k-1-i} De_I u(i).$$

$$(\text{III-4})$$

Assume stable system (III-1), leading

$$x_s^I(k+i) = A^k x_s^I(k) + \sum_{j=0}^{i-1} A^{i-1-j} De_I u(j), \quad i = 0, 1, \ldots, q-1. \quad (\text{III-5})$$

Using (III-1) and (III-5) we can have the following relationship:

$$Q_q y_q^I(k) = Q_q O_q x_s^I(k) + Q_Q H_q w_q^I(k), \quad (\text{III-6})$$

where Q_q is a block diagonal matrix with q blocks of positive definite output weighting matrix Q, and

$$y_q^I(k) = \begin{bmatrix} y_s^I(k) \\ y_s^I(k+1) \\ \vdots \\ y_s^I(k+q-1) \end{bmatrix}, \quad w_q^I(k) = \begin{bmatrix} e_I u(k) \\ e_I u(k+1) \\ \vdots \\ e_I u(k+q-1) \end{bmatrix},$$

$$O_q = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{q-1} \end{bmatrix}, \quad H_q = \begin{bmatrix} H_0 & 0 & \cdots & 0 \\ H_1 & H_0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ H_{q-1} & H_{q-2} & \cdots & H_0 \end{bmatrix}$$

$$(\text{III-7})$$

Taking the expectation over one PRBS period from both sides of Eq. (III-6) yields

$$Q_q R_q^I Q_q^{\mathrm{T}} = Q_q O_q X^I O_q^{\mathrm{T}} Q_q^{\mathrm{T}} + Q_q H_q W_q^I H_q^{\mathrm{T}} Q_q^{\mathrm{T}} + Q_q \Delta_q^I Q_q^{\mathrm{T}}, \quad \text{(III-8)}$$

where for any $q < n_r m$, matrix R_q^I has Toeplitz structure. Define

$$R_q = \sum_{I=1}^{n_w} R_q^I, \quad X = \sum_{I=1}^{n_w} X^I, \quad W_q = \sum_{I=1}^{n_w} W_q^I, \quad \Delta_q = \sum_{I=1}^{n_w} \Delta_q^I. \quad \text{(III-9)}$$

Then for any $q < n_r m$ we have

$$Q_q R_q Q_q^{\mathrm{T}} = Q_q O_q X O_q^{\mathrm{T}} Q_q^{\mathrm{T}} + Q_q H_q W_q H_q^{\mathrm{T}} Q_q^{\mathrm{T}} + Q_q \Delta_q Q_q^{\mathrm{T}}, \quad \text{(III-10)}$$

where

$$R_q = \begin{bmatrix} R_{yy}(0) & R_{yy}(1) & \dots & R_{yy}(q-1) \\ R_{yy}(1) & R_{yy}(0) & \dots & R_{yy}(q-2) \\ \vdots & \vdots & \ddots & \vdots \\ R_{yy}(q-1) & R_{yy}(q-2) & \dots & R_{yy}(0) \end{bmatrix}, \quad \text{(III-11)}$$

$$R_{yy}(\tau) = \sum_{i=1}^{n_w} \frac{1}{2 n_r m} \sum_{j=0}^{2 n_w m - 1} y_s^i(j+\tau)[y_s^i(j)]^{\mathrm{T}},$$

$$X = \sum_{I=1}^{n_w} \frac{1}{2 n_r m} \sum_{j=0}^{2 n_w m - 1} x_s^I(j+\tau)[x_s^I(j)]^{\mathrm{T}}, \quad \text{(III-12)}$$

$$W_q = \begin{bmatrix} W(0) & W(1) & \dots & W(q-1) \\ W(1) & W(0) & \dots & W(q-2) \\ \vdots & \vdots & \ddots & \vdots \\ W(q-1) & W(q-2) & \dots & W(0) \end{bmatrix}, \quad \text{(III-13)}$$

$$W(\tau) = \begin{cases} \left(n_r - \tau \dfrac{m-1}{m} \right) \text{block-diag}[a_1^2, \dots, a_{n_w}^2], & \tau < n_r, \\ 0 - n_r \times n_r \text{ matrix}, & \tau \geq n_r, \end{cases}$$

a_I is the PRBS magnitude used for the Ith input and Δ_q has the following property based upon Theorem 3.1 in Ref. [6]:

$$\lim_{m \to \infty} \|\Delta_q\|_F = 0, \qquad \text{(III-14)}$$

where $\|\cdot\|$ denotes Frobenius norm. Property of (III-14) is the key of allowing the use of the PRBS to develop a q-Markov Cover since for larger enough m,

$$Q_q R_q Q_q^{\mathrm{T}} = Q_q O_q X O_q^{\mathrm{T}} Q_q^{\mathrm{T}} + Q_q H_q W_q H_q^{\mathrm{T}} Q_q^{\mathrm{T}}. \qquad \text{(III-15)}$$

III.B. Weighted Multirate q-Markov Cover Theorem

Let the unknown (presumed nonlinear) system

$$\begin{aligned} x(k+1) &= f(x(k), w(k)), \\ y(k) &= g(x(k), w(k)), \end{aligned} \qquad \text{(III-16)}$$

be subjected to an input sequence $\{w(0), w(1), w(2), \ldots\}$ generating the output sequence $\{y(0), y(1), y(2), \ldots\}$. We say that (III-16) is *q-identifiable* if there exists a linear model of the form (III-1) that can reproduce the same output sequence $\{y(0), y(1), y(2), \ldots, y(q-1)\}$ in response to the same input sequence $\{w(0), w(1), w(2), \ldots, w(q-1)\}$. The following lemma (see Ref. [6] for a proof) provides the necessary and sufficient condition for *q-identifiability*.

LEMMA III-1   *Let R_q and $H_q W_q H_q^{\mathrm{T}}$ be constructed from the response of system* (III-16) *given the PRBS input w with period $m = \infty$. Then the system* (III-16) *generating the data $y(i)$ ($i = 0, 1, 2, \ldots$) is q-identifiable if and only if the data matrix $D_q = R_q - H_q W_q H_q^{\mathrm{T}} \geq 0$.*

If the system generating the data is not *q-identifiable*, then it is possible for the q-Markov Cover identification to construct a least squares fit to the data with a linear model, see Refs. [1,2], but there exists no linear model of any order that can produce the data exactly.

The computation of the weighted q-Markov Covers starts from the data matrices D_q, $M_q = [H_1^{\mathrm{T}}, H_2^{\mathrm{T}}, \ldots, H_{q-1}^{\mathrm{T}}]^{\mathrm{T}}$, and H_0. Based upon Lemma III-1 it is concluded that a state realization $\{\hat{A}, \hat{B}, \hat{C}, \hat{H}\}$ is a

q-Markov Cover of given matrices D_q, $M_q = [H_1^T, H_2^T, \ldots, H_{q-1}^T]^T$, and H_0 if the data matrices $\hat{D}_q$, $\hat{M}_q$, and $\hat{H}_q$ constructed from system matrices $\{\hat{A}, \hat{B}, \hat{C}, \hat{H}\}$ are identical to the data matrices D_q, M_q, and H_0, respectively. Define P_q to be a full column rank square root of $Q_q D_q Q_q^T$, i.e.,

$$Q_q D_q Q_q^T = Q_q(R_q - H_q W_q H_q^T)Q_q^T = P_q \cdot P_q^T, \tag{III-17}$$

and denote r to be the rank of $Q_q D_q Q_q^T$. Partition matrix P_q as

$$P_q = [P_0^T, P_1^T, \ldots, P_{q-1}^T], \quad P_i \in \mathfrak{R}^{n_y \times r}, \ i = 0, 1, \ldots, q - 1. \tag{III-18}$$

Define

$$P = [P_0^T, P_1^T, \ldots, P_{q-2}^T], \quad \bar{P} = [P_1^T, P_2^T, \ldots, P_{q-1}^T]. \tag{III-19}$$

The following lemma, which extends the result in Ref. [3], transfers the q-Markov Cover problem to a linear algebra problem.

LEMMA III-2   *Assume that the period of the PRBS is infinity, i.e., $m = \infty$. A realization $\{A, B, C, H\}$ with X equal to identity is a q-Markov Cover matching the data matrices D_q, M_q and H_0 if and only if there exists realization $\{\hat{A}, \hat{B}, \hat{C}, \hat{H}\}$ which is a unitary coordinate transformation of $\{A, B, C, H\}$ such that the following equations are satisfied:*

$$P[\hat{A}, \hat{D}] = [\bar{P}, Q_{q-1}M_q], \quad [\hat{A}, \hat{D}][\hat{A}, \hat{D}]^T = I,$$
$$\hat{C} = Q^{-1}P_0, \quad and \quad \hat{H} = Q^{-1}H_0. \tag{III-20}$$

Using the linear algebra result for all solutions of (III-20) all q-Markov Cover can be generated from the given data matrices $Q_q R_q Q_q^T$, $Q_{q-1}^T M_q$, and $Q^{-1}H_0$.

THEOREM III-1   *Suppose that the data matrices $Q_q R_q Q_q^T$, $Q_{q-1}^T M_q$, and $Q^{-1}H_0$ are generated by the steady state responses $y_s^l(k)$ $(l = 1, 2, \ldots, n_w)$ defined in (III-6) from the PRBS excitation with $m = \infty$. Then all minimal q-Markov Covers of the system with identity state covariance, data matrices D_q, M_q, and H_0 can be obtained by the*

following equations:

$$[\hat{A}, \hat{D}] = P^+[\bar{P}, Q_{q-1}M_q] + V_pUV_{\bar{P}}^{\mathrm{T}}; \quad \hat{C} = Q^{-1}P_0, \quad and \quad \hat{H} = Q^{-1}H_0,$$

$$\text{(III-21)}$$

where matrix U *is arbitrarily unitary with dimension* $(r - r_1) \times (r - r_1)$
*(r_1 is the rank of P), V_p is a column unitary matrix whose column space
belongs to the null of P with dimension* $r \times (r - r_1)$, *and $V_{\bar{p}}$ is also column
unitary, whose column space belongs to the null of* $[\bar{P}Q_{q-1}M_q]$ *with
dimension* $(r - n_u) \times (r - r_1)$. *Furthermore, the* q-*Markov Covers* (III-21)
are stable and exist for any $q > 0$.

The proof of Theorem III-1 follows the same procedure as the results
in Refs. [3,6].

III.C. Weighted Multirate q-Markov Cover Algorithm

The results of Theorem III-1 may not sound interesting because the
signal $y_s^I(k)$ $(I = 1, 2, \ldots, n_w)$ defined in (III-6) and the PRBS with
$m = \infty$ cannot be generated in practice. But Theorem III-1 suggests
that the q-Markov Cover can be approximated using the PRBS with
large enough period number m because in this case Eq. (III-10) can
be approximated by (III-15). Also, the steady state signal $y_s^I(k)$
$(I = 1, 2, \ldots, n_w)$ can be obtained approximately in the experiment by
taking the steady state responses of system (III-16). Following is the
weighted multirate q-Markov Cover algorithm using PRBS.

(1) Set parameters m and n_r (ratio of PRBS sampling period over sys-
tem output sampling one, see (II-7)), and then, conduct experiments
to obtain steady state output responses $y_s^I(k)$ $(I = 1, 2, \ldots, n_w)$.

(2) Compute autocorrelation matrices $R_{yy}(i)$ and cross-correlation
matrices $R_{yw}(i)$ $(i = 0, 1, \ldots, q - 1)$ by the following:

$$R_{yy}(i) = \frac{1}{2m} \sum_{I=1}^{n_w} \sum_{k=0}^{2n_rm-1} y_s^I(k+i)[y_s^I(k)]^{\mathrm{T}},$$

$$\text{(III-22)}$$

$$R_{yw}(i) = \frac{1}{2m} \sum_{I=1}^{n_w} \sum_{k=0}^{2n_rm-1} y_s^I(k+i)u_I(k)[e_I]^{\mathrm{T}},$$

where $u_I(k)$ $(I = 1, 2, \ldots, n_w)$ is the PRBS signal for the Ith input and $y_s^I(k)$ $(I = 1, 2, \ldots, n_w)$ is the steady state response of system (III-1) with only the Ith input is excited by $u_I(k)$. Then the system Markov matrices can be computed by

$$H_i = R_{yw}(i) W^{-1}, \qquad \text{(III-23)}$$

where matrix W is defined in (III-13), and notice that in (III-13) a_I $(I = 1, 2, \ldots, n_w)$ is the magnitude of the PRBS for the Ith input channel. See Ref. [6] for the derivation of Eqs. (III-22) and (III-23).

(3) Select parameter q, and form matrices R_q, M_q, and H_q by

$$R_q = \begin{bmatrix} R_{yy}(0) & R_{yy}(1) & \ldots & R_{yy}(q-1) \\ R_{yy}(1) & R_{yy}(0) & \ldots & R_{yy}(q-2) \\ \vdots & \vdots & \ddots & \vdots \\ R_{yy}(q-1) & R_{yy}(q-2) & \ldots & R_{yy}(0) \end{bmatrix},$$

$$M_q = \begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ H_{q-1} \end{bmatrix}, \quad H_q = \begin{bmatrix} H_0 & 0 & \ldots & 0 \\ H_1 & H_0 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ H_{q-1} & H_{q-2} & \ldots & H_0 \end{bmatrix} \qquad \text{(III-24)}$$

and Q_q is a block diagonal matrix with q blocks of positive definite matrix Q. Note that matrix Q is selected by user to scale the output.

(4) Form matrix P_q by
  - Computing $D_q = Q_q[R_q - H_q W_q H_q^T]Q_q^T$, where W_q is defined in (III-13).
  - Doing the Schur decomposition of D_q to obtain

$$D_q = \begin{bmatrix} \hat{P}_q & \tilde{P}_q \end{bmatrix} \begin{bmatrix} \Lambda & 0 \\ 0 & \tilde{\Lambda} \end{bmatrix} \begin{bmatrix} \hat{P}_q^T \\ \tilde{P}_q^T \end{bmatrix} \qquad \text{(III-25)}$$

such that the diagonal elements of block-diag $\lfloor \Lambda \ \tilde{\Lambda} \rfloor$ are in decreasing order, and Λ is chosen such that $\bar{\sigma}\|\tilde{\Lambda}\| \ll \underline{\sigma}\|\Lambda\|$,

where $\Lambda \in \mathfrak{R}^{r \times r}$, and $\bar{\sigma}\|M\|$ and $\underline{\sigma}\|M\|$ denotes the maximum and minimum singular values of M.

- Constructing P_q by

$$P_q = \hat{P}_q \Lambda^{1/2}. \tag{III-26}$$

(5) Construct matrices P, $\bar{P}$, and $V_p U V_p^{\mathrm{T}}$ by
   - Defining

$$P_q^{\mathrm{T}} = \lfloor P_0^{\mathrm{T}}, P_1^{\mathrm{T}}, \ldots, P_{q-1}^{\mathrm{T}} \rfloor,$$
$$P^{\mathrm{T}} = [P_0^{\mathrm{T}}, P_1^{\mathrm{T}}, \ldots, P_{q-2}^{\mathrm{T}}], \tag{III-27}$$
$$\bar{P}^{\mathrm{T}} = [P_1^{\mathrm{T}}, P_2^{\mathrm{T}}, \ldots, P_{q-1}^{\mathrm{T}}].$$

   - Making the singular value decomposition of P and $[\bar{P}, M_q]$

$$P = [\, U_a \quad U_b \,] \begin{bmatrix} \Sigma_a & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_a^{\mathrm{T}} \\ V_b^{\mathrm{T}} \end{bmatrix},$$
$$[\bar{P}, M_q] = [\, U_c \quad U_d \,] \begin{bmatrix} \Sigma_c & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_c^{\mathrm{T}} \\ V_d^{\mathrm{T}} \end{bmatrix}. \tag{III-28}$$

   - If P is full rank, letting, $V_p U V_{\bar{p}}^{\mathrm{T}} = 0$, and else,

$$V_p U V_{\bar{p}}^{\mathrm{T}} = V_b U V_d^{\mathrm{T}}, \tag{III-29}$$

where U is an arbitrary unitary matrix.

(6) Compute $\hat{A}$, $\hat{D}$, $\hat{C}$ and $\hat{H}$ by

$$\hat{A} = P^+ \bar{P} + V_p U V_{\bar{p}}^{\mathrm{T}} I_1, \quad I_1 = [I_r \quad 0];$$
$$\hat{D} = P^+ M_q + V_p U V_{\bar{p}}^{\mathrm{T}} I_2, \quad I_2 = [0 \quad I_{n_w}]; \tag{III-30}$$
$$\hat{C} = Q^{-1} P_0, \quad \hat{H} = Q^{-1} H_0.$$

III.D. Matlab GUI and Selection of PRBS System Identification Parameters

Subsection III.C describes the mathematical details of the multirate q-Markov system identification algorithm using PRBS. In practical application, in order to obtain a good system identification model,
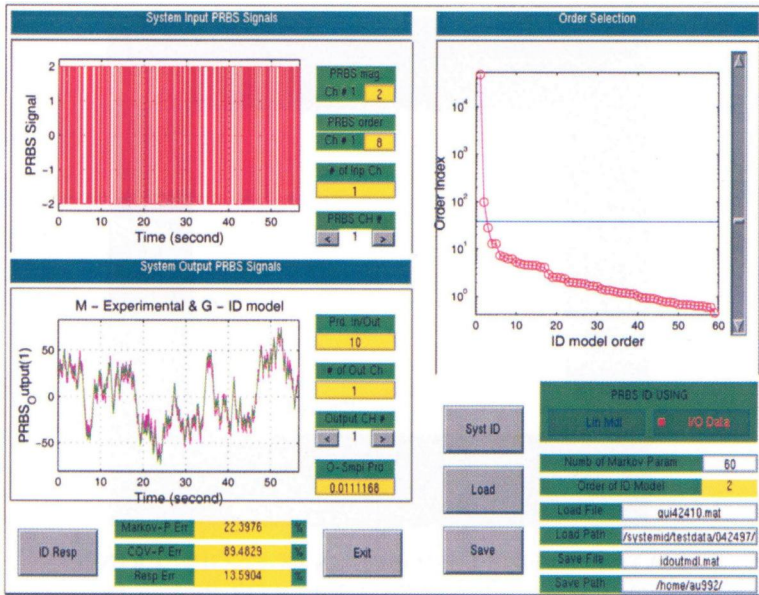
FIGURE 2   Weighted multirate PRBS system identification GUI. (See Color Plate II at the back of the issue.)

parameters like q and r defined in Step 4 of the algorithm may need to be tuned to have a good fit of the test data corresponding to the identified model. In order to make the system identification tuning process (varying parameters q and r) easy, an Matlab GUI is developed. Figure 2 shows the Matlab system identification GUI.

The functionality of the PRBS system identification GUI is listed below:

• *System input block* (upper-left portion) specifies the input PRBS. The GUI allows the user to view number of input channels, PRBS magnitude and order for any selected input channel. The selected PRBS signal is also plotted on the GUI.
• *System output block* (middle-left portion) specifies and displays the system output response of a loaded data file. The GUI allows the user to view the number of output channels, output sample period, and the input/output sample period ratio $n_r = T_p/T$.

- *Loading/saving data file block* (lower-right portion) allows the user to load a test data file for PRBS system identification and save the identified model into a specified data file. This block also make it possible to modify the selected identification parameter q (see Step 1 of the PRBS system identification algorithm in Subsection III.C).
- *Order selection block* (upper-right portion) plots the diagonal elements of Λ (see (III-25)). Note that each plotted point corresponding to one order of the identified model. Moving the slider on right allows the user to select parameter r (see Step 4 of the PRBS system identification algorithm in Subsection III.C).
- *System response block* (lower-left portion) calculates the response error and plots the model response over the test response in the system output block. This provides the feeling of the system identification quality with selected system identification parameters q and r.

The general procedure of weighted multirate system identification using PRBS and its parameter selection can be described in the following steps:

Step 1: Design the PRBS system identification experiments
To design PRBS system identification experiments, one needs to select the PRBS magnitudes a_i $(i = 1, 2, \ldots, n_w)$ and the order of PRBS corresponding to parameter m, the system output sample period T and the integer sample ratio $n_r = T_p/T$, where T_p is the sample period for generating PRBS. In general, the magnitude of PRBS is selected to maximize the system output signal to noise ratio, and the order of the PRBS shall cover the dominated frequencies of the identified system such that these modes can be excited by PRBS. The sample period ratio is selected such that the actuator is able to respond to the PRBS command. For instance, consider the diesel engine system from the fuel command to the engine speed output. Note that in a diesel engine, fuel is directly injected into the individual cylinder at a proper crank angle, or in other words, the fuel injection is a discrete event. Identifying an engine model from the fuel command to the engine speed requires that the PRBS fuel excitation can be executed by the fuel system. For a six-cylinder four-stroke diesel engine running at 600 RPM, the fuel injection frequency is 30 Hz. If the desired system output sample rate $(1/T)$ is at 100 Hz, the input PRBS excitation cannot be generated at 100 Hz since the PRBS fuel excitation cannot

be executed by the fuel system. This requires a lower PRBS generation frequency ($1/T_p$) than 30 Hz to make sure that the PRBS fuel command can be executed by the fuel system actuators. If the PRBS generation sample period is selected to $T_p = 0.1$ (s), the selected sample ratio is $n_r = T_p/T = 10$.

Step 2: Run the system identification test and collect experimental data
After designing the system identification experiment, the next step is to run the test and collect the experimental data. From the derivation of the PRBS system identification theorem, steady state responses (defined in (III-6)) of one PRBS cycle ($2n_r m$ points) are used for system identification. After the PRBS perturbation for a given channel is applied, the response data shall not be collected until the correspond-ing system responses are periodic and have the same period as the PRBS (this indicates that steady state has been reached). For the system with multiple input channels, the PRBS input excitation needs to be applied individually, and the system output responses ($y_q^I(k)$) ($I = 1, 2, \ldots, n_w$), see (III-6)), corresponding to each PRBS input channel shall be collected.

Step 3: System identification using GUI
After the experimental data is collected and converted into a Matlab data file ready to be loaded into Matlab, the PRBS system identifica-tion GUI can be used to obtain a linearized system model. To have a good system identification result, the following parameters need to be determined: output weighting matrix Q, number q of Markov and covariance parameter to be preserved, and the identified system order r. In general, the output weighting matrix is used to balance the output response scale for different output channels. For example, if the magnitude of output channel one is ten times larger than channel two for a two-output system, the relative system identification error for output channel two could be bigger than channel one even though the absolute system identification error for both channels are the same. In this case, the output weighting matrix Q can be used to adjust the output magnitude ratio for different output channels to improve relative modeling error for each individual output channel. The selection of parameter q is related to how well the identified model responses match actual system ones. Increasing q improves transient and steady state response match, but for a given PRBS order (given m) increasing

q introduces more relative system identification error due to the fact that finite PRBS order is used for system identification. Selection of model order parameter r is described in Step 4 of the PRBS system identification algorithm. The good news is that the PRBS system identification GUI allows the user to select the parameters q and r and validate the identification quality through clicking the buttons on the GUI.

IV. AN APPLICATION TO DIESEL ENGINE SPEED CONTROL SYSTEM

This section describes the application of the weighted multirate q-Markov Cover system identification to the diesel engine system from the engine fuel command to the engine speed output. The system identification is conducted using a Cummins 95 M11 diesel engine in a test cell with PC controller (PC with extended I/O capable of controlling a diesel engine) instead of an ECM (Engine Control Module). The PC controller provides flexibility of implementing new control logic. The PRBS system identification software was implemented into the PC controller for experiment purpose.

The purpose of this system identification is to obtain a proper linearized engine model for model-based control design, especially, for the engine speed control system. In general, engine output torque is controlled by the fuel quantity injected into the individual cylinder, and the other engine control input such as engine timing (start of injection) affects the engine torque output a little which will be treated as disturbance input. An engine speed control system can be simplified in Fig. 3, where engine timing and load are treated as disturbance. The subsections below follow the system identification procedure defined in the previous section.

IV.A. Design the PRBS System Identification Experiments

The system identification experiments is design based upon rules stated in Step 1 of the system identification process in Subsection III.D. Selection of parameters a_1 (PRBS magnitude), $m = 2^n - 1$ (where n is the order of PRBS), T (system output sample period), and T_p (PRBS generation sample period) is shown in Table II.
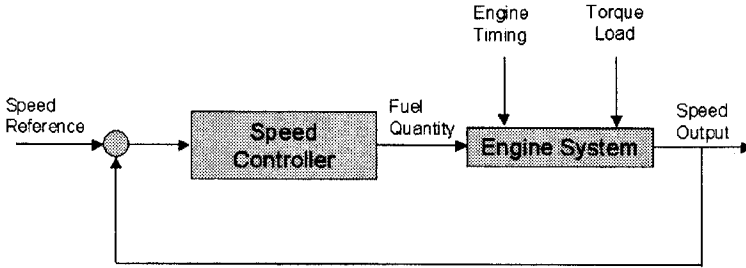
FIGURE 3   Engine speed control system.

TABLE II   Parameters for PRBS system identification experiment

Parameter	Selected values	Why
a_1	2 (mm^3/stroke)	2 mm^3/stroke fueling fluctuation may generate about 50 RPM speed variation for a bared engine.
m	255 ($n = 8$)	With selected T and T_p the period of PRBS in continuous time domain is 51 s ($2 \times m \times T_p$) which shall be able to excite the system dominated mode as low as 0.025 Hz.
T	0.01 (s)	T is selected based upon the controller sample rate (control algorithm executed at about 100 Hz in PC controller).
T_p	0.1 (s)	T_p is selected such that the engine fuel system is able to execute the PRBS fuel command over 600 RPM. Note that over the given engine speed range, the lowest engine fuel injection frequency (at 600 RPM) is 30 Hz for a six-cylinder four-stroke engine. The PRBS generation sample period is selected three times slower than fuel injection period.

IV.B. Experiment Setup and Data Collection

The experiment used a bare M11 engine (engine is not connected to output) with a PC engine controller. The experiment setup (see Fig. 4) and data collection process can be described as follows: (a) default engine timing is used for this test, (b) engine DC fueling is used to run the engine at desired engine speed, (c) after engine speed is stable PRBS fuel excitation is added to perturb the engine system, and finally, (d) after the engine reaches its steady state corresponding to
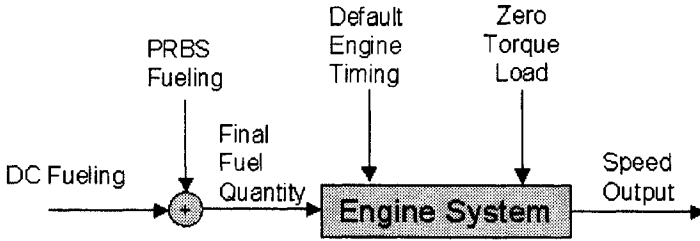
FIGURE 4   Experiment setup.

the PRBS perturbation, one PRBS cycle of the input PRBS perturba-
tion and output engine speed response are collected at sample rate $1/T$
(about 100 Hz). Note that each collected data file is associated to a
given engine speed. Therefore, the identified engine model is a function
of the engine operational speed.

Note that due to engine friction torque fluctuation, holding a
constant engine speed with constant fueling is not always feasible. In
fact, closed-loop system identification (with a stable engine speed
controller in the loop) is recommended. In this case, the speed con-
troller holds the engine at the desired speed when the PRBS fueling
disturbance is applied, and the identified model contains both engine
and speed controller dynamics. The engine system model can be
obtained by subtracting the controller dynamics. Both open- and
closed-loop system identification were conducted and similar results
are obtained. For this paper, only the open-loop system identification
results are presented.

Many system identification tests were conducted, but only two
system identification tests are described in this paper. That is, the
system identification test at engine speed equal to 1150 and 1500 RPM.
Figure 5 shows both the PRBS fueling disturbance in mm^3/stroke
(upper plot) and the corresponding engine speed fluctuation in RPM
(gray line of lower plot). In the system identification experiments, due
to the PC control operational system the actual system sample period
is about 0.0113 s, leading one PRBS period equals to 57.6 s. One of the
important aspect during the system identification tests is judging if the
collected system response is at the steady state or not. An easy way is
using the inverse symmetric property ($u(k) = -u(k + m)$) of the inverse
PRBS signal. Therefore, at the steady state the system output shall
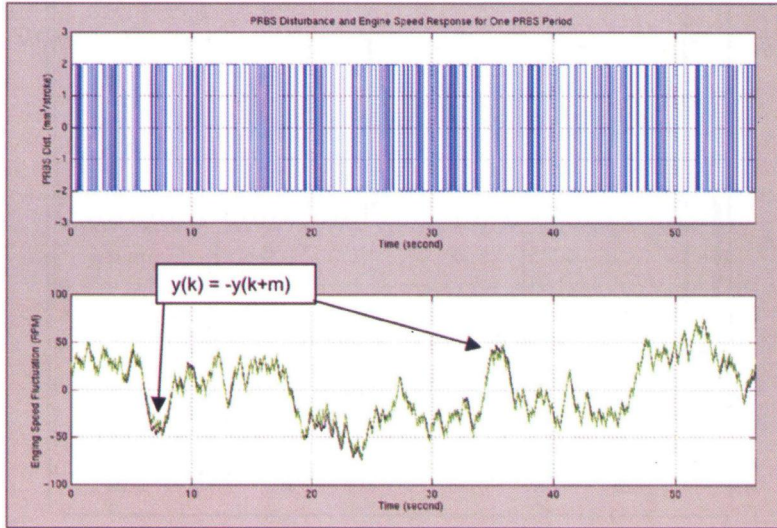
FIGURE 5   PRBS disturbance and corresponding engine speed response. (See Color Plate III at the back of the issue.)

have the similar property, that is,

$$y(k) = -y(k + m), \qquad (IV\text{-}1)$$

see the two points marked by arrows in Fig. 5. With the help of this technique, the author was able to collect the system identification data successfully without repeating any experiment.

IV.C. System Identification Using Weighted Multirate q-Markov Cover

The PRBS system identification GUI in Fig. 2 was used to obtain a linear engine model at a given engine speed. Step 3 of the procedure described in Subsection III.D was used for obtaining discrete time linear models. Since the system is an SISO one, the output weighting matrix is set to one. The number of Markov and covariance parameters to be preserved is selected to be 60. That is, the first 60 Markov

and covariance parameters of the identified model are close to these calculated ones based on (III-23) and (III-24). For practical purposes, the parameter q is upper limited by the numerical accuracy of q-Markov Cover and the constraint $q \ll m$ due to a finite PRBS period. On the other hand, small q may cost the accuracy of the identified model. The selected q is based upon the concerns mentioned above, and especially, the accuracy of the identified model quality, such as how well the identified model response matches with the test one.

The order of the identified model is determined using the order selection portion (see Fig. 6) of the system identification GUI in Fig. 2. From Fig. 6 we can see that the largest gap exists between the first (top) dots and second one. This indicates that there exists one dominated first-order dynamics. For the diesel engine system from fuel command to engine speed, this dominated mode is obviously the dynamics from engine torque to engine speed. The second largest gap is between second and third modes. The identified model order was selected to be two since a third order model does not improve the identified model accuracy and the gap between second and third modes is the second largest. The steady state response of the identified engine
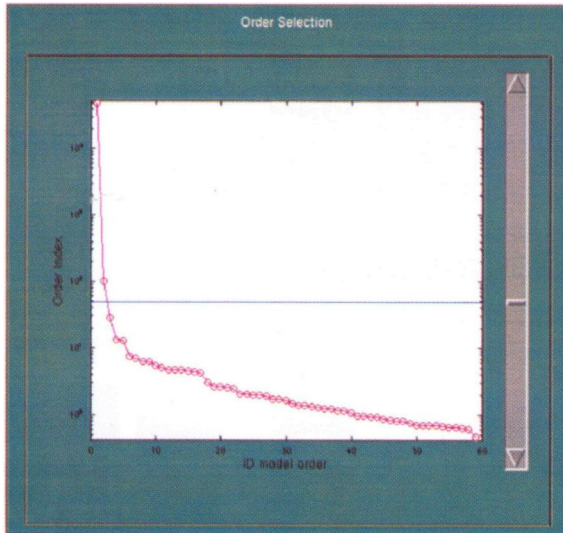

FIGURE 6   Model order selection. (See Color Plate IV at the back of the issue.)

TABLE III   Identified engine models

Engine speed	1150 (RPM)	1500 (RPM)
Discrete transfer function	$(0.0198z^2 - 0.1949z + 0.1562)/$ $(z^2 - 1.834z + 0.835)$	$(0.0567z^2 - 0.1888z + 0.2297)/$ $(z^2 - 1.732z + 0.7335)$
Continuous transfer function	$114 \times (3.2 \times 10^{-5}s^2 - 0.0159s + 1)/$ $((s/0.348 + 1)(s/15.88 + 1))$	$85.6 \times (6.4 \times 10^{-5}s^2 - 0.0147s + 1)/$ $((s/0.383 + 1)(s/26.95 + 1))$
Output sample period	0.0113	0.0113
PRBS generation period	0.113	0.113
Selected model order r	2	2
Selected parameter q	60	60

model with the same PRBS excitation as experimental one is drawn in the lower plot of Fig. 5 (black line). One can see that both responses are very close, indicating good system identification model.

Table III shows the two identified engine models for engine running at 1150 and 1500 RPM in both discrete and continuous time domains. The identified models are in discrete state space format, and then, transferred into the discrete transfer function format and continuous transfer function assuming zero order sampling. Consider the continuous time transfer function for engine running at 1150 RPM. The quadratic coefficient of the transfer function numerator is very close to zero. By setting it to zero we can rewrite the transfer function into the following format:

$$G(s) = 114 \times \frac{1 - s/62.9}{1 + s/15.88} \times \frac{1}{1 + s/0.348}. \qquad \text{(IV-2)}$$

It is clear that the transfer function consists of two first-order dynamic components. The first portion of dynamics $(1 - s/62.9)/(1 + s/15.88)$ is a non-minimum phase transfer function approximating the pure delay dynamics due to the diesel engine fuel system, and the second portion of dynamics $1/(1 + s/0.348)$ represents the engine mechanical dynamics from engine combustion torque to engine speed. From Table III one can see that the engine model DC gain decreases as the engine speed increases due to the quadratic increment of the engine friction torque. For the same reason, the time constant of the mechanical dynamics increases as engine speed. Figure 7 shows the step response of the
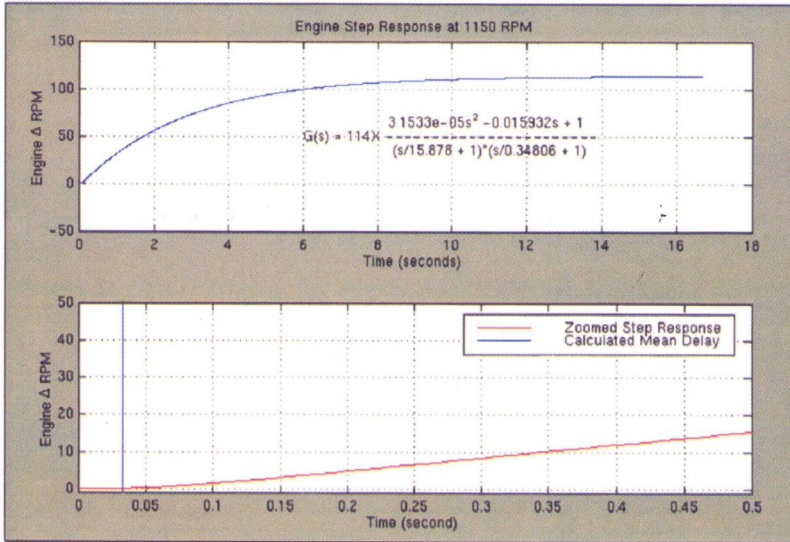
FIGURE 7 Step response and delay of identified model. (See Color Plate V at the back of the issue.)

engine transfer function at 1150 RPM. The lower plot of Fig. 7 shows a blow-up of the step response at the first 0.5 s, where the vertical line indicated the calculated fuel system delay at 1150 RPM and given fuel command. We can see it clearly that identified transfer function provides almost identical time delay. This indicates that the PRBS system identification is able to use a first-order transfer function with non-minimum phase numerator to represent a pure time delay dynamics.

As a summary of this section, the PRBS q-Markov Cover is successfully applied to a diesel engine system from the fuel command to the engine speed output. The identified transfer functions have order of two. The pure time delay of the diesel fuel system is approximated by a first-order transfer function with a non-minimum phase numerator. The identified engine model is a function of engine speed due to non-linear engine friction torque and fuel system delay. The identified engine model was used for designing engine idle speed governor using the OCC (Output Covariance Constraint, see Ref. [7]) control design method. The designed controller stabilizes the engine speed with proper response and over/under shoot performance at the first try.

V. CONCLUSIONS

This paper successfully extend the PRBS q-Markov Cover system identification technique to a weighted multirate case. The extension allows the construction of a discrete time state space model at system output sample rate while the PRBS excitation sample rate is slower than the output one. The PRBS q-Markov Cover is successfully applied to a diesel engine system from the fuel command to the engine speed. The identified engine model is a function of engine speed due to a nonlinear engine friction torque and fuel system delay. The identified engine model was used for designing a engine idle speed controller using the OCC Ref. [7] control design method. The designed controller stabilizes the engine speed with proper response and over/under shoot performance at the first try. This provides confidence of using identified engine models for engine governor design.

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