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Performance Analysis of Multi-Hop Wireless Packet Networks

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In this paper, a unified analytical framework for performance analysis of multi-hop wireless packet networks is developed. The effect of coupling between the hops on the degradation of the delay-throughput characteristics and the probability of blocking is investigated. The issue of hop decoupling is addressed.

Keywords: Wireless packet networks; multi-hop networks; hops decoupling

Classification Categories: 90B12, 90B15

1. INTRODUCTION

The goal of this paper is to present a unified, analytical framework for performance analysis of multi-hop wireless packet networks. Such networks may be viewed as a model for wireless communication networks where the users within a cluster communicate with each other directly through the common base and with the users from other clusters through the second hop (sub)network formed by the bases. Performance of such networks has been discussed for a long time. In particular, in [2], the maximum throughput of two-hop systems has been evaluated and design problems related to the number of bases have been addressed. In [3], the effect of routing in multi-hop networks with a regular structure, i.e., in grid network, has been studied. A model for performance analysis of transmission strategies in a multi-hop network, where each user has adjustable transmission radius, has been presented in [4]. In [5]–[6], the throughput-delay performance has been analyzed and its dependancy on the access control protocol and the storage capacity of the bases has been investigated for the ALOHA and CSMA systems. However, the process of packet arrivals to the bases was assumed to be a Bernoulli sequence. Thus, the performance of the two-hop system has been approximated by a single-hop network. In [7], a simple model of multi-hop network with CSMA input control has been presented and the throughput and blocking probability have been evaluated. In [8]–[9], the performance of multi-channel multi-hop lightwave networks has been studied under uniform and non-uniform traffic conditions.

In spite of these achievements, no unified performance analysis technique, applicable to a sufficiently broad class of multi-hop wireless networks, is at present available. This paper is intended to contribute to this end. Specifically, using the approach developed in [1], we present a technique for analysis of the coupling between various hops and, on this basis, investigate the resulting throughput-delay characteristics and the probability of blocking. In addition, we analyze the conditions under which the complete decoupling can be achieved.

The structure of this paper is as follows: In Section 2, the channel and the network under consideration are defined. In Section 3 and 4, respectively, the evolution equations are constructed and their asymptotic approximations are derived under the assumptions that the number of users in each hop is sufficiently large and the access control protocols are Markovian. In Section 5, the performance characteristics of multi-hop networks are analyzed, and in Section 6 the hop decoupling issues are addressed. The conclusions are formulated in Section 7.

2. THE CHANNEL AND THE NETWORK

A. The Channel

A wireless channel consisting of $2 \leq S < \infty$ hops (sub)channels is considered. It is assumed that each hop channel is formed by a noiseless collision feedforward subchannel and noiseless, collisionless feedback (acknowledgement) subchannel. In addition, for each k-th hop channel, k = 1,…, S, it is assumed that

(a) the channel propagation delay is zero;

(b) transmission of a packet across the feedforward channel requires a unit interval of time $[n, n + 1)$, $n = 0, 1,\ldots,$ where time is slotted with a slot duration equal to 1;

(c) the feedback signal at time $n + 1$ is received by all users that have been active during the time slot $[n, n + 1)$, $n = 0, 1,\ldots$

B. The Network

The network is assumed to be formed by S hop (sub)networks, each operating in a particular hop (sub)channel. The term "a user in the k-th hop network", k = 1,…, S, is used to imply "a base in the k-th hop network." In addition, for each k-th hop network, k = 1,…, S, it is assumed that

(a_1) the k-th hop network has M_{k+1} clusters of users which are associated with M_{k+1} users in the (k + 1)-th hop network, respectively, where $M_{S+1} = 1$, and the k-th hop network associated with a user in the (k + 1)-th hop network consists of $1 << M_k < \infty$ users communicating through the collision channel, introduced above;

(b_1) every user in the k-th hop network has a buffer capable of storing $1 \leq N_k < \infty$ packets;

(c_1) the input traffic to the first hop network is a Bernoulli sequence with a parameter p_a^1, i.e., during each time slot [n, n + 1), every user in the first hop network generates a packet with probability p_a^1. The interhop traffic between the k-th and the (k + 1)-th hop networks is defined by the parameter $0 \leq p_a^k \leq 1, k = 2,\ldots, S$, where p_a^k is the probability that a packet in the buffer of the k-th hop user is, in fact, an interhop packet. Since only one packet can be successfully transmitted across the feedforward channel during a time slot, to avoid triviality, it is assumed that $M_k p_a^k \leq \alpha$, $k = 1,\ldots, S$, where α is a constant of order 1;

(d_1) if a packet is generated by a user in the first hop network having its buffer full, the newly generated packet is rejected. Otherwise, it is stored in the first available cell of the buffer. If a positive acknowledgment is received by a user in the k-th hop network, k = 2,…, S, at time $n + 1$, the packet transmitting during $[n, n + 1)$ is eliminated from the first cell of its buffer. Otherwise, it is stored and attempted for transmission at a later time, according to an access control protocol of the k-th hop network. Transmissions of packets in the k-th hop network associated with a user in the (k + 1)-th hop network don't interfere with transmission of packets in any of other networks;

(e_1) a packet which has been successfully transmitted in the k-th hop network during $[n, n+1)$ is stored in a buffer of the (k + 1)-th hop network to decide whether it is to be relayed or not. The packet which is not to be relayed leaves the buffer of the (k + 1)-th hop network at the end of time slot $[n, n+1)$. Thus, the successfully transmitted packet in the k-th hop network joins the (k + 1)-th hop network with probability p_a^{k+1};

(f_1) if a packet is eliminated from the first cell of a buffer in the k-th hop network, a packet stored in its mth cell, $m = 2,\ldots, N_k$, instantaneously moves down to be stored in cell $m - 1$. Thus, a higher cell can't be occupied if one of the lower cell is empty.

3. EVOLUTION EQUATIONS

Let $h_i^k(n)$, $i = 1,\ldots, N_k$, $k = 1, 2,\ldots, S$, be the occupancy of the i-th layer of buffers in the k-th hop network associated with a user in the (k + 1)-th hop network at time n. Due to the assumptions formulated above, the evolution of h_i^k can be characterized as follows:

$$h_i^k(n+1) = h_i^k(n) + \zeta_i^k(n, n+1) - \psi_{i,i-1}^k(n+1) + \psi_{i+1,i}^k(n+1), \tag{1}$$

$$i = 1,\ldots, N_k,\ k = 1,\ldots, S,$$

$$\psi_{N_k+1,N_k}(n) = 0,\ \forall\, n, k,\ M_k \geq h_1^k \geq \cdots \geq h_{N_k}^k,\ \forall\, k.$$

While the distributions of $\zeta_i^1(n, n+1)$, $i = 1,\ldots, N_1$, and $\psi_{i,i-1}^k(n+1)$, $i = 2,\ldots, N_k$, $k = 1,\ldots, S$, remain same as in [1], the distributions of $\psi_{1,0}^k(n+1)$, $k = 1,\ldots, S$, and $\zeta_i^k(n, n+1)$, $i = 1,\ldots, N_k$, $k = 2,\ldots, S$, are different.

The sequence of $\psi_{1,0}^k(n+1)$, $n = 0, 1,\ldots,$ represents the process of successful transmission in the k-th hop network associated with a user in the (k + 1)-th hop network during $[n, n+1)$. The event $\psi_{1,0}^{k}(n+1) = 1$ means that a successful transmission has occurred during $[n, n+1)$ in the k-th hop network associated with a user in the (k + 1)-th hop network. The conditional probability distribution of $\psi_{1,0}^{k}(n+1)$ can be represented as follows:

$$P[\psi_{1,0}^{k}(n+1) = 1 | h_i^k(n), \forall i \in \{1, N_k\}, k \in \{1,..., S\}]$$

$$= F_1^k(h_1^k(n))\,(1 - \frac{h_{N_{k-1}}^{k+1}(n)}{M_{k+1}}),\; h_{N_{S-1}}^{S+1}(n) = 0, \forall n, \tag{2}$$

where function $F_1^k(\cdot)$ is defined by the Markovian access control protocol employed in the k-th hop network. Consider the ALOHA whereby every busy user attempts a transmission at the beginning of each time slot with probability p_k and the TDMA whereby at every n, a user chosen randomly and equiprobably, is given the right to transmit a packet during $[n, n+1)$. Then

$$F_1^k(h_1^k(n)) = \begin{cases} h_1^k(n) p_k (1-p_k)^{h_1^k(n)-1} & \text{for ALOHA,} \\ \dfrac{h_1^k(n)}{M_k} & \text{for TDMA.} \end{cases}$$

The sequence of $\zeta_i^k(n, n+1)$, $n = 0, 1, 2, \ldots$, represents the process of packet arrivals into the i-th layer of buffers in the k-th hop network associated with a user in the (k + 1)-th hop network during $[n, n+1)$. The event $\zeta_i^k(n, n+1) = l$, $l = 0, 1, \ldots, M_K$, means that l packets have arrived into the i-th layer of buffers in the k-th hop network associated with a user in the (k + 1)-th hop network during $[n, n+1)$. Due to assumptions (c_1) and (d_1),

$$P[\zeta_i^k(n, n+1) = l | \ldots] = \binom{h_{i-1}^k(n) - h_i^k(n)}{l} [F_1^{k-1}(h_1^{k-1}(n)) p_a^k]^l$$

$$\cdot [1 - F_1^{k-1}(h_1^{k-1}(n)) p_a^k]^{h_{i-1}^k(n) - h_i^k(n) - l}, \tag{3}$$

$$l = 0, 1,..., h_{i-1}^k(n) - h_i^k(n),\; i = 1,..., N_k,\; k = 1,..., S,$$

$$h_0^k(n) = M_k, F_1^0(h_1^0(n)) = 1, \forall n.$$

Thus, the performance of multi-hop network is described by the non-linear, stochastic, finite-difference equations (1)–(3). Analysis of these equations is, in general, prohibitively difficult. However, based on the assumption (a_1) which states, essentially, that the number of users in each hop is large, an asymptotic technique of [1] can be employed in order to obtain a relatively simple and complete analysis procedure.

4. ASYMPTOTIC APPROXIMATIONS

In terms of the normalized variables $x_i^k = h_i^k/M_k$, equation (1) can be rewritten as

$$x_i^k(n+1) = x_i^k(n) + \frac{1}{M_k}[\zeta_i^k(n, n+1) - \psi_{i,i-1}^k(n+1) + \psi_{i+1,i}^k(n+1)], \tag{4}$$
$$i = 1,\ldots, N_k, k = 1,\ldots, S,$$

$$\psi_{N_k+1,N_k}^k(n+1) = 0, \forall n, k, 1 \geq x_1^k \geq x_2^k \geq \cdots \geq x_{N_k}^k, \forall k.$$

To simplify this equation, we use the averaging theory of [1]. First we show that the conditions of Theorem 1 of [1] are met. Indeed, since

$$E[\zeta_i^k(n, n+1)|\ldots] = M_k p_a^k(x_{i-1}^k(n) - x_i^k(n))F_1^{k-1}(h_1^{k-1}(n)), \forall k \in \{2,\ldots, S\},$$
$$Var[\zeta_i^k(n, n+1)|\ldots] = M_k p_a^k(x_{i-1}^k(n) - x_1^k(n))F_1^{k-1}(h_1^{k-1}(n))$$
$$\cdot (1 - p_a^k F_1^{k-1}(h_1^{k-1}(n))), \forall k \in \{2,\ldots, S\}$$

and since $M_k p_a^k$, $k = 1,\ldots, S$, is of the order 1, Theorem 1 of [1] is applicable. Second, using this Theorem, we construct the asymptotic approximation of (1)–(3) in the form:

$$y_i^k(n+1) = y_i^k(n) + \epsilon\alpha_k[(y_{i-1}^k(n) - y_i^k(n))M_{kp}p_a^kF_1^{k-1}(y_1^{k-1}(n))$$
$$- \frac{y_i^k(n) - y_{i+1}^k(n)}{y_1^k(n)} \cdot F_1^k(y_1^k(n))(1 - y_{N_{k-1}}^{k+1}(n))], \tag{5}$$

$$i = 1,\ldots, N_k,\ k = 1,\ldots, S,\ \epsilon\alpha_k = \frac{1}{M_k},\ y_0^k(n) = 1,\ y_{N_k+1}^k(n) = 0,\ \forall n, k,$$

$$y_i^{S+1}(n) = 0,\ \forall i, n,\ y_i^k \in [0, 1],\ 1 \geq y_1^k \geq y_2^k \geq \cdots \geq y_{N_k}^k,\ \forall k,\ F_1^0(y_1^0(n)) = 0,\ \forall n,$$

where, due to (a_1), $0 < \epsilon << 1$ and α_k, $k = 1,\ldots, S$, is independent of ϵ. The simplified equation (5) constitutes the basis of the analysis that follows.

Remark 1. Comparing the averaged equation (5) with the corresponding equation for the single-hop network (equation (15) of [1]), we see that the only effect of the multi-hop architecture is in the coupling term represented by the expression in the last parenthesis of the right hand side of (5). This term describes the influence of the (k + 1)-th hop on the behavior of the k-th hop. The effect of this coupling on the performance characteristics of the multi-hop network is described next.

5. PERFORMANCE OF MULTI-HOP NETWORKS

To simplify the presentation, we concentrate below on two-hop networks; generalizations for S-hop networks are straightforward. For specific examples, we consider ALOHA and TDMA access control protocols, in particular, we study the ALOHA-TDMA and TDMA-TDMA architectures. Although it may not have a practical significance, for methodological reasons we illustrate also the TDMA-ALOHA architecture. (In all these architectures, the first access control protocol is used in the first hop of the network and the second protocol is employed in the second hop.)

Steady States.

Let y_{is}^k denote the steady state value of the averaged normalized occupancy of the i-th layer of buffers in the k-th hop network associated with a user in the (k + 1)-th hop network, $i = 1,\ldots, N_k$, $k = 1, 2$. Then from system (5) and Theorem A of [1] we obtain:

$$y^1_{is} = y^1_{1s} \cdot \frac{y^1_{(i-1)s} - y^1_{N_1 s}}{1 - y^1_{N_1 s}}, \; i = 1, \ldots, N_1, \tag{6}$$

$$y^2_{is} = y^2_{1s} \cdot \frac{y^2_{(i-1)s} - y^2_{N_2 s}}{1 - y^2_{N_2 s}}, \; i = 1, \ldots, N_2,$$

where y^1_{1s}, $y^1_{N_1 s}$, y^2_{1s} and $y^2_{N_2 s}$ are related by

$$(1 - y^1_{N_1 s}) M_1 p^1_a = F^1_1(y^1_{1s})(1 - y^2_{N_2 s}), \tag{7}$$

$$(1 - y^2_{N_2 s}) M_2 p^2_a F^1_1(y^1_{1s}) = F^2_1(y^2_{1s}).$$

or, in other words, by

$$(1 - y^1_{N_1 s}) M_1 p^1_a = \frac{1}{M_2 p^2_a} F^2_1(y^2_{1s}). \tag{8}$$

Remark 2. As in the case of a single-hop network [1], $y^1_{N_1 s}$ can be represented as a function of y^1_{1s}. Specifically, from (6) we obtain:

$$y^1_{N_1 s}(y^1_{1s}) = \begin{cases} \dfrac{y^1_{1s} + 1}{2} - \dfrac{\sqrt{1 + 2y^1_{1s} - 3(y^1_{1s})^2}}{2} & \text{for } N_1 = 2, \\ \approx (y^1_{1s})^{10} & \text{for } N_1 = 10. \end{cases}$$

Analogous expressions could be derived for $y^2_{N_2 s}(y^2_{1s})$. These functions are illustrated in Fig. 1.

Remark 3. The steady state value y^2_{1s} can also be represented as a function of y^1_{1s}. Specifically, from (6) and (7) it follows that

$$y^2_{1s} = \begin{cases} {}^{1}\;{}^{2}[1 - DM_1 p^1_a + \sqrt{(1 - DM_1 p^1_a)(1 - 3DM_1 p^1_a)}] & \text{for } N_2 = 2, \\ (1 - DM_1 p^1_a)^{\frac{1}{10}} & \text{for } N_2 = 10, \end{cases}$$

where

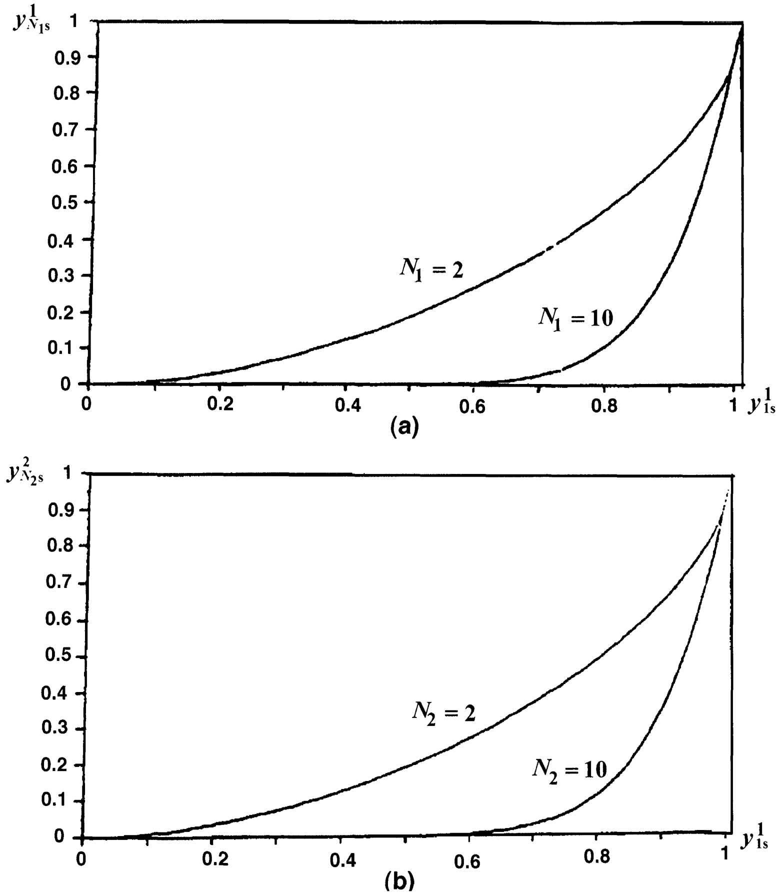

FIGURE 1 (a) Relationship between $y^1_{N_1s}$ and y^1_{1s}. (b) Relationship between $y^2_{N_2s}$ and y^2_{2s}.

$$D = \begin{cases} \left[\dfrac{1 - y^1_{1s}}{2} + \dfrac{\sqrt{1 + 2y^1_{1s} - 3(y^1_{1s})^2}}{2}\right] / F^1_1(y^1_{1s}) & \text{for } N_1 = 2, \\ [1 - (y^1_{1s})^{10}]/F^1_1(y^1_{1s}) & \text{for } N_1 = 10. \end{cases} \tag{9}$$

Function $y^2_{1s}(y^1_{1s})$ is illustrated in Fig. 2.

Remark 4. In terms of the single variable y^1_{1s}, equation (8) becomes:

$$[1 - y^1_{N_1s}(y^1_{1s})]M_1 p^1_a M_2 p^2_a = F^2_1(y^2_{1s}(y^1_{1s})). \tag{10}$$

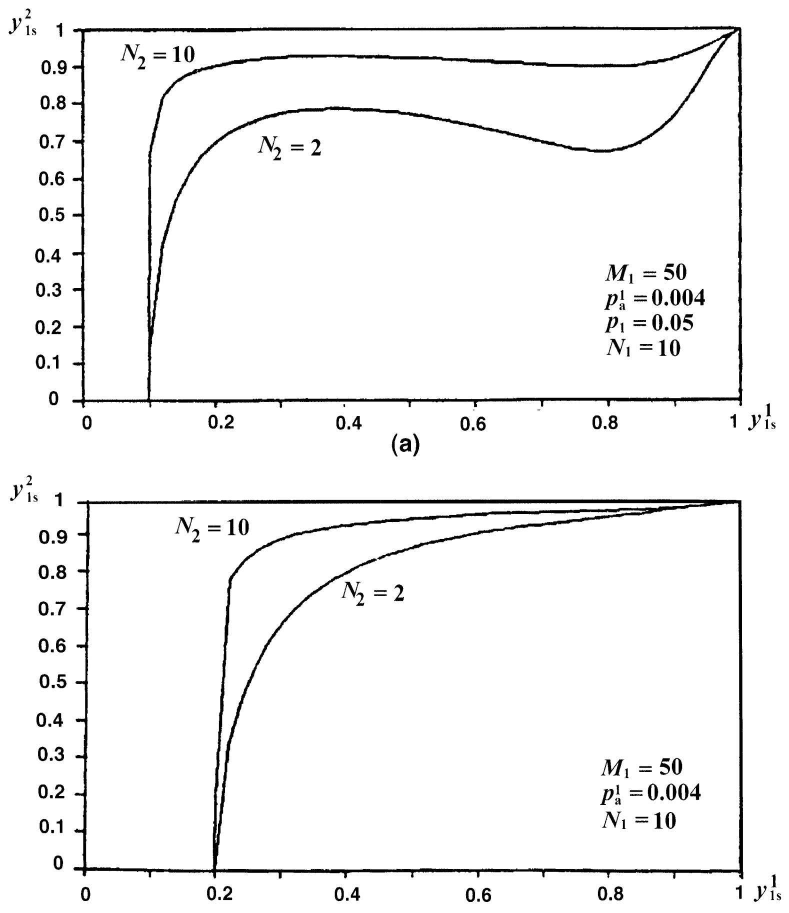

FIGURE 2 (a) Relationship between y^2_{1s} and y^1_{1s} with ALOHA in the first hop. (b) Relationship between y^2_{1s} and y^1_{1s} with TDMA in the first hop.

The left hand side of (10) is referred to as a load line and the right hand side of (10) is referred to as a transmission line. Thus, every intersection of the load line and the transmission line defines a steady state of the network y^1_{1s}. The load and the transmission lines of two-hop networks with ALOHA-TDMA, TDMA-TDMA and TDMA-ALOHA architectures are illustrated in Fig. 3. (Compare with Fig. 4 of [1].)

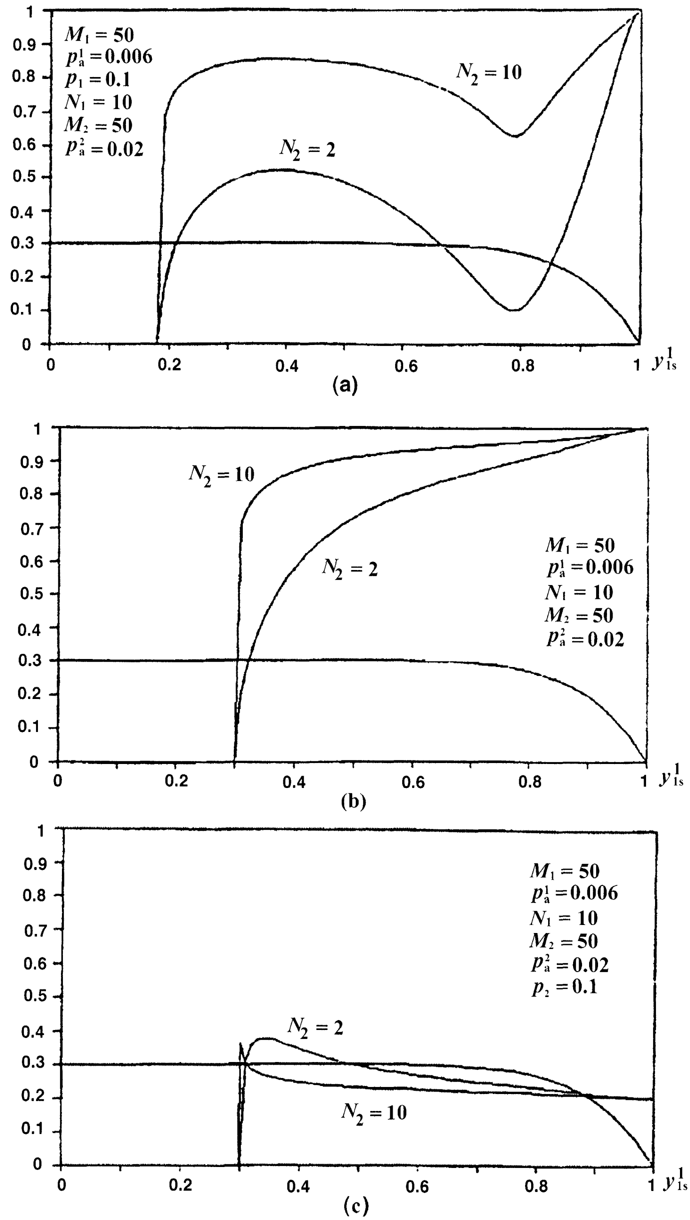

FIGURE 3 (a) The load and transmission lines for ALOHA-TDMA architecture. (b) The load and transmission lines for TDMA-TDMA architecture. (c) The load and transmission lines for TDMA-ALOHA architecture.

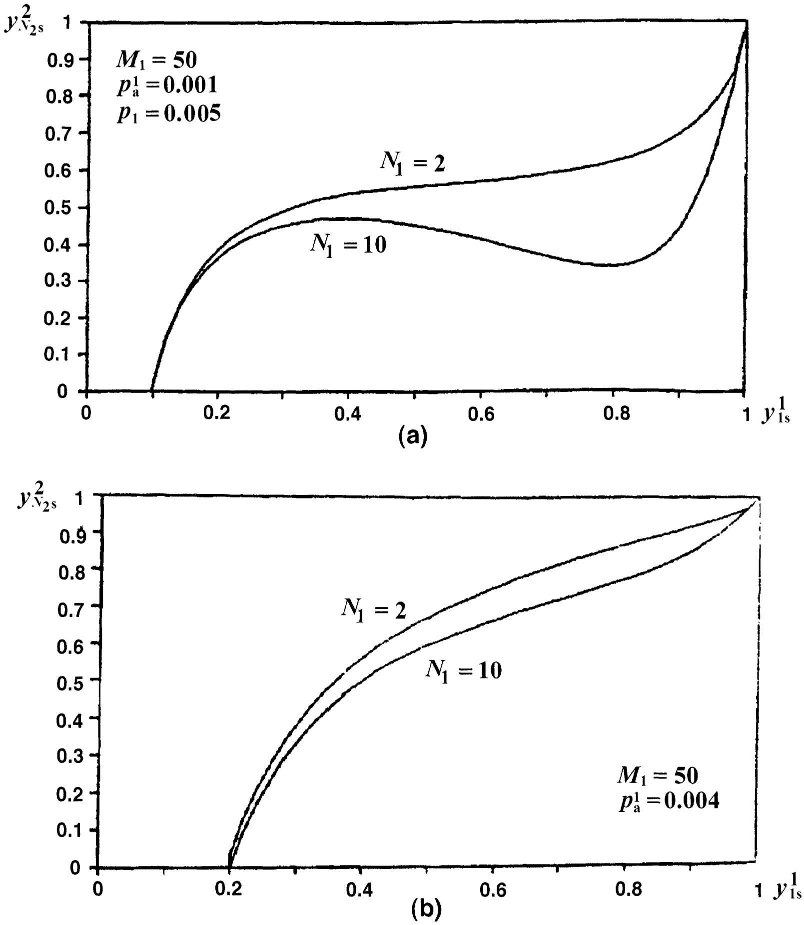

FIGURE 4 (a) Relationship between $y^2_{N_2s}$ and y^1_{1s} with ALOHA in the first hop. (b) Relationship between $y^2_{N_2s}$ and y^1_{1s} with TDMA in the first hop.

Local Stability of the Equilibria.

The slope of the load line and the transmission line at the intersection defines the stability properties, i.e., Theorem B of [1] holds.

Steady State Performance Characteristics.

If equation (10) has a unique solution then, as it follows from Theorem A of [1], there exists M_0, such that for $M_1 \geq M_0$, $M_2 \geq M_0$, the two-hop network has the following performance characteristics:

$$TP^1 = F_1^1(y_{1s}^1)\,(1 - y_{N_2s}^2(y_{1s}^1)),$$

$$TP^2 = F_1^2(y_{1s}^2(y_{1s}^1)),$$

where y_{1s}^1 is the solution of (10).

Remark 5. From (7), the relationship between $y_{N_2s}^2$ and y_{1s}^1 can be found to be:

$$y_{N_2s}^2 = 1 - \frac{F_1^2(y_{1s}^2)}{M_2 p_a^2 F_1^1(y_{1s}^1)},$$

where y_{1s}^2 and $y_{N_2s}^2$ are related by (6). Examples of this relationship are illustrated in Fig. 4. Using this relationship, $TP^1(y_{1s}^1)$ can be represented as shown in Fig. 5, where the corresponding throughput characteristic of a single hop network is also shown for comparison purposes. Note that $TP^2(y_{1s}^1) = M_2 p_a^2 TP^1(y_{1s}^1)$.

(ii) The steady state time delay of the first and the second hops, TD^1 and TD^2, respectively, are

$$TD^1 = \frac{M_1 y_s^1}{F_1^1(y_{1s}^1)\,(1 - y_{N_2s}^2(y_{1s}^1))},$$

$$TD^2 = \frac{M_2 y_s^2}{F_1^2(y_{1s}^2(y_{1s}^1))},$$

where $y_s^1 = \sum_{i=1}^{N_1} y_{is}^1$, $y_s^2 = \sum_{i=1}^{N_2} y_{is}^2$. Functions $TD^1(y_{1s}^1)$ and $TD^2(y_{1s}^1)$ are illustrated in Fig. 6.

(iii) The probability of blocking (packet rejection) of the first and the second hops, PB^1 and PB^2, are

$$PB^1 = y_{N_1s}^1(y_{1s}^1),$$

$$PB^2 = y_{N_2s}^2(y_{1s}^1).$$

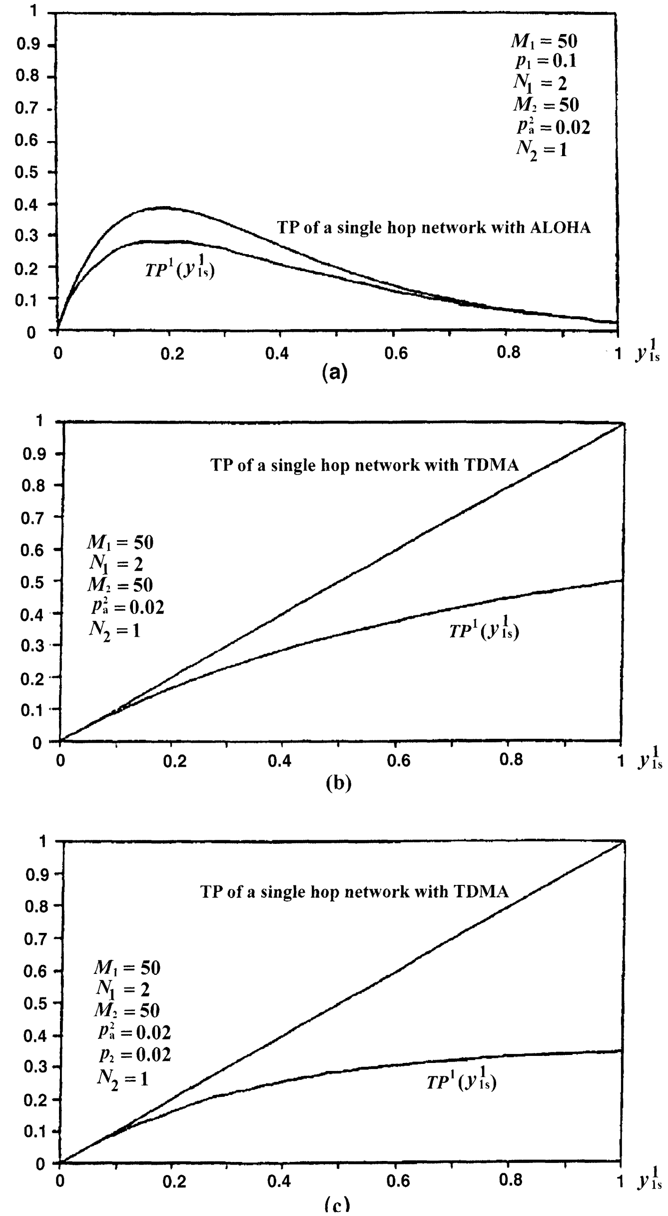

FIGURE 5 (a) The first hop throughput in the ALOHA-TDMA architecture. (b) The first hop throughput in the TDMA-TDMA architecture. (c) The first hop throughput in the TDMA-ALOHA architecture.

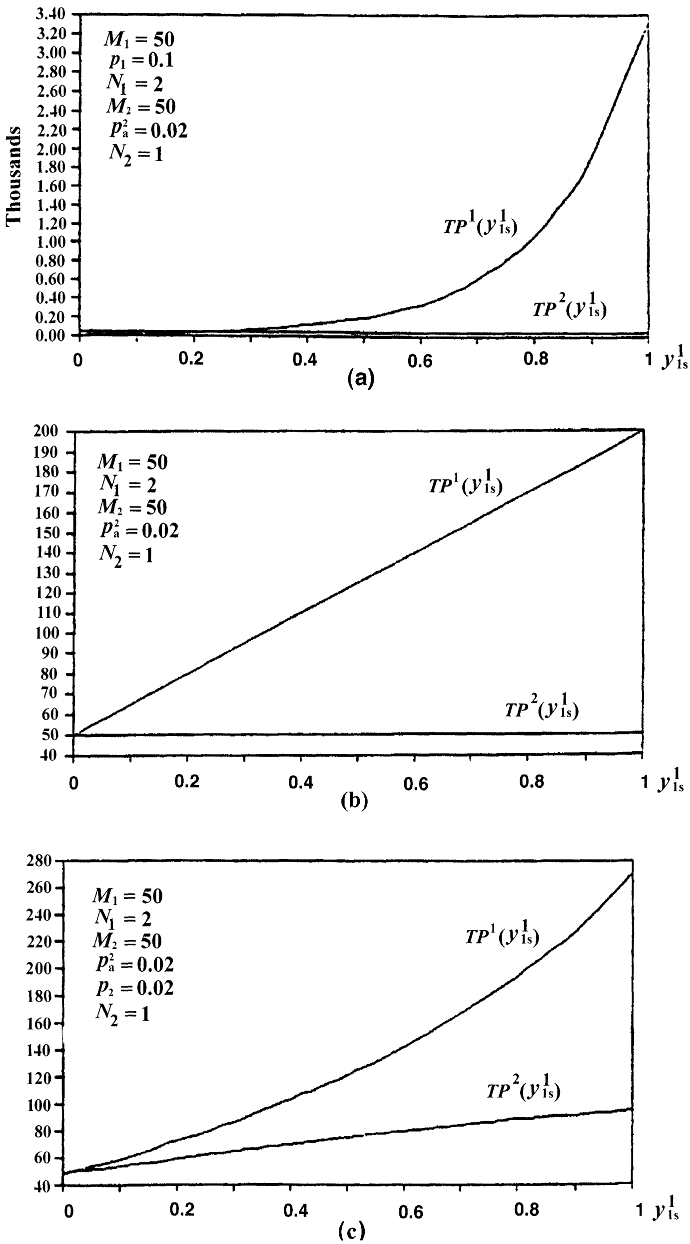

FIGURE 6 (a) Time delay in the ALOHA-TDMA architecture. (b) Time delay in the TDMA-TDMA architecture. (c) Time delay in the TDMA-ALOHA architecture.

These probabilities are illustrated in Fig. 7.

Remark 6. If equation (10) has multiple solutions, global analysis of TP^1, TP^2, TD^1, TD^2, PB^1 and PB^2 can be conducted using the large deviations approach as in [1].

6. DECOUPLING ISSUES

As it follows from the results described above, the coupling between the hops degrades the first hop performance as compared with a single hop system. Therefore, the issue of hops decoupling is of importance. It is intuitively appealing to think that if the buffer capacity of the second hop users tends to infinity, the coupling effect diminishes to zero. Unfortunately, this is not always true: under a heavy interhop traffic, the coupling persists even when $N_2 \to \infty$. To show this, consider first the ALOHA-TDMA and TDMA-TDMA architectures. From (7) it follows that

$$y^2_{N_2s} \approx \frac{M_2 p_a^2 F_1^1(y^1_{1s}) - (y^2_{N_2s})^{1/N_2}}{M_2 p_a^2 F_1^1(y^1_{1s})}, N_2 \geq 10.$$

Let $k = M_2 p_a^2 F_1^1(y^1_{1s})$. Since $0 \leq y^2_{N_2s} \leq 1$, $(y^2_{N_2s})^{1/N_2} \to \min\{1, k\}$ as $N_2 \to \infty$. Thus as $N_2 \to \infty$,

$$y^2_{N_2s} \to \begin{cases} 0, & \text{if } k \leq 1, \\ \frac{k-1}{k}, & \text{otherwise.} \end{cases}$$

Since $\lim_{N_2 \to \infty} y^2_{N_2s} \neq 0$ for $k > 1$, the probability of blocking does not tend to 0 and, therefore, the coupling between the first and the second hops persists. Obviously, $k > 1$ if the interhop traffic is sufficiently high. For the TDMA-ALOHA architecture we write:

$$y^2_{N_2s} \approx \frac{M_2 p_a^2 F_1^1(y^1_{1s}) - M_2 p_2 (y^2_{N_2s})^{1/N_2} (1-p_2)^{M_2 (y^2_{N_2s})^{1/N_2} - 1}}{M_2 p_a^2 F_1^1(y^1_{1s})}, N_2 \geq 10.$$

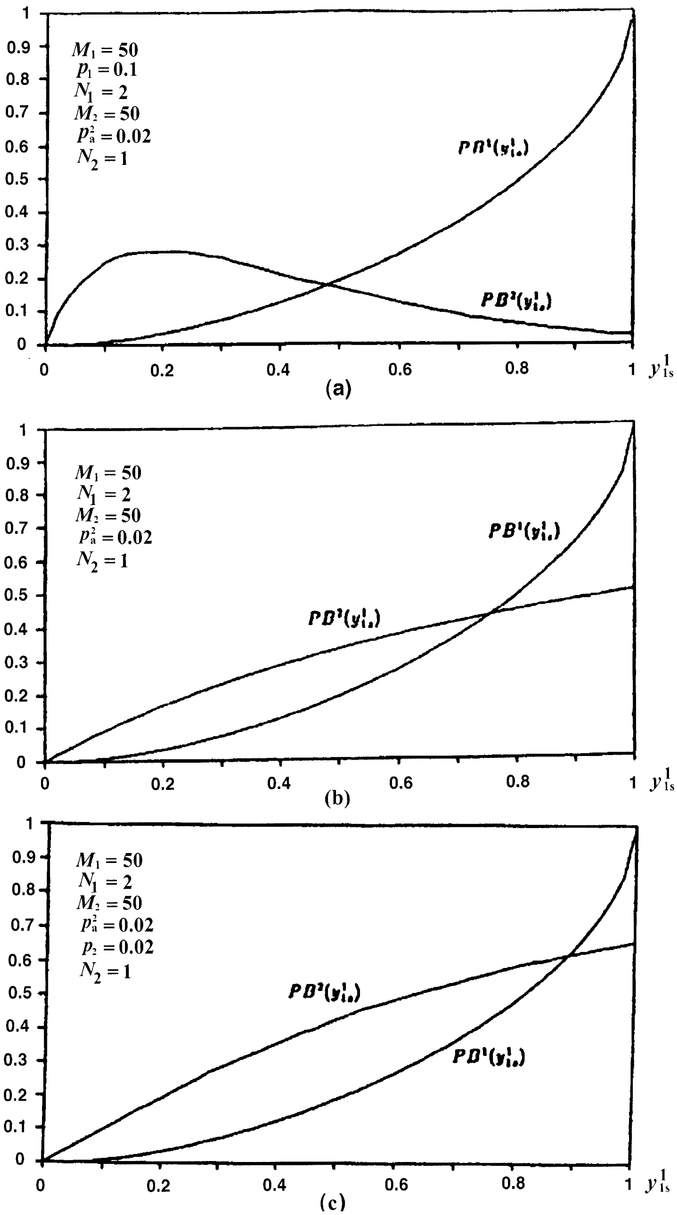

FIGURE 7 (a) Probability of blocking in the ALOHA-TDMA architecture. (b) Probability of blocking in the TDMA-TDMA architecture. (c) Probability of blocking in the TDMA-ALOHA architecture.

With large M_2,

$$y^2_{N_2s} \approx \frac{M_2 p^2_a F^1_1(y^1_{1s}) - Ge^{-G}}{M_2 p^2_a F^1_1(y^1_{1s})}, N_2 \geq 10,$$

where $G = M_2 p_2 (y^2_{N_2s})^{1/N_2}$. From here, as $N_2 \to \infty$,

$$y^2_{N_2s} \to \begin{cases} 0, & if\, k \leq M_2 p_2 e^{-M_2 p_2}, \\ \frac{k-1}{k}, & \text{otherwise.} \end{cases}$$

Thus, in each of the architectures, the first and second hop can be decoupled by large buffers only if the interhop traffic is sufficiently light, i.e.,

$$k \leq Q = \begin{cases} 1, & \text{for the ALOHA(TDMA)-TDMA architecture,} \\ e^{-1}, & \text{for the TDMA-ALOHA architecture,} \end{cases}$$

since $\max\{M_2 p_2 e^{-M_2 p_2} : 0 \leq p_2 \leq 1, 1 << M_2 < \infty\} = e^{-1}$. It is expected that if the packet arrival rate in the k-th hop network is higher than the throughput of the k-th hop network, then all the buffers of the k-th hop network will be utilized no matter what the buffer size is. Fig. 8 illustrates the strength of coupling as a function of N_2 and Fig. 9 shows the maximal achievable throughput in the first hop of the network with $N_2 = \infty$.

When the decoupling based on long buffers fails, the decoupling based on the introduction of the third hop may prove to be efficient. Indeed, if the second hop is partitioned into two hops, the smaller second hop and the new third hop, equations (7) take the form

$$(1 - y^1_{N_1s}) M_1 p^1_a = F^1_1(y^1_{1s})(1 - y^2_{N_2s}),$$

$$(1 - y^2_{N_2s}) M^*_2 p^2_a F^1_1(y^1_{1s}) = F^2_1(y^2_{1s})(1 - y^3_{N_3s}),$$

$$(1 - y^3_{N_3s}) M_3 p^3_a F^2_1(y^2_{1s}) = F^3_1(y^3_{1s})$$

where $M_2 = M_3 M^*_2$, and $p^3_a < p^2_a$. If M_3 is chosen so that

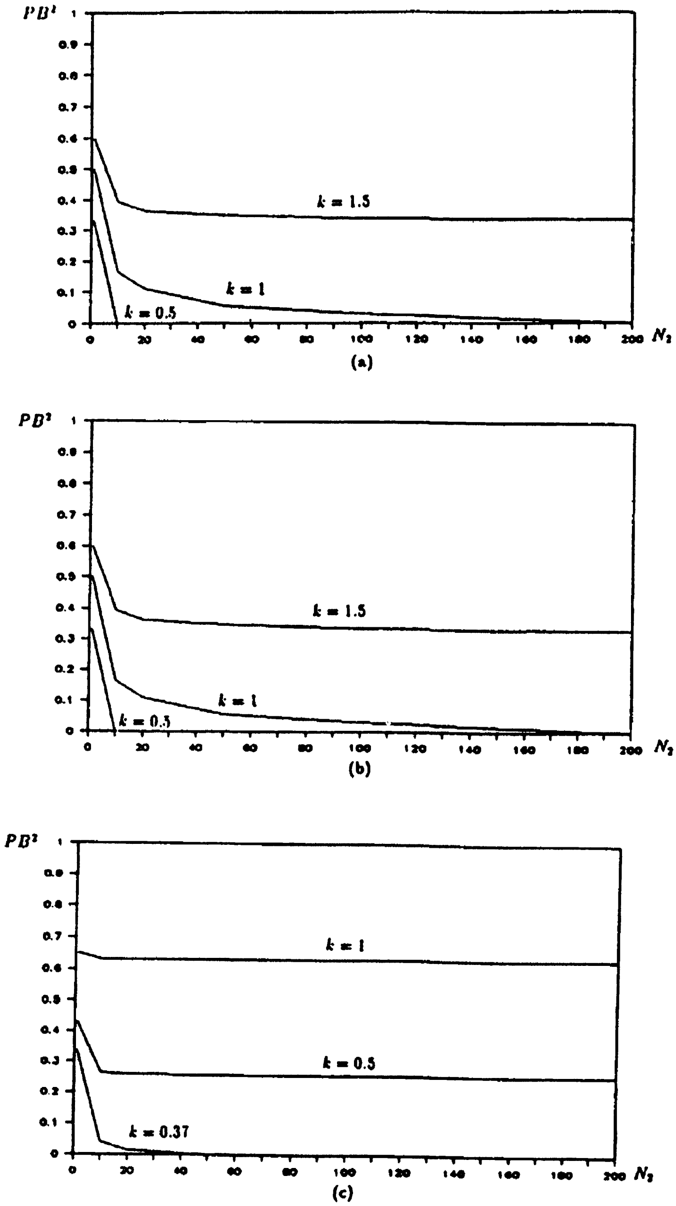

FIGURE 8 (a) Probability of blocking in the second hop in the ALOHA-TDMA architecture. (b) Probability of blocking in the second hop in the TDMA-TDMA architecture. (c) Probability of blocking in the second hop in the TDMA-ALOHA architecture.

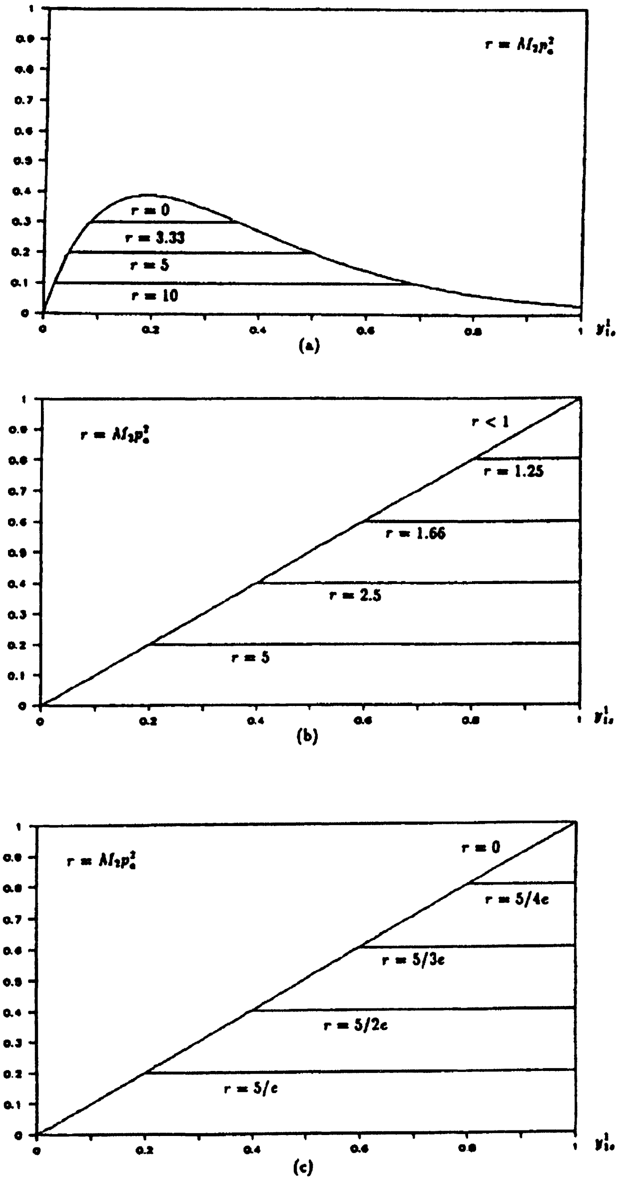

FIGURE 9 (a) Maximum achievable throughput in the first hop in the ALOHA-TDMA architecture. (b) Maximum achievable throughput in the first hop in the TDMA-TDMA architecture. (c) Maximum achievable throughput in the first hop in the TDMA-ALOHA architecture.

$$M_2^* p_a^2 F_1^1(y_{1s}^1) \leq Q, \tag{11}$$

$$M_3 p_a^3 F_1^2(y_{1s}^2) \leq Q,$$

all three hops can be decoupled by sufficiently large N_2 and N_3. This can be viewed as a justification for having a higher hop networks, other than the limitations of the transmission power of the users.

7. CONCLUSIONS

In this paper, a unified, analytical framework for performance analysis of multi-hop wireless networks with Markovian access control protocols is developed. For two-hop networks, the method developed can be utilized as follows:

(a) Find the intersections of the load and transmission lines (10). These intersections define the number and the position of the steady states as well as their stability properties.

(b) If the intersection is unique, the steady state performance characteristics are calculated according to (i)–(iii).

(c) If the intersection is not unique, estimate the residence time, as in [1], and the evaluate the global (average) performance characteristics as in Corollary 2 of [1].

Applied to the problem of hops decoupling, the method may be used as follows:

(α) If the intensity of the interhop traffic is light, utilize the buffer size in the second hop large enough to decouple the network. Calculate the decoupled throughput-delay and probability of blocking characteristics using Corollaries 1 and 2 of [1].

(β) If the intensity of the interhop traffic is heavy, introduce the third hop network and choose the number of bases in the third hop so as to satisfy (11). Choose N_2 and N_3 large enough and evaluate the resulting (decoupled) performance characteristics using Corollaries 1 and 2 of [1].

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