Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2007, Article ID 98532, 9 pages
doi:10.1155/2007/98532

Research Article

Orbital Maneuvers Using Low Thrust to Place a Satellite in a Constellation

Vivian Martins Gomes, Antonio Fernando Bertachini de Almeida Prado, and Helio Koiti Kuga

Received 19 April 2006; Revised 20 September 2006; Accepted 19 October 2006

Recommended by José Manoel Balthazar

This paper considers the problem of low thrust suboptimal maneuvers to insert a satellite in a constellation. It is assumed that a satellite constellation is given with all the Keplerian elements of the satellite members having known values. Then, it is necessary to maneuver a new satellite from a parking orbit to its position in the constellation. The control available to perform this maneuver is the application of a low thrust to the satellite and the objective is to perform this maneuver with minimum fuel consumption.

1. Introduction

The problem of orbital maneuvers is very important in astronautics and it is under investigation for a long time. Looking in the more recent literature, there are important papers, like the one written by Stewart and Melton [1], that presented a multivariable perturbation solution to the equations of a vehicle moving under the influence of constant low-level thrust, using a fixed-angle steering law and moving in an inverse-square gravitational field, in a single plane. Comparisons with direct numerical integrations show relatively low errors and sizeable advantages in speed for the analytical approximation.

After that, Geffroy and Epenoy [2] presented a study of optimal low-thrust transfers based on the use of averaging techniques. Both minimum time and fuel-saving strategies using nonconstant acceleration are investigated to optimize the trajectory between arbitrary distant orbits. Optimal low-thrust rendezvous are treated in the same way. In each case, several constraints are included, either environmental such as Earth oblateness and shadow effect or technological such as thrust direction limits. The different problems are treated by a generalized averaging method and for each of them a theoretical justification of the approximation is discussed.

Kluever and Oleson [3] developed a method for computing near optimal minimum time Earth orbit transfers for solar electric propulsion spacecraft. A direct optimization approach is used to solve the optimal control problem. The thrust direction time history is parameterized by combining three extremal feedback control laws and the optimal thrust steering is obtained by computing the optimal weights for each feedback law. The optimal trajectories computed via the direct approach exhibit a very close match with optimal trajectories computed via the calculus of variations approach.

Kechichian [4] analyzed the problem of minimum time orbit transfer using intermediate acceleration and using both accurate integration and averaging. Continuous constant accelerations of the order of 10^{-2} g are considered for applications using nuclear propulsion upper stages. The acceleration vector is optimized in direction with its magnitude held constant throughout the flight.

In Herman and Spencer's work [5], it is used a trajectory optimization technique based upon higher-order collocation to solve optimal, low-thrust, Earth orbit transfer problems. A comparison is made between the optimal transfers found in this work and the transfer found by using analytical blended control methods.

Kiforenko et al. [6] did a study about the problem of minimum time low-thrust transfer arbitrary elliptic orbits in the central gravitational field. The rocket engine is assumed to produce constant in magnitude and controllable in direction thrust, which is essentially smaller than the gravity force. Approximate solution is obtained with the use of the averaging method. Simple control algorithms for practical realization are proposed and their efficiency is compared with that of the optimal control.

In [7], Haberkorn et al. described the study of an Earth orbital transfer with a low-thrust propulsion system. The objective is the maximization of the final mass, which leads to a discontinuous control with a large number of thrust arcs. The method to solve this problem is based on single shooting, combined with a homotopic approach in order to cope with the problem of the initial guess, which is actually critical for nontrivial problems.

In the present paper, based on previous papers from one of the authors (Prado [8], Prado and Rios-Neto [9]), where a detailed analysis of the alternatives available in the literature in the problem of performing orbital maneuvers with low thrust was made; and in bibliographical studies to consider papers written after that, the suboptimal parameterization method is selected to be implemented and used to simulate the maneuvers required by a satellite to be inserted in a constellation. The reason for this choice is the simple implementation, in terms of hardware for the satellite, together with the fact that the results are very close to the optimal approach, as shown in the references cited above.

In this problem, the spacecraft is supposed to be in Keplerian motion controlled only by the thrusts, whenever they are active. This hypothesis is not very restrictive because you can consider an osculating orbit just before the maneuver, and the duration of the maneuver is not too long to have important perturbations on the satellite during the transfer. This means that there are two types of motion.

(i) A Keplerian orbit, that is an orbit obtained by assuming that the Earth's gravity (assumed to be a point of mass) is the only force acting on the spacecraft. This motion occurs when the thrusts are not firing.

(ii) The motion governed by two forces: the Earth's gravity field (also assumed to be a point of mass) and the force delivered by the thrusts. This motion occurs during the time the thrusts are firing.

The thrusts are assumed to have the following characteristics.

(i) Fixed magnitude: the force generated by them is always of constant magnitude during the maneuver. The value of this constant is a free parameter (an input for the algorithm developed here) that can be high or low.

(ii) Constant ejection velocity: meaning that the velocity of the gases ejected from the thrusts is constant.

(iii) Constrained angular motion: this means that the direction of the force given by the thrusts can be modified during the transfer. This direction can be specified by the angles α and β, called pitch (the angle between the direction of the thrust and the perpendicular to the line Earth-spacecraft) and yaw (the angle with the orbital plane), respectively. The motions of those angles are constrained (constant, linear variations, forbidden regions for firing the thrusts, etc.).

(iv) Operation in on-off mode: it means that intermediate states are not allowed. The thrusts are either at zero or maximum level all the time.

The solution is given in terms of the constants that specify the control to be applied and the fuel consumed. Several numbers of "thrusting arcs" (arcs with the thrusts active) can be used for each maneuver.

Instead of time, the "range angle" (the angle between the radius vector of the spacecraft and an arbitrary reference line in the orbital plane) is used as the independent variable.

The contribution of the present paper is to implement the algorithm combined with a new numerical method to solve the problem. This new implementation showed to have a better numerical behavior regarding convergence in the situations simulated. Another point is the use of this technique to place a satellite in a constellation.

2. Definition of the problem

The basic problem discussed in this paper is the problem of orbit transfer maneuvers to place a satellite in a constellation. The objective of this problem is to modify the orbit of a given spacecraft, from an initial parking orbit to a specific position in a final orbit. In the case considered in this paper, an initial and a final orbit around the Earth are completely specified, as well as the final desired position of the satellite in that orbit. The problem is to find how to transfer the spacecraft between the first of those two orbits to a specific position in a final orbit, in such way that the consumed fuel is minimum. There is no time restriction involved here and the spacecraft can leave from any point in the initial orbit. The maneuver is performed with the use of an engine that is able to deliver a thrust with constant magnitude and linearly variable direction. The mechanism, time, and fuel consumption to change the direction of the thrust are not considered in this paper, because the optimization of the maneuver does not depend on those parameters and the cost is very small compared with the cost of the maneuver itself.

3. Formulation of the optimal control problem

This is a typical optimal control problem, and it is formulated as follows: objective function: M_f, where M_f is the final mass of the vehicle and it has to be maximized with respect to the control $\mathbf{u}(\cdot)$; subject to: equations of motion, constraints in the state (initial and final orbit), and control (limits in the angles of "pitch" and "yaw," forbidden region of thrusting, etc.); and given: all parameters (gravitational force field, initial values of the satellite, etc.).

The equations of motion are the ones suggested by Biggs [10] that avoid the singularities in circular and/or planar orbits. They are given by

$$\begin{aligned}
\frac{dX_1}{ds} &= f_1 = \mathrm{Si}X_1F_1,\\
\frac{dX_2}{ds} &= f_2 = \mathrm{Si}\{[(\mathrm{Ga}+1)\cos(s)+X_2]F_1+\nu F_2\sin(s)\},\\
\frac{dX_3}{ds} &= f_3 = \mathrm{Si}\{[(\mathrm{Ga}+1)\sin(s)+X_3]F_1-\nu F_2\cos(s)\},\\
\frac{dX_4}{ds} &= f_4 = \frac{\mathrm{Si}\nu F(1-X_4)}{X_1W},\\
\frac{dX_5}{ds} &= f_5 = \frac{\mathrm{Si}\nu(1-X_4)m_0}{X_1},\\
\frac{dX_6}{ds} &= f_6 = -\frac{\mathrm{Si}F_3[X_7\cos(s)+X_8\sin(s)]}{2},\\
\frac{dX_7}{ds} &= f_7 = \frac{\mathrm{Si}F_3[X_6\cos(s)-X_9\sin(s)]}{2},\\
\frac{dX_8}{ds} &= f_8 = \frac{\mathrm{Si}F_3[X_9\cos(s)+X_6\sin(s)]}{2},\\
\frac{dX_9}{ds} &= f_9 = \frac{\mathrm{Si}F_3[X_7\sin(s)-X_8\cos(s)]}{2},
\end{aligned} \tag{3.1}$$

where

$$\begin{aligned}
\mathrm{Ga} &= 1+X_2\cos(s)+X_3\sin(s),\\
\mathrm{Si} &= \frac{(\mu X_1^4)}{\mathrm{Ga}^3m_0(1-X_4)},\\
F_1 &= F\cos(\alpha)\cos(\beta),\\
F_2 &= F\sin(\alpha)\cos(\beta),\\
F_3 &= F\sin(\beta),
\end{aligned} \tag{3.2}$$

and F is the magnitude of the thrust, W is the velocity of the gases when leaving the engine, ν is the true anomaly of the spacecraft.

In those equations, the state was transformed from the Keplerian elements (a = semi-major axis, e = eccentricity, i = inclination, Ω = argument of the ascending node, ω = argument of periapsis, ν = true anomaly of the spacecraft) in the variables X_i, to avoid singularities, by the following relations:

$$
\begin{aligned}
X_1 &= \left[\frac{a(1-e^2)}{\mu}\right]^{1/2}, \\
X_2 &= e\cos(\omega-\phi), \\
X_3 &= e\sin(\omega-\phi), \\
X_4 &= \frac{\text{Consumed Fuel}}{m_0}, \\
X_5 &= t = \text{time}, \\
X_6 &= \cos\left(\frac{i}{2}\right)\cos\left(\frac{\Omega+\phi}{2}\right), \\
X_7 &= \sin\left(\frac{i}{2}\right)\cos\left(\frac{\Omega-\phi}{2}\right), \\
X_8 &= \sin\left(\frac{i}{2}\right)\sin\left(\frac{\Omega-\phi}{2}\right), \\
X_9 &= \cos\left(\frac{i}{2}\right)\sin\left(\frac{\Omega+\phi}{2}\right), \\
\phi &= \nu+\omega-s,
\end{aligned}
\tag{3.3}
$$

s is the range angle of the spacecraft, μ is the gravitational parameter of the central body, and m_0 is the initial mass of the satellite.

The number of state variables defined above is greater than the minimum required to describe the system, which implies that they are not independent and relations between them exist, like: $X_6^2+X_7^2+X_8^2+X_9^2=1$. This system is also subject to the constraints in state and to some of the Keplerian elements of the initial and the final orbit. All the parameters (gravitational force field, initial values of the satellite, etc.) are assumed to be known.

4. Suboptimal method

In this approach (Biggs [10]; Prado [8]), a linear parameterization is used as an approximation for the control law (angles of pitch (α) and yaw (β)),

$$
\begin{aligned}
\alpha &= \alpha_0+\alpha' * (s-s_s), \\
\beta &= \beta_0+\beta' * (s-s_s),
\end{aligned}
\tag{4.1}
$$

where $\alpha_0, \beta_0, \alpha', \beta'$ are parameters to be found, s is the instantaneous range angle, and s_s is the range angle when the motor is turned on.

These equations are the mathematical representation of the "a priori" hypothesis that α and β vary linearly with the "range angle" s. This is done to explore the possibility of having a model easy to implement in terms of hardware.

Considering these assumptions, there is a set of six variables to be optimized (start and end of thrusting and the parameters $\alpha_0, \beta_0, \alpha', \beta'$) for each "burning arc" in the maneuver. Note that this number of arcs is given "a priori" and it is not an "output" of the algorithm. This is the control available to maneuver the satellite.

By using parametric optimization, this problem is reduced to one of nonlinear programming, which can be solved by several standard methods.

In the references cited before (Prado [8]; Prado and Rios-Neto [9]), the gradient projection method was used to solve the numerical problem. It showed to be a useful tool, but convergence was difficult in some situations. In the present paper, the Powell's quadratically convergent method (Press et al. [11]) was used for the optimization steps of the algorithm. This method showed to be better, in terms of convergence, in the performed simulations.

5. Simulations

To validate the developed and implemented algorithm, the same maneuvers available in Biggs [10] were simulated and the results showed a good agreement. After that, a transfer maneuver is simulated. It is assumed that the satellite starts its motion in a parking orbit. This orbit has the following data: semimajor axis of 7000 km, eccentricity of 0, inclination of 5 degrees, ascending node of 35 degrees, argument of perigee of 0, mean anomaly of 90 degrees.

From this orbit, the satellite has to go to a final orbit, that has the following orbital elements: semimajor axis of 26560 km, eccentricity of 0.0131, inclination of 55 degrees, ascending node of 90 degrees, argument of perigee of 180 degrees, mean anomaly of 270 degrees.

It is also assumed that the initial mass of the satellite is 500 kg and the thrust level to perform the maneuver is 20 N.

The results are shown in Table 5.1 for three different situations: a maneuver with two, four, and eight thrusting arcs. For each maneuver the table shows, for every thrusting arc, s_s(deg) the range angle of the instant the engine is turned on; s_e(deg) the range angle of the instant the engine is turned off; α_0(deg) the initial value for the pitch angle; β_0(deg) the initial value for the yaw angle; α' the value of the derivative of the pitch angle with respect to the range angle; β' the value of the derivative of the yaw angle with respect to the range angle; the fuel consumed by the maneuver.

In a second set of simulations, the same maneuvers were performed with the additional constraints that the control angles must be fixed ($\alpha' = \beta' = 0$); and, in a third set, the constraint $\alpha_0 = 0$ was added (only β_0 is a free parameter for the control law). The solution shown in Table 5.1 was used as a first guess for those new simulations. The objective is to know how much more fuel is required to compensate a simpler implementation of

Table 5.1. Maneuvers with 2, 4, and 8 “thrusting arcs.”

Arc	s_s(deg)	s_e(deg)	α_0(deg)	β_0(deg)	α'	β'	Fuel (kg)
1	17.2	1736.3	5.5	62.3	0.016	0.304	—
2	1965.1	3737.1	6.7	47.4	−0.234	−0.038	53.65
1	19.2	850.1	0.79	50.4	0.031	0.076	—
2	1009.7	1853.1	10.1	44.3	−0.020	−0.178	—
3	2036.3	2900.9	11.5	29.4	−0.012	0.317	—
4	3078.3	3989.1	5.3	42.4	−0.120	0.178	51.49
1	40.2	439.7	1.6	46.4	0.003	−0.024	—
2	610.2	1059.1	4.9	54.1	−0.119	0.109	—
3	1157.4	1579.4	3.2	48.5	−0.007	0.653	—
4	1711.9	2135.1	4.3	60.2	−0.127	−0.096	—
5	2325.3	2773.4	5.1	52.5	−0.009	−0.065	—
6	2949.3	3360.2	4.4	42.5	0.119	−0.097	—
7	3470.6	3910.6	3.2	37.6	−0.101	−0.457	—
8	4090.2	4510.4	3.1	54.7	−0.141	−0.101	49.97

Table 5.2. Maneuvers with 2, 4, and 8 “thrusting arcs” ($\alpha' = \beta' = 0$).

Arc	s_s(deg)	s_e(deg)	α_0(deg)	β_0(deg)	Fuel (kg)
1	10.1	1750.5	4.5	75.1	—
2	1955.3	3747.5	4.3	52.2	55.01
1	14.1	855.3	1.09	54.2	—
2	1004.3	1859.1	9.2	47.2	—
3	2030.2	2906.7	10.3	32.3	—
4	3072.1	3995.3	6.4	47.8	52.97
1	37.2	443.2	3.2	50.3	—
2	606.1	1062.9	6.3	57.2	—
3	1153.2	1583.2	5.3	50.6	—
4	1707.1	2139.4	2.1	66.4	—
5	2320.1	2779.3	7.2	50.2	—
6	2943.2	3362.1	6.3	47.8	—
7	3465.3	3913.2	4.1	42.5	—
8	4085.1	4514.3	5.6	57.6	51.02

the control device and to satisfy the constraints of keeping some equipment (e.g., antennas) pointed toward Earth. Tables 5.2 and 5.3 show the results and Table 5.4 shows the comparison in fuel expenditure for all cases studied.

Table 5.3. Maneuvers with 2, 4, and 8 “thrusting arcs” ($\alpha_0 = \alpha' = \beta' = 0$).

Arc	s_s(deg)	s_e(deg)	β_0(deg)	Fuel (kg)
1	7.4	1763.2	77.2	—
2	1945	3759.3	57.3	56.27
1	9.1	859.8	55.2	—
2	1000.2	1863.8	47.1	—
3	2026.2	2909.2	33.2	—
4	3070.9	3998.9	44.5	54.97
1	35.1	445.1	52.3	—
2	600.4	1070.2	57.2	—
3	1148.9	1580.2	52.3	—
4	1700.8	2144.3	58.1	—
5	2315.2	2782.9	59.4	—
6	2939.1	3369.9	38.7	—
7	3462.3	3919.9	39.1	—
8	4081	4519.9	59.2	52.11

Table 5.4. Fuel expenditure (kg) for all simulated maneuvers.

Method	2 arcs	4 arcs	8 arcs
Suboptimal	53.65	51.49	49.97
Suboptimal ($\alpha' = \beta' = 0$)	55.01	52.97	51.02
Suboptimal ($\alpha' = \beta' = \alpha_0 = 0$)	56.27	54.22	52.11

It is clear from the results that the increase of the number of arcs of burning reduces the fuel consumed and that the increase of constraints forces the spacecraft to spend more fuel.

6. Conclusions

Suboptimal control was explored to generate an algorithm to obtain solutions for the minimum fuel maneuvers required to insert a satellite in a constellation.

The method has a good numerical behavior. Process time (CPU) is short (around one minute, in an IBM-PC computer) for simple maneuvers, but when several constraints and/or “thrusting arcs” are present the process time can be larger (around 15 minutes in some cases). The method used to solve the numerical problem in this paper (Powell’s quadratically convergent method) is better then the one used in previous cited papers, because convergence is easier (it occurs in more situations).

The simulations show that an increase in the number of “thrusting arcs” reduces the consumed fuel, a reduction of the order of 5% to 10%. They also show that the additional restrictions added to the problem generate an increase in the consumed fuel, in the order of 2% to 4%.

Acknowledgments

The authors are grateful to National Council for Scientific and Technological Development (CNPq), Brazil, for the Contract 300221/95-9, and to Foundation to Support Research in São Paulo State (FAPESP) for the Contract 03/03262-4.

References

[1] D. J. Stewart and R. G. Melton, "Approximate analytic representations for low-thrust trajectories," *Advances in the Astronautical Sciences*, vol. 76, no. 2, pp. 1455–1467, 1991.
[2] S. Geffroy and R. Epenoy, "Optimal low-thrust transfers with constraints—generalization of averaging techniques," *Acta Astronautica*, vol. 41, no. 3, pp. 133–149, 1997.
[3] C. A. Kluever and S. R. Oleson, "A direct approach for computing near optimal low-thrust transfers," *Advances in the Astronautical Sciences*, vol. 97, no. 2, pp. 1783–1800, 1997.
[4] J. A. Kechichian, "Optimal low-earth-orbit-geostationary-earth-orbit intermediate acceleration orbit transfer," *Journal of Guidance, Control and Dynamics*, vol. 20, no. 4, pp. 803–811, 1997.
[5] A. L. Herman and D. B. Spencer, "Optimal, low-thrust earth orbit transfers using higher-order collocation methods," *Journal of Guidance, Control and Dynamics*, vol. 25, no. 1, pp. 40–47, 2002.
[6] B. M. Kiforenko, Z. V. Pasechnik, S. B. Kyrychenko, and I. Y. Vasiliev, "Minimum time transfers of a low-thrust rocket in strong gravity fields," *Acta Astronautica*, vol. 52, no. 8, pp. 601–611, 2003.
[7] T. Haberkorn, P. Martinon, and J. Gergaud, "Low-thrust minimum-fuel orbital transfer: a homotopic approach," *Journal of Guidance, Control and Dynamics*, vol. 27, no. 6, pp. 1046–1059, 2004.
[8] A. F. B. A. Prado, "Análise, Seleção e Implementação de Procedimentos que Visem Manobras Ótimas em Órbitas de Satélites Artificiais," Master Thesis, INPE, São José dos Campos, Brazil, 1989.
[9] A. F. B. A. Prado and A. Rios-Neto, "Um estudo bibliográfico sobre o problema de transferências de órbitas," *Revista Brasileira de Ciências Mecânicas*, vol. 15, no. 1, pp. 65–78, 1993.
[10] M. C. B. Biggs, *The Optimisation of Spacecraft Orbital Manoeuvres. Part I: Linearly Varying Thrust Angles*, The Hattfield Polytechnic Numerical Optimisation Centre, Hartford, Conn, USA, 1978.
[11] W. H. Press, B. P. Flannery, S. A. Teukolsky, and W. T. Vetterling, *Numerical Recipes*, Cambridge University Press, NewYork, NY, USA, 1989.

Vivian Martins Gomes: Division of Space Dynamics and Control, Instituto Nacional de Pesquisas Espaciais (INPE), Avenue dos Astronautas, 1758 Jardim da Granja, 12227-010 São José dos Campos, SP, Brazil
Email address: vivian.gomes@uol.com.br

Antonio Fernando Bertachini de Almeida Prado: Division of Space Dynamics and Control, Instituto Nacional de Pesquisas Espaciais (INPE), Avenue dos Astronautas, 1758 Jardim da Granja, 12227-010 São José dos Campos, SP, Brazil
Email address: prado@dem.inpe.br

Helio Koiti Kuga: Division of Space Dynamics and Control, Instituto Nacional de Pesquisas Espaciais (INPE), Avenue dos Astronautas, 1758 Jardim da Granja, 12227-010 São José dos Campos, SP, Brazil
Email address: hkk@dem.inpe.br

Special Issue on Time-Dependent Billiards

Call for Papers

This subject has been extensively studied in the past years for one-, two-, and three-dimensional space. Additionally, such dynamical systems can exhibit a very important and still unexplained phenomenon, called as the Fermi acceleration phenomenon. Basically, the phenomenon of Fermi acceleration (FA) is a process in which a classical particle can acquire unbounded energy from collisions with a heavy moving wall. This phenomenon was originally proposed by Enrico Fermi in 1949 as a possible explanation of the origin of the large energies of the cosmic particles. His original model was then modified and considered under different approaches and using many versions. Moreover, applications of FA have been of a large broad interest in many different fields of science including plasma physics, astrophysics, atomic physics, optics, and time-dependent billiard problems and they are useful for controlling chaos in Engineering and dynamical systems exhibiting chaos (both conservative and dissipative chaos).

We intend to publish in this special issue papers reporting research on time-dependent billiards. The topic includes both conservative and dissipative dynamics. Papers discussing dynamical properties, statistical and mathematical results, stability investigation of the phase space structure, the phenomenon of Fermi acceleration, conditions for having suppression of Fermi acceleration, and computational and numerical methods for exploring these structures and applications are welcome.

To be acceptable for publication in the special issue of Mathematical Problems in Engineering, papers must make significant, original, and correct contributions to one or more of the topics above mentioned. Mathematical papers regarding the topics above are also welcome.

Authors should follow the Mathematical Problems in Engineering manuscript format described at http://www.hindawi.com/journals/mpe/. Prospective authors should submit an electronic copy of their complete manuscript through the journal Manuscript Tracking System at http://mts.hindawi.com/ according to the following timetable:

Manuscript Due	March 1, 2009
First Round of Reviews	June 1, 2009
Publication Date	September 1, 2009

Guest Editors

Edson Denis Leonel, Department of Statistics, Applied Mathematics and Computing, Institute of Geosciences and Exact Sciences, State University of São Paulo at Rio Claro, Avenida 24A, 1515 Bela Vista, 13506-700 Rio Claro, SP, Brazil; edleonel@rc.unesp.br

Alexander Loskutov, Physics Faculty, Moscow State University, Vorob'evy Gory, Moscow 119992, Russia; loskutov@chaos.phys.msu.ru