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Research Article

Patrol Mobile Robots and Chaotic Trajectories

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This paper presents a study of special trajectories attainment for mobile robots based on the dynamical features of chaotic systems. This method of trajectories construction is envisaged for missions for terrain exploration, with the specific purpose of search or patrol, where fast scanning of the robot workspace is required. We propose the imparting of chaotic motion behavior to the mobile robot by means of a planner of goal positions sequence based on an area-preserving chaotic map. As a consequence, the robot trajectories seem highly opportunistic and unpredictable for external observers, and the trajectories's characteristics ensure the quick scanning of the patrolling space. The kinematic modeling and the closed-loop control of the robot are described. The results and discussion of numerical simulations close the paper.

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1. Introduction

Mobile robotics, after decades of continuous development, keeps up as an intensive research issue because of its ever-increasing application to different domains and its economical and technological relevances. Interesting applications can be seen in robots performing floor-cleaning tasks [1], executing industrial transportation [2], exploring volcanos, scanning areas to find explosive devices, and so on [3].

This work deals with the problem in which a mobile robot is used for searching or patrolling a defined region. To avoid risks to human life, it is very wise to use autonomous or remotely operated robots to deal with the hazardous tasks of detecting dangerous materials or an intruder. For these applications, the physical robot and its systems architecture

and software have been studied extensively for military and civil operations. Some mobile patrol robots are already commercially available (e.g., [4]). Many examples of studies involving vigilance robots can be found in [5–9]. These works mainly focus on target perception and identification, robot localization, terrain map updating, optimization of communication with the operation center or other robots. Here, we are specifically interested in investigating a very critical and still open issue that is paramount for the success of these applications: the path planning. The main goal is to generate a very convenient trajectory to be followed by the rover so that it increases as much as possible the probability of finding the intruder inside the surveillance region. This trajectory must be as opportunistic as possible so that its developed thread on the region cannot be easily understood or predicted by the intruder. Otherwise, he can come up with a way that allow him to avoid the rover. As so, this problem of using a robot to patrol a defined region is actually a problem of conceiving a proper strategy that generates an opportunistic and properly crafted trajectory to be followed by the robot. We can affirm that high unpredictability for the robot trajectories as well as fast scanning of the workspace area are strongly required. In this work, we introduce a very convenient strategy to accomplish these basic requirements. Our strategy exploits the dynamical behavior of a conservative chaotic system to generate trajectories to be followed by the rover. As a spin-off of this approach, previous terrain mapping is no longer necessary.

The interaction between mobile robotics and chaos theory has been studied only recently, as can be seen in [10–12]. For instance, the integration between the robot motion system and a chaotic system, the Arnold dynamical system, is used to impart chaotic behavior to a robot in [13]. An extension of this motion control strategy, applying diverse chaotic systems on integration with the robot kinematics model, can be found in [14]. In [15], the same principle of systems integration is used, however a Van der Pol equation is associated with the target. In [16], an open-loop control approach is proposed to produce unpredictable trajectories so that the state variables of the Lorenz chaotic system are used to command the velocities of the robot's wheels.

Here, we introduce a new strategy that generates an opportunistic and proper crafted trajectory that works as follows. We associate a path-planning generator module with a closed-loop locomotion control module. At each step, the first one generates a position goal defined by its coordinates in the phase space. This position goal is provided to the second module, which drives the mobile rover from its actual position to the desired one. When the rover arrives at the desired position, the path-planning generator module is used again to give another position goal, which is subsequently provided to the second module. This sequence of action is repeated over and over again. The path-planning generator module is implemented by exploiting the dynamics of an area-preserving map in a chaotic regime. Different to a dissipative chaotic map, in which the chaotic evolution takes place on attractors, the chaotic region of an area-preserving map for specific parameter values extends practically over all of its phase spaces. For these parameter values, the entire phase space is covered by a single chaotic orbit. It is therefore possible to make an association between the physical region that we wish to scan with the phase space defined for the area-preserving map. Thus, the position goals are generated as transformed iterations of an area-preserving map and so present a chaotic dynamics with its remarkable

characteristic that makes long-time prediction based on measurements very difficult. Between the position goals, the rover trajectory is driven by a closed-loop locomotion control module which allows a short trajectory between the points and introduces an element of regularity to the strategy. By properly combining the parameters of both modules, we can exploit a multitude of possibilities between a quick scanning of a region by using a trajectory with "small" level of unpredictability to a "slow" scanning with a highly unpredictable trajectory. Actually, by properly combining the parameters, we are able to come up with a very efficient, opportunistic, and properly crafted trajectory that fits the desired requirements for patrol missions.

This paper presents the proposed path planning (Section 2). The robot model and the adopted control are described in Section 3. Section 4 presents the simulation results and discussions, and Section 5 concludes the paper.

2. Chaotic trajectory planning

In the context of deterministic systems, sensitive dependence on initial conditions is the main well-known characteristic of the chaotic behavior [17]. This means that arbitrarily close initial conditions imply trajectories that move far away from each other after some time. This property of a chaotic system makes long-term prediction of a chaotic trajectory based on finite-time measurements practically impossible because of the limited accuracy associated to measurement sensors. Another intrinsic characteristic of a chaotic evolution is the transitivity [17]. A deterministic system is transitive on an invariant set if for any two open subsets U and V of this invariant set, there exist trajectories originating from U that pass through V after some time. This property means that we always can use a chaotic trajectory as a transportation path between regions belonging to the chaotic invariant set. It implies the "mixing property" founded in chaotic systems that ensures that the system cannot be broken down into subsystems that do not interact with each other. From the perspective of an external observer, a chaotic trajectory presents a complicated behavior that does not exhibit any recurrent pattern and seems to be random. In other words, a chaotic trajectory is reported by an external observer as an erratic trajectory that quickly moves among different regions of a certain invariant set. It is precisely this behavior that we exploit in this work to orient the movement of a robot to make it very suitable to be used as an opportunistic patrolling engine.

The space to be scanned by the robot can be viewed as a kind of a continuous subset with an integer dimension 2. In order to accomplish our goal, the chaotic trajectory must fill densely this integer dimension subset. This requirement provides an extra ingredient that delimits the class of chaotic system to be used. In a dissipative chaotic system, the chaotic invariant set is an attractor. In general, this attractor has a noninteger dimension, that is, its geometric picture on the phase space is a fractal. However, in our problem, the robot must cover densely its patrol region. Consequently, we consider it more appropriate to choose another class of chaotic systems: an area-preserving chaotic system. One of the basic properties of an area-preserving system is that it preserves volumes in the phase space [18]. As a consequence of this property, these systems do not have attractors, and the chaotic regions spread densely over regions of the phase space. Note that this fact individualizes our approach in the scenario of previous works that uses a chaotic

system to run the robot dynamics. Because previous approaches use dissipative chaotic systems (e.g., [13]), they require subterfuges to make the robot wander opportunistically through the patrolling area. This is not necessary in this approach. For the knowledge of the authors, this is the first time that this concept is applied in the area of mobile robots.

2.1. The standard map-based path planning. The planning procedure is based on determining a sequence of intermediary goal points (coordinates x and y) that will compose the robot trajectory. The path-planning generator module uses an area-preserving map that is considered as a paradigm for area-preserving chaotic systems: the standard map, also called Taylor-Chirikov map [19]. It is a two-dimensional map which results from a periodic impulsive kicking of a rotor. This map was firstly proposed by Brian Taylor and then independently obtained by Chirikov [20] to describe the dynamics of magnetic field lines on the kicked rotor [18]. The dynamic effect of this system is expressed mathematically through the map equations, given by

$$\begin{aligned} x_{n+1} &= x_n + K \sin y_n, \\ y_{n+1} &= y_n + x_{n+1}, \end{aligned} \tag{2.1}$$

where x is a periodic configuration variable (angular position) and y is the momentum variable (angular speed). These map variables are both computed mod(2π). The map parameter K represents the strength of the nonlinear kick applied in the rotor mechanism. In its phase space and according to the value associated with the parameter K, it has stable and unstable periodic orbits, Kolmogorov-Arnold-Moser (KAM) surfaces, and chaotic regions. Depending on the nonlinear parameter K, the regions of regular motion and the regions of chaotic motion are complexly interwoven, but the chaotic regions are confined between KAM tori. As this parameter is increased, the KAM surfaces start to be destroyed, chaotic regions occupy increasingly large areas until, for a specific value of K, the last KAM torus is destroyed and the entire region of the phase space appears to be densely covered by a single chaotic orbit. Our path-planning generator module is implemented based on the standard map that presents this dynamics.

Let us now show an example of how the standard map is used in the context of our path-planning generator module. By numerically simulating the map equations, we can analyze the properties of terrain covering considering the basic mission requirements for fast terrain scanning. We define a square terrain with dimensions 100×100 in a normalized measurement unit. The map simulation begins with an arbitrary initial position, and considers the gain value $K = 7$. We can see that the third case can cover completely the considered terrain (in fact, the necessary condition for the complete scan is $K > 6$ [20]). The results of passage locations planned for 100 and 3000 iterations can be seen in Figure 2.1.

The terrain covering can be judged through a performance index. This index is defined using a terrain division on square unit cells (e.g., 1×1, i.e., 10 000 cells), and computing the visited cells percentage after the robot locations planning. This index of terrain covering is presented in the form of a plotting index *versus* planning evolution, as can be seen in Figure 2.2 (where index = 1 represents 100% of cells visited). However, this analysis does not consider the robot trajectories between two subsequent locations, that will be

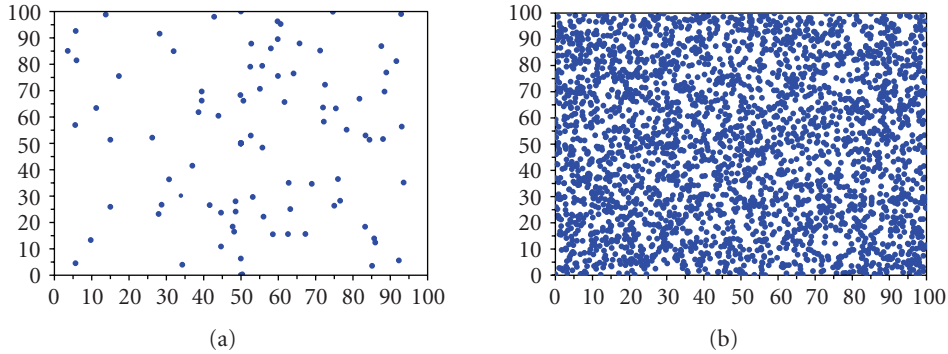

Figure 2.1. Terrain covering by subgoals planned points using standard maps after 100 and 3000 planning iterations (considered map gain value $K = 7$).

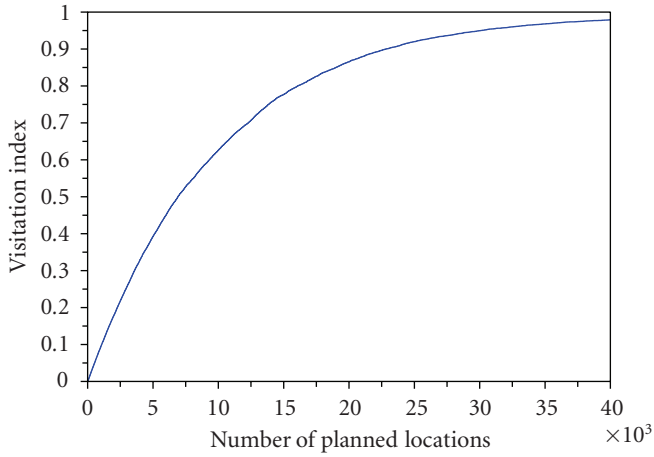

Figure 2.2. Index of terrain covering (visited cells portion) for 40 000 iterations, with map gain $K = 7$.

taken in account later in this paper. It is quite evident that a faster complete area covering could be obtained using a systemic scan without passing two or more times at one same terrain cell, but this classic strategy is absolutely predictable, and inadequate to the patrolling mission.

Another strategy to plan these points could be defined using a random numbers generator. Considering a uniform distribution random sequence, we obtain a very similar terrain space covering. The results for this alternative planning strategy are shown in Figures 2.3 and 2.4. Even if the appearance and the terrain covering are similar, the planning nature is quite different to the conservative standard map. We will discuss this fundamental difference in Section 4.

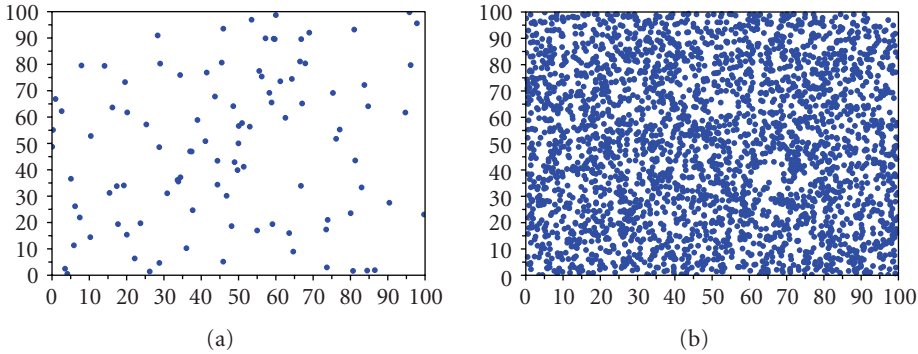

Figure 2.3. Terrain covering by subgoals planned points using a random numbers generator after 100 and 3000 planning iterations.

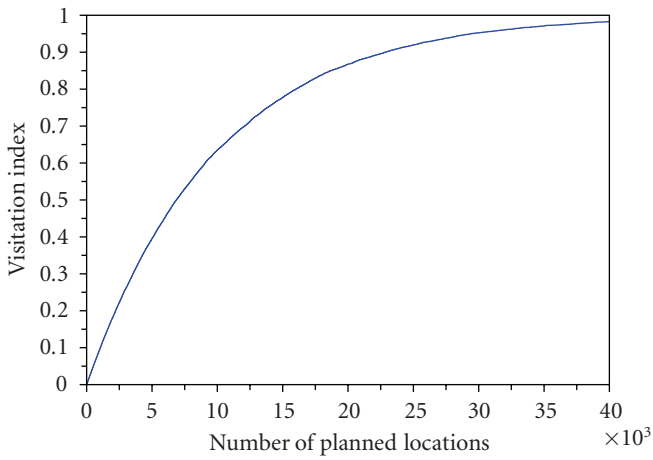

Figure 2.4. Terrain covering by subgoals planned points using a random numbers generator (40 000 planning iterations).

3. Kinematic control of the mobile robot

The mobile robot considered in this work is a typical differential motion robot with two degrees of freedom, composed by two active, parallel, and independent wheels, a third passive wheel with exclusive equilibrium functions (a sort of free steered standard wheel), and proximity sensors capable of obstacles detection. The active wheels are independently controlled on velocity and sense of turning. The sensors provide short-range distances to obstacles. For instance, these sensors could be sonar or infrared devices commonly used in mobile robots, with adequate accuracy. Additionally, the robot is assumed to be equipped with specific sensors for detection and recognition of the searched objects or intruders. This robot model represents an interesting compromise between control simplicity and degrees of freedom that allow the robot to accomplish mobility requirements

[21], and it is largely adopted in several researches on mobile robotics, for example, [22–24].

The robot chassis is considered as a rigid body operating on the horizontal plane. Its motion is obtained by driving the active wheels. The resultant motion is described in terms of linear velocity $v(t)$ and direction $\theta(t)$, representing an instantaneous linear motion of the medium point of the wheel axis and a rotational motion (rotational velocity $\omega(t)$) of the robot body over this same point.

The robot motion control can be done providing the wheels velocities, $\omega_l(t)$ and $\omega_r(t)$, or equivalently the body linear and angular velocities, $v(t)$ and $\omega(t)$, called input or control variables. The mathematical model of this kinematics problem considers these two input variables and also three state variables: the robot position and orientation ($x(t)$, $y(t)$, $\theta(t)$) [21]:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}. \tag{3.1}$$

These equations constitute a nonholonomic dynamical system. The control of this system has been studied extensively by various research groups, and diverse solutions are available, for example, [22, 23]. The motion control strategy adopted in this work involves a state feedback controller, proposed in [24], which is an appropriate approach to produce a desired trajectory described by a sequence of coordinates (x_p, y_p). This means that the path-planning task is given by a specialized robot module, independent of the motion control module, that sets intermediate positions lying on the requested path.

The adopted control law considers the geometric situation shown in Figure 3.1. In this figure, the robot is placed at an arbitrary configuration (position x, y and orientation θ), and a desired position (the target x_p, y_p: the origin of frame $X_G Y_G$) is defined by the robot path-planner. In the robot reference frame $X_R Y_R$, the configuration error vector is defined by $e = [\rho\ \varphi]^T$, where ρ and φ localize the target position, and provide a coordinate change:

$$\begin{aligned} \rho &= \left(\Delta x^2 + \Delta y^2\right)^{1/2}, \\ \varphi &= 180 + \theta - \psi. \end{aligned} \tag{3.2}$$

The robot kinematics model is described by (3.1), where $\dot{x}(t)$ and $\dot{y}(t)$ are the linear velocity components in the absolute reference frame (fixed on the workspace). We define the angle φ between the X_R axis of body reference frame and the vector connecting the robot center and the desired position. The other configuration variables ρ and ψ represent, respectively, the distance between present and desired positions, and the angle between the direction to the target and the axis X_0.

The description of the motion in the new coordinates becomes

$$\begin{bmatrix} \dot{\rho} \\ \dot{\varphi} \end{bmatrix} \begin{bmatrix} -\cos\varphi & 0 \\ \dfrac{1}{\rho}\sin\varphi & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}. \tag{3.3}$$

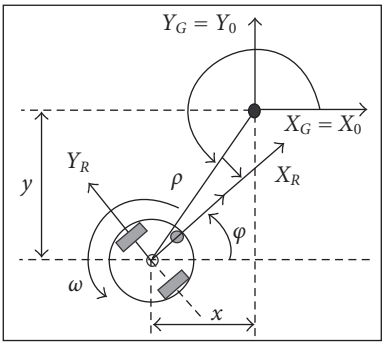

Figure 3.1. The robot control problem configuration, where the position/orientation error (ρ, φ), the linear and angular velocities (v, ω), the robot frame $(X_R Y_R)$, and the desired position frame $(X_G Y_G \equiv X_0 Y_0)$ can be seen.

Concerning these polar coordinates system descriptions, it is necessary to remark that the coordinate transformation is not defined at $x = y = 0$, that is, when the robot achieves the goal location. The adopted control law [24], in terms of error feedback (ρ, φ) to determine the value of system inputs v, ω, is given by

$$\begin{aligned} v &= k_1 \rho \cos\varphi, \\ \omega &= -k_1 \sin\varphi \cos\phi - k_2 \varphi. \end{aligned} \tag{3.4}$$

Using the Lyapunov stability analysis, we can verify the robot kinematic system with the application of this nonlinear control law [25]. Composing a quadratic Lyapunov function with the state variables ρ and φ given by $V = (1/2)(\rho^2 + \varphi^2)$, the time derivative of this Lyapunov function, considering the control law, is

$$\dot{V} = \rho(-v\cos\varphi) + \varphi\left(\omega + \frac{v}{\rho}\sin\varphi\right) = -k_1(\rho\cos\varphi)^2 - k_2\varphi^2. \tag{3.5}$$

The constant control gains k_1 and k_2 are exclusively positive. As a consequence, the value of $\dot{V}$ is negative for all nonnull value of (ρ, φ), and null at the origin of state space. That is a sufficient condition for the asymptotic convergence of (3.3).

If an obstacle is found on the trajectory, a specific navigation competence, obstacle avoidance, must be used to drive the robot from this obstacle. Is this work, the obstacle avoidance problem is not treated, nevertheless a simple solution could be implemented, for example, using the algorithm proposed in [26].

Using this feedback control law, we intend to validate the proposed path planning, examining the trajectory unpredictability, the terrain covering by the robot motion, and the general characteristics of the obtained trajectories. Nevertheless, we note that any stable control law can provide adequate closed-loop motion respecting the planned sequence of passage points, and obtaining the chaotic robot motion.

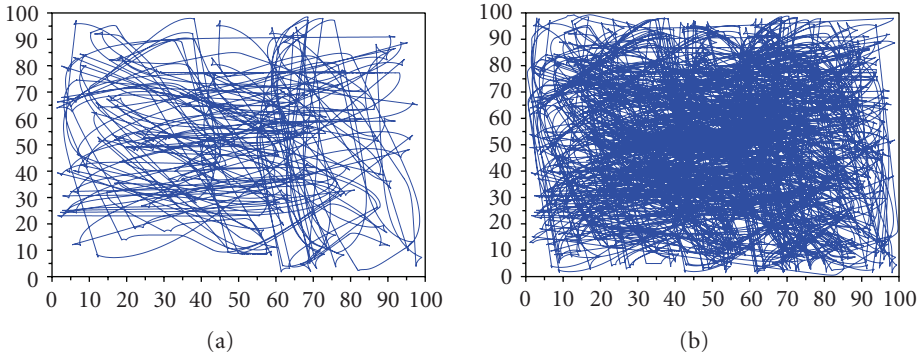

Figure 4.1. Two instants of the mobile robot trajectory evolution, after 200 and 1000 planned passage locations, using the adopted continuous control law (considering a terrain with dimension 100×100 in a normalized measurement unit).

4. Numerical simulations

To test our mobile robot patrolling approach, we have simulated the robot kinematic motion applying the closed-loop control law discussed in the previous section to track a sequence of planned objective points, provided by the standard map. The adopted control law obtains smooth trajectories between two subsequent locations, reinforcing the apparently erratic nature of the movement, which constitute an interesting feature for patrol missions. The trajectory results of the application of this control law are shown in Figure 4.1. For any other terrain shape, the planning process can fit the area of interest inside a square standard map, ignoring the points planned outside the terrain but ensuring the desired fast workspace scan.

We analyze the terrain covering of this executed (or effective) trajectory using the same performance index proposed for the planned sequence of points (Section 2), that is, the terrain is divided in 10 000 square unit cells, and the index represents the visited cells percentage (index = 1 represents 100%). The time evolution of the terrain covering index is shown in Figure 4.2. We do not take in to account here the extra area covered by the sensors perception range region that certainly could augment the area covering indexes. The terrain scan analysis presented here is based on a worst case in which the sensor perception field is considered as a single point.

We can see that the central part of the terrain will be visited more often than the regions close to the borders. This is a consequence of frequent changes of motion direction and it is in accordance with the strategy which is to cover completely the patrolled terrain all the time so that an intruder appearing in an arbitrary time/location inside the terrain is quickly detected by the rover. Evidently, the complete terrain covering will be obtained only after a long-term execution of the robot planning/motion procedure.

Considering the basic requirements of patrol missions, and the main ideas of our approach, we can discuss different ways to obtain the construction of mobile robot trajectories by combining the adoption of a motion control law and a strategy for the determination of the regions sequence to be inspected by the robot.

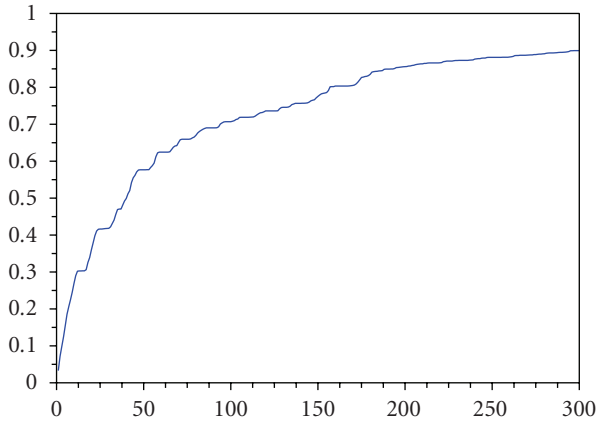

Figure 4.2. The time evolution of the index of terrain covering for the executed robot trajectory (300 simulation time intervals).

In the example presented, we adopt a continuous control law that provides smooth trajectories and minimizes unnecessary maneuvers and consequent control switches. Another very different but intuitive control strategy could be adopted: a discontinuous control law based on an initial rotational maneuver around the robot wheel axis center to orientate the robot towards the next desired position, followed by a straight trajectory to the objective.

In Section 2, we mentioned an alternative approach to be used as our path-planning generator module: to use a series of random locations, uniformly distributed in the patrolled space. This sort of random planning strategy results in a similar terrain covering to the chaotic planning. However, the nature of locations planning is quite different: the chaotic one is deterministic.

We can conclude our proposed approach by analyzing the advantages and disadvantages of these aspects of the patrolling scenario: the type of control law and the inspection planning strategy. Firstly, we compare the two options of control laws presented here, continuous and discontinuous, and we can verify that the continuous one offers advantages, because the smoother trajectories save control switches and maneuvers; moreover, they contribute to perform unpredictable trajectories for external observers. On the other hand, the discontinuous control produces a sequence of piecewise predictable straight trajectories (an example can be seen in Figure 4.3). However, in terms of time of terrain scanning, the advantageous control approach is the discontinuous one, because it will evidently cover faster the terrain than the discontinuous control when using straight trajectories.

If we compare the two options of scanning plan, chaotic and random approaches, both of them provide very similar results in terms of terrain scan appearance and patrol covering. In spite of that, chaotic trajectories have an important advantage, they are based on a deterministic sequence of objective points. This means that the behavior of the rover can be predicted in advance for the system designer. In terms of navigation expertise, this

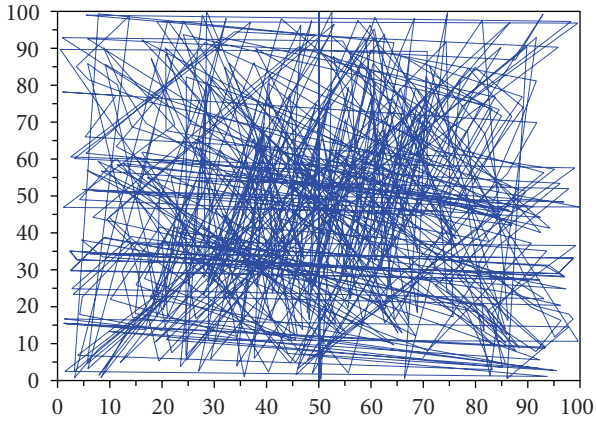

Figure 4.3. An example of trajectories obtained using the discontinuous control law (500 planned passage locations, standard map planning).

path-planning determinism represents an important advantage over navigation based on a sort of random walk trajectory. This feature can facilitates the frequent robot localization procedure, which is a crucial function because the knowledge of the robot position with appropriate precision constitutes very necessary information for the robot itself and also for the mission operation center. This determinism can be also advantageous to other mission aspects, for example, executed trajectory supervision, terrain's scanning information, and precise localization of intruders or targets.

5. Conclusion

The presented strategy to deal with terrain exploration missions for mobile robots can achieve adequately the main requirements for patrol missions: we are able to achieve very efficient, opportunistic, and proper crafted trajectory that fits the desired requirements for patrol missions. Imparting chaotic motion behavior to the robot motion through the utilization of an area-preserving chaotic map as a path planner ensures high unpredictability of robot trajectories, resembling a nonplanned erratic motion from external observers' point of view. Validation tests, based on numerical simulations of closed-loop motion control to follow the sequence of objective points on the robot trajectory, confirm that the chaotic planning procedure can obtain adequate results. The advantageous property of the proposed chaotic motion planning over unplanned or randomly planned motion resides mainly in the deterministic nature of chaotic behavior, which can be useful for important functions of the robot motion control, for example, the robot localization and the terrain mapping.

This study shows that the application of dynamical behaviors of nonlinear systems to solutions for mobile robots control problems represents an interesting interdisciplinary interface for researchers of both scientific domains, opening promising perspectives of future works including experimental realizations.

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