HEAT TRANSFER ANALYSIS ON ROTATING FLOW OF A SECOND-GRADE FLUID PAST A POROUS PLATE WITH VARIABLE SUCTION

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We deal with the study of momentum and heat transfer characteristics in a second-grade rotating flow past a porous plate. The analysis is performed when the suction velocity normal to the plate, as well as the external flow velocity, varies periodically with time. The plate is assumed at a higher temperature than the fluid. Analytic solutions for velocity, skin friction, and temperature are derived. The effects of various parameters of physical interest on the velocity, skin friction, and temperature are shown and discussed in detail.

1. Introduction

The study of non-Newtonian fluids has attracted much attention, because of their practical applications in engineering and industry particularly in extraction of crude oil from petroleum products, food processing, and construction engineering. Due to complexity of fluids, various models have been proposed. The equations of motion of non-Newtonian fluids are highly nonlinear and one order higher than the Navier-Stokes equations. Finding accurate analytic solutions to such equations is not easy. There is a particular class of non-Newtonian fluids namely the second-grade fluids for which one can reasonably hope to obtain an analytic solution. Important studies of second-grade fluids in various contexts have been given in the references [1, 3, 6, 7, 9, 10, 11, 12, 13, 17, 19, 20, 21, 22, 24].

Since the pioneering work of Lighthill [16] there has been a considerable amount of research undertaken on the time-dependent flow problems dealing with the response of the boundary layer to external unsteady fluctuations about a mean value. Important contributions to the topic with constant and variable suction include the work of Stuart [25], Messiha [18], Kelley [15], Soundalgekar and Puri [23], and Hayat et al. [8].

Despite the above studies, no attention has been given to the study of the simultaneous effects of the rotation and heat transfer characteristics on the non-Newtonian flow with variable suction. Such work seems to be important and useful for gaining our basic understanding of such flow and partly for possible applications to geophysical and astrophysical problems. Also, heat transfer plays an important role during the handling and processing of non-Newtonian fluids. The understanding of heat transfer in boundary

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layer flows of non-Newtonian fluids is of importance in many engineering applications such as the design of thrust bearings and radial diffusers, transpiration cooling, drag reduction, thermal recovery of oils, and so forth. The primary purpose of the present paper is to make an investigation of the combined effects of rotation, and heat transfer characteristics on the flow of a second-grade fluid past a porous plate with variable suction. This work is concerned with a boundary value problem in a rotating flow. The analytical solution of the velocity field, skin-friction, and temperature distribution is obtained. Special attention is given to finding the analytical solutions and to describe the physical nature. Finally, in order to see the variations of different emerging parameters, the graphs are sketched and discussed.

2. Mathematical formulation

Let us consider an incompressible second-grade fluid past a porous plate. The plate and the fluid rotate in unison with an angular velocity $\mathbf{\Omega}$ about the z'-axis normal to the plate. The plate is located at $z' = 0$ having temperature T_0. The flow far away from the plate is uniform and temperature of the fluid is T_∞.

For the problem under question, we consider the velocity and temperature fields as

$$\mathbf{V} = \left(u'(z',t'), v'(z',t'), w'(z',t')\right), \tag{2.1}$$

$$T = T(z',t'), \tag{2.2}$$

in which u', v', and w' are the velocity components in x', y', and z' directions, respectively, and T indicates the temperature.

The governing equations in absence of body forces and radiant heating are

$$\operatorname{div}\mathbf{V} = 0, \tag{2.3}$$

$$\rho'\left[\frac{d\mathbf{V}}{dt'} + 2\mathbf{\Omega}\times\mathbf{V} + \mathbf{\Omega}\times(\mathbf{\Omega}\times\mathbf{r})\right] = \operatorname{div}\mathbf{T}, \tag{2.4}$$

$$\rho'\frac{de}{dt'} = \mathbf{T}\cdot\mathbf{L} - \operatorname{div}\mathbf{q}. \tag{2.5}$$

In above equations d/dt', ρ', e, $\mathbf{L}$, and $\mathbf{q}$ are, respectively, the material derivative, density, the specific internal energy, the gradient of velocity, the heat flux vector, and the radial distance $r^2 = x^2 + y^2$. The Cauchy stress $\mathbf{T}$ in an incompressible homogeneous fluid of second grade is of the form

$$\mathbf{T} = -p\mathbf{I} + \mu\mathbf{A}_1 + \alpha_1\mathbf{A}_2 + \alpha_2\mathbf{A}_1^2, \tag{2.6}$$

$$\mathbf{A}_1 = (\operatorname{grad}\mathbf{V}) + (\operatorname{grad}\mathbf{V})^\top, \tag{2.7}$$

$$\mathbf{A}_2 = \frac{d\mathbf{A}_1}{dt} + \mathbf{A}_1(\operatorname{grad}\mathbf{V}) + (\operatorname{grad}\mathbf{V})^\top\mathbf{A}_1, \tag{2.8}$$

where μ, $-p\mathbf{I}$, α_j $(j = 1,2)$, $\mathbf{A}_1$, and $\mathbf{A}_2$ are, respectively, the dynamic viscosity, spherical stress, normal stress moduli, and first two Rivlin-Ericksen tensors. The thermodynamic analysis of model (2.6) has been discussed in detail by Dunn and Fosdick [4]. The Clausius-Duhem inequality and the assumption that the Helmholtz free energy is a minimum in equilbrium provide the following restrictions [5]:

$$\mu \geq 0, \qquad \alpha_1 \geq 0, \qquad \alpha_1 + \alpha_2 = 0. \tag{2.9}$$

It is evident from (2.1) and (2.3) that

$$\frac{\partial w'}{\partial z'} = 0. \tag{2.10}$$

The above equation shows that w' is a function of time. Following Messiha [18] and Soundalgekar and Puri [23] we take

$$w' = -W_0'\left(1 + \epsilon A e^{i\omega' t'}\right). \tag{2.11}$$

In above equation W_0' is nonzero constant mean suction velocity, A is real positive constant, ϵ is small such that $\epsilon A \leq 1$, and negative sign indicates that suction velocity normal to the plate is directed towards the plate. From (2.1), (2.4), (2.6), (2.8), and (2.11) we get

$$\begin{aligned}\frac{\partial u'}{\partial t'} - W_0'\left(1 + \epsilon A e^{i\omega' t'}\right)\frac{\partial u'}{\partial z'} - 2\Omega v' &= -\frac{1}{\rho'}\frac{\partial \hat{p}}{\partial x'} + \upsilon\frac{\partial^2 u'}{\partial z'^2} + \alpha^*\frac{\partial^3 u'}{\partial z'^2 \partial t'} \\ &\quad - \alpha^* W_0'\left(1 + \epsilon A e^{i\omega' t'}\right)\frac{\partial^3 u'}{\partial z'^3},\end{aligned} \tag{2.12}$$

$$\begin{aligned}\frac{\partial v'}{\partial t'} - W_0'\left(1 + \epsilon A e^{i\omega' t'}\right)\frac{\partial v'}{\partial z'} 2\Omega u' &= -\frac{1}{\rho'}\frac{\partial \hat{p}}{\partial y'} + \upsilon\frac{\partial^2 v'}{\partial z'^2} + \alpha^*\frac{\partial^3 v'}{\partial z'^2 \partial t'} \\ &\quad - \alpha^* W_0'\left(1 + \epsilon A e^{i\omega' t'}\right)\frac{\partial^3 v'}{\partial z'^3},\end{aligned} \tag{2.13}$$

$$\frac{\partial w'}{\partial t'} = -\frac{1}{\rho'}\frac{\partial \hat{p}}{\partial z'}, \tag{2.14}$$

subject to the boundary conditions

$$u' = v' = 0 \quad \text{at } z' = 0, \tag{2.15}$$

$$u' \longrightarrow U'(t'), \quad v' \longrightarrow 0 \quad \text{as } z' \longrightarrow \infty, \tag{2.16}$$

where $U'(t')$ is the free stream velocity and modified pressure

$$\begin{aligned}\hat{p} &= p - \frac{1}{2}\rho'\Omega^2 r^2 - (2\alpha_1 + \alpha_2)\left[\left(\frac{\partial u'}{\partial z'}\right)^2 + \left(\frac{\partial v'}{\partial z'}\right)^2\right], \\ \upsilon &= \frac{\mu}{\rho'}, \qquad \alpha^* = \frac{\alpha_1}{\rho'}.\end{aligned} \tag{2.17}$$

In view of (2.11) and (2.14), $\partial\hat{p}/\partial z'$ is small in the boundary and hence can be ignored [8, 18, 23]. The modified pressure $\hat{p}$ is assumed constant along any normal and is given by its value outside the boundary layer. Equations (2.12) and (2.13) for the free stream yields

$$-\frac{1}{\rho'}\frac{\partial\hat{p}}{\partial x'}=\frac{dU'}{dt'}, \tag{2.18}$$

$$-\frac{1}{\rho'}\frac{\partial\hat{p}}{\partial y'}=2\Omega U'. \tag{2.19}$$

Making use of (2.18) and (2.19) into (2.12) and (2.13), we have

$$\begin{aligned}\frac{\partial u'}{\partial t'}-W_0'\left(1+\epsilon Ae^{i\omega' t'}\right)\frac{\partial u'}{\partial z'}-2\Omega v'&=\frac{dU'}{dt'}+\upsilon\frac{\partial^2 u'}{\partial z'^2}+\alpha^*\frac{\partial^3 u'}{\partial z'^2\partial t'}\\&\quad-\alpha^* W_0'\left(1+\epsilon Ae^{i\omega' t'}\right)\frac{\partial^3 u'}{\partial z'^3},\end{aligned} \tag{2.20}$$

$$\begin{aligned}\frac{\partial v'}{\partial t'}-W_0'\left(1+\epsilon Ae^{i\omega' t'}\right)\frac{\partial v'}{\partial z'}2\Omega u'&=2\Omega U'+\upsilon\frac{\partial^2 v'}{\partial z'^2}+\alpha^*\frac{\partial^3 v'}{\partial z'^2\partial t'}\\&\quad-\alpha^* W_0'\left(1+\epsilon Ae^{i\omega' t'}\right)\frac{\partial^3 v'}{\partial z'^3},\end{aligned} \tag{2.21}$$

where U' is periodic free stream velocity given by

$$U'(t')=U_0'\left(1+\epsilon e^{i\omega' t'}\right), \tag{2.22}$$

where U_0' is the reference velocity.

With the help of (2.22), (2.20), (2.21), and boundary conditions (2.15) become

$$\begin{aligned}&\frac{\partial F'}{\partial t'}-W_0'\left(1+\epsilon Ae^{i\omega' t'}\right)\frac{\partial F'}{\partial z'}+2i\Omega F'\\&\quad=U_0' i\omega'\epsilon e^{\iota\omega' t'}+\upsilon\frac{\partial^2 F'}{\partial z'^2}+2i\Omega U_0'\left(1+\epsilon e^{i\omega' t'}\right)+\alpha^*\frac{\partial^3 F'}{\partial z'^2\partial t'}\\&\quad\quad-\alpha^* W_0'\left(1+\epsilon Ae^{i\omega' t'}\right)\frac{\partial^3 F'}{\partial z'^3},\end{aligned} \tag{2.23}$$

$$\begin{aligned}F'&=0 \quad \text{at } z'=0,\\ F'&=U_o'\left(1+\epsilon e^{i\omega' t'}\right) \quad \text{as } z'\longrightarrow\infty,\end{aligned} \tag{2.24}$$

where

$$F'=u'+iv'. \tag{2.25}$$

Introducing the nondimensional variables

$$\begin{aligned}\eta=\frac{z'W_o'}{\upsilon},\qquad t=\frac{W_o'^2 t'}{4\upsilon},\qquad \omega=\frac{4\upsilon\omega'}{W_o'^2},\qquad U=\frac{U'}{U_o'},\\ u=\frac{u'}{U_o'},\qquad v=\frac{v'}{U_o'},\qquad F=\frac{F'}{U_o'},\end{aligned} \tag{2.26}$$

the boundary value problem consisting of (2.23) and conditions (2.24) yields

$$\begin{aligned}\frac{1}{4}\frac{\partial F}{\partial t}-\left(1+\epsilon Ae^{i\omega t}\right)\frac{\partial F}{\partial \eta}+2iNF=\frac{1}{4}\left(i\omega\epsilon e^{\iota\omega t}\right)+2iN\left(1+\epsilon e^{i\omega t}\right)+\frac{\partial^2 F}{\partial \eta^2}\\ +\alpha\left(\frac{1}{4}\frac{\partial^3 F}{\partial \eta^2\partial t}-\left(1+\epsilon Ae^{i\omega t}\right)\frac{\partial^3 F}{\partial \eta^3}\right),\end{aligned} \tag{2.27}$$

$$\begin{gathered}F=0 \quad \text{at } \eta=0,\\ F\longrightarrow 1+\epsilon e^{i\omega t} \quad \text{as } \eta\longrightarrow\infty,\end{gathered} \tag{2.28}$$

where

$$\alpha=\frac{\alpha^* W_o^{'2}}{\upsilon^2}, \qquad N=\frac{\Omega\nu}{W_o^{'2}}. \tag{2.29}$$

3. Analytical solution

The solution of (2.27) subject to conditions (2.28) is written as

$$F(\eta,t)=f_1(\eta)+\epsilon e^{i\omega t}f_2(\eta). \tag{3.1}$$

Using above equation into (2.27) and separating the harmonic and nonharmonic terms we obtain

$$\begin{gathered}\alpha\frac{d^3 f_1}{d\eta^3}-\frac{d^2 f_1}{d\eta^2}-\frac{df_1}{d\eta}+iNf_1=iN,\\ \alpha\frac{d^3 f_2}{d\eta^3}-\left(1+\frac{i\alpha\omega}{4}\right)\frac{d^2 f_2}{d\eta^2}-\frac{df_2}{d\eta}+iN_1 f_2=iN_1+A\frac{df_1}{d\eta}-\alpha A\frac{d^3 f_1}{d\eta^3},\end{gathered} \tag{3.2}$$

where

$$N_1=N+\frac{\omega}{4}. \tag{3.3}$$

The corresponding boundary conditions are

$$\begin{gathered}f_1=0 \quad \text{at } \eta=0,\\ f_1\longrightarrow 1 \quad \text{as } \eta\longrightarrow\infty,\\ f_2=0 \quad \text{at } \eta=0,\\ f_2\longrightarrow 1 \quad \text{as } \eta\longrightarrow\infty.\end{gathered} \tag{3.4}$$

It is worth emphasizing that (3.2) for second-grade fluid are third order (one order higher than the Navier-Stokes equation). Thus, one needs three conditions for the unique solution whereas two conditions are prescribed. One possible way to overcome this difficulty is to employ a perturbation analysis (as in Beard and Walters [2], Soundalgekar and Puri [23], Kaloni [14], and Hayat et al. [8]) and take the solution as follows

$$\begin{aligned} f_1 &= f_{01} + \alpha f_{11} + o(\alpha^2), \\ f_2 &= f_{02} + \alpha f_{12} + o(\alpha^2). \end{aligned} \tag{3.5}$$

Substituting (3.5) into (3.2), (3.4), equating the coefficients of α, and then solving the corresponding problems we have for f_1 and f_2

$$\begin{gathered} f_1 = 1 - (1 + \alpha\eta L)e^{-h\eta}, \\ f_2 = 1 - Se^{-g\eta} - (1-S)e^{-h\eta} + \alpha\left[c_5 e^{-g\eta} - \eta M e^{-g\eta} - (\eta c_{3+} c_5)e^{-h\eta}\right], \end{gathered} \tag{3.6}$$

and so from (3.1),

$$F = 1 - (1 + \alpha\eta L)e^{-h\eta} + \epsilon e^{i\omega t}\begin{bmatrix} 1 - Se^{-g\eta} - (1-S)e^{-h\eta} \\ +\alpha\{c_5 e^{-g\eta} - \eta M e^{-g\eta} - (\eta c_{3+} c_5)e^{-h\eta}\} \end{bmatrix}, \tag{3.7}$$

which upon separating real and imaginary parts gives

$$\begin{aligned} u = u_0 + \epsilon e^{i\omega t}u_1 &= 1 - e^{-h_r\eta}\left((1 + \alpha\eta L_r)\cosh_i\eta + \alpha\eta L_i \sinh_i\eta\right) \\ &+ \epsilon e^{i\omega t}\begin{bmatrix} 1 - e^{-g_r\eta}\left(\left(1 - \dfrac{4Ah_i}{\omega}\right)\cos g_i\eta + \dfrac{4Ah_r}{\omega}\sin g_i\eta\right) \\ +e^{-h_r\eta}\left(\dfrac{4Ah_i}{\omega}\cosh_i\eta - \dfrac{4Ah_r}{\omega}\sinh_i\eta\right) \\ +\alpha e^{-g_r\eta}\left(c_{5r}\cos g_i\eta + c_{5i}\sin g_i\eta\right) \\ -\alpha\eta e^{-g_r\eta}\left(M_r\cos g_i\eta + M_i\sin g_i\eta\right) \\ -\alpha e^{-h_r\eta}\left((\eta c_{3r} + c_{5r})\cos h_i\eta + (\eta c_{3i} + c_{5i})\sin h_i\eta\right) \end{bmatrix}, \end{aligned} \tag{3.8}$$

$$v = v_0 + \epsilon e^{i\omega t} v_1 = e^{-h_r\eta}((1+\alpha\eta L_r)\sinh_i\eta + \alpha\eta L_i\cosh_i\eta) + \epsilon e^{i\omega t}\left[\begin{array}{c} e^{-g_r\eta}\left(\left(1-\frac{4Ah_i}{\omega}\right)\sin g_i\eta - \frac{4Ah_r}{\omega}\cos g_i\eta\right) \\ -e^{-h_r\eta}\left(\frac{4Ah_i}{\omega}\sinh_i\eta + \frac{4Ah_r}{\omega}\cosh_i\eta\right) \\ +\alpha e^{-g_r\eta}(c_{5i}\cos g_i\eta - c_{5r}\sin g_i\eta) \\ -\alpha\eta e^{-g_r\eta}(M_i\cos g_i\eta - M_r\sin g_i\eta) \\ -\alpha e^{-h_r\eta}((\eta c_{3i}+c_{5i})\cosh_i\eta - (\eta c_{3r}+c_{5r})\sinh_i\eta) \end{array}\right], \tag{3.9}$$

where

$$h = h_r + ih_i = \frac{1+\sqrt{1+4iN}}{2},$$

$$h_r = \frac{1}{2} + \frac{1}{2}\left[\frac{1}{2}(1+\sqrt{1+16N^2})\right]^2, \qquad h_i = \frac{1}{2}\left[\frac{1}{2}\left(-1+\sqrt{1+16N^2}\right)\right]^2,$$

$$a = \left[\frac{1}{2}\left(1+\sqrt{1+16N^2}\right)\right]^2, \qquad b = \left[\frac{1}{2}\left(-1+\sqrt{1+16N^2}\right)\right]^2,$$

$$r = a^2 + b^2 = \sqrt{1+16N^2}, \qquad g = g_r + ig_i = \frac{1+\sqrt{1+4iN_1}}{2},$$

$$g_r = \frac{1}{2} + \frac{1}{2}\left[\frac{1}{2}\left(1+\sqrt{1+16N_1^2}\right)\right]^2, \qquad g_i = \frac{1}{2}\left[\frac{1}{2}\left(-1+\sqrt{1+16N_1^2}\right)\right]^2,$$

$$a_1 = \left[\frac{1}{2}\left(1+\sqrt{1+16N_1^2}\right)\right]^2, \qquad b_1 = \left[\frac{1}{2}\left(-1+\sqrt{1+16N_1^2}\right)\right]^2,$$

$$r_1 = a_1^2 + b_1^2 = \sqrt{1+16N_1^2}, \qquad S = S_r + iS_i = 1 - \frac{4iAh}{\omega},$$

$$S_r = 1 + \frac{4Ah_i}{\omega}, \qquad S_i = \frac{4Ah_r}{\omega}, \qquad L = L_r + iL_i = \frac{h^3}{\sqrt{1+4iN}},$$

$$L_r = \frac{1}{r}\left[\begin{array}{c} \frac{1}{4}\left(\frac{r+1}{2}\right)^{1/2}\left(1+\left(\frac{r+1}{2}\right)\right) + \frac{1}{2} - 2N^2 \\ -\frac{7}{4}\left(\frac{r-1}{2}\right)\left(\frac{r+1}{2}\right)^{1/2} \end{array}\right],$$

$$L_i = \frac{1}{r}\left[\begin{array}{c} \frac{1}{4}\left(\frac{r-1}{2}\right)^{1/2}\left(1-\left(\frac{r-1}{2}\right)\right) + \frac{5N}{2} \\ +\frac{3}{2}\left(\frac{r+1}{2}\right)\left(\frac{r-1}{2}\right)^{1/2} \end{array}\right],$$

$$M = M_r + iM_i = \frac{g^2(g+i\omega/4)(1-4iAh/\omega)}{\sqrt{1+4iN_1}},$$

$$M_r = \frac{1}{r_1}\left[\begin{array}{c} \frac{1}{4}\left(\frac{r_1+1}{2}\right)^{1/2} + \frac{1}{8}(r_1+1)\left(\frac{r_1+1}{2}\right)^{1/2} + \frac{r_1}{2} \\ +\frac{3}{8}(r_1-1)\left(\frac{r_1+1}{2}\right)^{1/2} - N_1^2\left(\frac{r_1+1}{2}\right)^{1/2} \\ +\frac{Ah_i}{\omega}\left(\begin{array}{c} \left(\frac{r_1+1}{2}\right)^{1/2} + \left(\frac{r_1+1}{2}\right)\left(\frac{r_1+1}{2}\right)^{1/2} \\ +2r_1 + \frac{3}{2}(r_1-1)\left(\frac{r_1+1}{2}\right)^{1/2} \\ -4N_1^2\left(\frac{r_1+1}{2}\right)^{1/2} \end{array}\right) \\ +\frac{Ahr}{\omega}\left(\begin{array}{c} 2N_1(r_1+1) + \frac{3}{2}(r_1+1)\left(\frac{r_1-1}{2}\right)^{1/2} \\ -\left(\frac{r_1-1}{2}\right)^{1/2} + \left(\frac{r_1-1}{2}\right)\left(\frac{r_1-1}{2}\right)^{1/2} \\ +4N_1^2\left(\frac{r_1-1}{2}\right)^{1/2} \end{array}\right) \end{array}\right],$$

$$M_i = \frac{1}{r_1}\left[\begin{array}{c} 2N_1r_1 - \frac{1}{4}\left(\frac{r_1-1}{2}\right)^{1/2} + \frac{3}{8}(r_1+1)\left(\frac{r_1+1}{2}\right)^{1/2} \\ +\frac{3}{8}(r_1-1)\left(\frac{r_1-1}{2}\right)^{1/2} + N_1^2\left(\frac{r_1-1}{2}\right)^{1/2} \\ +\frac{Ah_i}{\omega}\left(\begin{array}{c} 8N_1 + 4N_1r_1 + \left(\frac{r_1-1}{2}\right)^{1/2} \\ +\left(\frac{r_1-1}{2}\right)\left(\frac{r_1-1}{2}\right)^{1/2} + 4N_1^2\left(\frac{r_1-1}{2}\right)^{1/2} \\ +\frac{5}{2}(r_1+1)\left(\frac{r_1-1}{2}\right)^{1/2} \end{array}\right) \\ +\frac{Ahr}{\omega}\left(\begin{array}{c} -3N_1(r_1-1) - 2r_1 \\ -\left(\frac{r_1+1}{2}\right)^{1/2}\left(1 + \left(\frac{r_1+1}{2}\right)\right) \\ +4N_1^2\left(\frac{r_1+1}{2}\right)^{1/2} \end{array}\right) \end{array}\right],$$

$$c_5 = c_{5r} + ic_{5i} = (c_{1r} + c_{2r} + c_{4r}) + i(c_{1i} + c_{2i} + c_{4i}),$$

$$c_1 = c_{1r} + ic_{1i} = \frac{4h^3}{\iota\omega}(A - (1-S)),$$

$$c_{1r} = 4(h_i^3 - 3h_r^2h_i)\left(A + \frac{4Ah_i}{\omega}\right) - 16\frac{Ah_r}{\omega}(h_r^3 - 3h_i^2h_r),$$

$$c_{1i} = -4(h_r^3 - 3h_i^2h_r)\left(A + \frac{4Ah_i}{\omega}\right) - 16\frac{Ah_r}{\omega}(h_r^3 - 3h_r^2h_i),$$

$$c_2 = c_{2r} + ic_{2i} = \frac{4A(h^3 + L)}{i\omega},$$

$$c_{2r} = \frac{4A}{\omega}(L_i(h_r^3 - 3h_i^2h_r) - L_r(h_i^3 - 3h_r^2h_i)),$$

$$
\begin{gathered}
c_{2i} = -\frac{4A}{\omega}\left(L_r\left(h_r^3 - 3h_i^2 h_r\right) + L_i\left(h_i^3 - 3h_r^2 h_i\right)\right), \\
c_3 = c_{3r} + ic_{3i} = \frac{4AhL}{i\omega}, \\
c_{3r} = \frac{4A}{\omega}\left(h_i L_r + h_r L_i\right), \qquad c_{3i} = -\frac{4A}{\omega}\left(h_r L_r - h_i L_i\right), \\
c_4 = c_{4r} + ic_{4i} = \frac{16AhL(1-2h)}{\omega^2}, \\
c_{4r} = \frac{16A}{\omega^2}\left(\left(1-2h_r\right)\left(h_r L_r - h_i L_i\right) + 2h_i\left(h_r L_i + h_i L_r\right)\right), \\
c_{4i} = \frac{16A}{\omega^2}\left(\left(1-2h_r\right)\left(h_r L_i + h_i L_r\right) - 2h_i\left(h_r L_r - h_i L_i\right)\right).
\end{gathered} \tag{3.10}
$$

The drag P_{xz} and lateral stress P_{yz} at the plate in nondimensional form can be written, respectively, as

$$
\begin{aligned}
P_{xz} &= \frac{P'_{x'z'}}{U'_0 W'_0 \rho'} = \frac{\partial u}{\partial \eta} - \frac{\alpha}{4}\left[\frac{\partial^2 u}{\partial \eta \partial t} - 4\left(1 + \epsilon A e^{i\omega t}\right)\frac{\partial^2 u}{\partial \eta^2}\right], \\
P_{yz} &= \frac{P'_{y'z'}}{U'_0 W'_0 \rho'} = \frac{\partial v}{\partial \eta} - \frac{\alpha}{4}\left[\frac{\partial^2 v}{\partial \eta \partial t} - 4\left(1 + \epsilon A e^{i\omega t}\right)\frac{\partial^2 v}{\partial \eta^2}\right].
\end{aligned} \tag{3.11}
$$

The above equations after using (3.8) and (3.9) give

$$
P_{xz} = \alpha\left(h_i^2 - h_r^2\right) - h_r - \alpha L_r + \epsilon e^{i\omega t}
\begin{bmatrix}
g_r - \dfrac{4A}{\omega}\left(h_i g_r + h_r g_i\right) - \dfrac{8Ah_i h_r}{\omega} \\
-\alpha\left(g_r c_{5r} - g_i c_{5i}\right) + \alpha A\left(h_i^2 - h_r^2\right) \\
-\alpha M_r + \alpha\left(h_r c_{5r} - h_i c_{5i}\right) - \alpha c_{3r} \\
-i\alpha\omega\left(\dfrac{g_r}{4} - \dfrac{A}{\omega}\left(h_i g_r + h_r g_i\right) - \dfrac{2Ah_i h_r}{\omega}\right) \\
+\alpha\begin{pmatrix}\left(g_i^2 - g_r^2\right)\left(1 - \dfrac{4Ah_i}{\omega}\right) + \dfrac{8Ah_r g_i g_r}{\omega} \\ + \dfrac{12Ah_i h_r^2}{\omega} - \dfrac{4Ah_i^3}{\omega}\end{pmatrix}
\end{bmatrix}, \tag{3.12}
$$

$$
P_{yz} = h_i - \alpha L_i - 2\alpha h_i h_r + \epsilon e^{i\omega t}
\begin{bmatrix}
g_i + \dfrac{4A}{\omega}\left(h_r g_r - h_i g_i\right) + \dfrac{4A}{\omega}\left(h_r^2 - h_i^2\right) \\
-\alpha\left(g_r c_{5i} + g_i c_{5r}\right) - 2\alpha A h_i h_r \\
-\alpha M_i + \alpha\left(h_r c_{5i} + h_i c_{5r}\right) - \alpha c_{3i} \\
-i\alpha\omega\left(g_i + \dfrac{4A}{\omega}\left(h_r g_r - h_i g_i\right) + \dfrac{4A}{\omega}\left(h_r^2 - h_i^2\right)\right) \\
+\alpha\begin{pmatrix}-2g_i g_r + \dfrac{8Ah_i g_i g_r}{\omega} - \dfrac{4Ah_r}{\omega}\left(g_r^2 - g_i^2\right) \\ + \dfrac{12Ah_r h_i^2}{\omega} - \dfrac{4Ah_r^3}{\omega}\end{pmatrix}
\end{bmatrix}. \tag{3.13}
$$

The above equations can also be written as

$$P_{xz} = \alpha(h_i^2 - h_r^2) - h_r - \alpha L_r + \epsilon|B|\cos(\omega t + \beta),$$
$$P_{yz} = h_i - \alpha L_i - 2\alpha h_i h_r + \epsilon\,|B_1|\cos(\omega t + \gamma), \tag{3.14}$$

where

$$B = B_r + iB_i = \left[\begin{array}{c} g_r - \dfrac{4A}{\omega}(h_i g_r + h_r g_i) - \dfrac{8Ah_i h_r}{\omega} \\ -\alpha(g_r c_{5r} - g_i c_{5i}) + \alpha A(h_i^2 - h_r^2) \\ -\alpha M_r + \alpha(h_r c_{5r} - h_i c_{5i}) - \alpha c_{3r} \\ -i\alpha\omega\left(\dfrac{g_r}{4} - \dfrac{A}{\omega}(h_i g_r + h_r g_i) - \dfrac{2Ah_i h_r}{\omega}\right) \\ +\alpha\left(\begin{array}{c}(g_i^2 - g_r^2)\left(1 - \dfrac{4Ah_i}{\omega}\right) + \dfrac{8Ah_r g_i g_r}{\omega} \\ + \dfrac{12Ah_i h_r^2}{\omega} - \dfrac{4Ah_i^3}{\omega}\end{array}\right)\end{array}\right],$$

$$B_r = \left[\begin{array}{c} g_r - \dfrac{4A}{\omega}(h_i g_r + h_r g_i) - \dfrac{8Ah_i h_r}{\omega} \\ -\alpha(g_r c_{5r} - g_i c_{5i}) + \alpha A(h_i^2 - h_r^2) \\ -\alpha M_r + \alpha(h_r c_{5r} - h_i c_{5i}) - \alpha c_{3r} \\ +\alpha\left(\begin{array}{c}(g_i^2 - g_r^2)\left(1 - \dfrac{4Ah_i}{\omega}\right) + \dfrac{8Ah_r g_i g_r}{\omega} \\ + \dfrac{12Ah_i h_r^2}{\omega} - \dfrac{4Ah_i^3}{\omega}\end{array}\right)\end{array}\right],$$

$$B_i = \alpha\omega\left(\frac{g_r}{4} - \frac{A}{\omega}(h_i g_r + h_r g_i) - \frac{2Ah_i h_r}{\omega}\right),$$

$$B_1 = B_{1r} + iB_{1i} = \left[\begin{array}{c} g_i + \dfrac{4A}{\omega}(h_r g_r - h_i g_i) + \dfrac{4A}{\omega}(h_r^2 - h_i^2) \\ -\alpha(g_r c_{5i} + g_i c_{5r}) - 2\alpha A h_i h_r \\ -\alpha M_i + \alpha(h_r c_{5i} + h_i c_{5r}) - \alpha c_{3i} \\ -i\alpha\omega\left(g_i + \dfrac{4A}{\omega}(h_r g_r - h_i g_i) + \dfrac{4A}{\omega}(h_r^2 - h_i^2)\right) \\ +\alpha\left(\begin{array}{c}-2g_i g_r + \dfrac{8Ah_i g_i g_r}{\omega} - \dfrac{4Ah_r}{\omega}(g_r^2 - g_i^2) \\ + \dfrac{12Ah_r h_i^2}{\omega} - \dfrac{4Ah_r^3}{\omega}\end{array}\right)\end{array}\right],$$

$$\beta = \tan^{-1}\left(\frac{B_i}{B_r}\right), \qquad \gamma = \tan^{-1}\left(\frac{B_{1i}}{B_{1r}}\right),$$

$$B_{1r} = \begin{bmatrix} g_i + \dfrac{4A}{\omega}(h_r g_r - h_i g_i) + \dfrac{4A}{\omega}(h_r^2 - h_i^2) \\ -\alpha(g_r c_{5i} + g_i c_{5r}) - 2\alpha A h_i h_r \\ -\alpha M_i + \alpha(h_r c_{5i} + h_i c_{5r}) - \alpha c_{3i} \\ +\alpha \begin{pmatrix} -2g_i g_r + \dfrac{8Ah_i g_i g_r}{\omega} - \dfrac{4Ah_r}{\omega}(g_r^2 - g_i^2) \\ +\dfrac{12Ah_r h_i^2}{\omega} - \dfrac{4Ah_r^3}{\omega} \end{pmatrix} \end{bmatrix},$$

$$B_{1i} = \alpha\omega\left(g_i + \frac{4A}{\omega}(h_r g_r - h_i g_i) + \frac{4A}{\omega}(h_r^2 - h_i^2)\right). \tag{3.15}$$

We now proceed to derive the energy equation appropriate for the problem under consideration. We start with the energy equation (2.5). It follows from (2.5), (2.6), (2.7), (2.8), and (2.9) and $\mathbf{L} = \operatorname{grad}\mathbf{V}$ that

$$\mathbf{T}\cdot\mathbf{L} = \mu\left[\left(\frac{\partial u'}{\partial z'}\right)^2 + \left(\frac{\partial v'}{\partial z'}\right)^2\right] + \alpha\begin{bmatrix} \dfrac{\partial u'}{\partial z'}\left(\dfrac{\partial^2 u'}{\partial t'\partial z'} + w'\dfrac{\partial^2 u'}{\partial z'^2}\right) \\ +\dfrac{\partial v'}{\partial z'}\left(\dfrac{\partial^2 v'}{\partial t'\partial z'} + w'\dfrac{\partial^2 v'}{\partial z'^2}\right) \end{bmatrix}. \tag{3.16}$$

Following the thermodynamical considerations given in Dunn and Fosdick [4] for fluids of second grade and representing $\mathbf{q}$ by Fourier's law with a constant thermal conductivity, k, (2.5) reduces to

$$\rho' c\left[\frac{\partial T}{\partial t'} + w'\frac{\partial T}{\partial z'}\right] - k\frac{\partial^2 T}{\partial z'^2} = \mu\left[\left(\frac{\partial u'}{\partial z'}\right)^2 + \left(\frac{\partial v'}{\partial z'}\right)^2\right] + \alpha_1\begin{bmatrix} \dfrac{\partial u'}{\partial z'}\left(\dfrac{\partial^2 u'}{\partial t'\partial z'} + w'\dfrac{\partial^2 u'}{\partial z'^2}\right) \\ +\dfrac{\partial v'}{\partial z'}\left(\dfrac{\partial^2 v'}{\partial t'\partial z'} + w'\dfrac{\partial^2 v'}{\partial z'^2}\right) \end{bmatrix}, \tag{3.17}$$

where c is the specific heat. The boundary conditions for the temperature are

$$\begin{aligned} T &= T_0 \quad \text{at } z' = 0, \\ T &\longrightarrow T_\infty \quad \text{as } z' \longrightarrow \infty. \end{aligned} \tag{3.18}$$

Using

$$\theta = \frac{T - T_0}{T_\infty - T_0}, \tag{3.19}$$

(3.17) and boundary conditions (3.18) become

$$-\frac{\partial^2\theta}{\partial\eta^2}-P_r\left(1+\epsilon Ae^{i\omega t}\right)\frac{\partial\theta}{\partial\eta}+\frac{P_r}{4}\frac{\partial\theta}{\partial t}=E_c\left[\left(\frac{\partial u}{\partial\eta}\right)^2+\left(\frac{\partial v}{\partial\eta}\right)^2\right]+P\begin{bmatrix}\dfrac{\partial u}{\partial\eta}\dfrac{\partial^2 u}{\partial\eta\partial t}+\dfrac{\partial v}{\partial\eta}\dfrac{\partial^2 v}{\partial\eta\partial t}\\ -\left(1+\epsilon Ae^{\iota\omega t}\right)\dfrac{\partial u}{\partial\eta}\dfrac{\partial^2 u}{\partial\eta^2}\\ -\left(1+\epsilon Ae^{\iota\omega t}\right)\dfrac{\partial v}{\partial\eta}\dfrac{\partial^2 v}{\partial\eta^2}\end{bmatrix},\tag{3.20}$$

$$\begin{aligned}\theta&=0\quad\text{at }\eta=0,\\ \theta&\longrightarrow 1\quad\text{at }\eta\longrightarrow\infty,\end{aligned}\tag{3.21}$$

in which

$$P_r=\frac{\mu c}{k},\qquad E_c=\frac{k^*U_0^2}{\left(T_\infty-T_0\right)},\tag{3.22}$$

$$P=\frac{\alpha U_0^{'2}\mu}{k\left(T_\infty-T_0\right)}.\tag{3.23}$$

We further assume that

$$\theta=\theta_0+\epsilon e^{i\omega t}\theta_1.\tag{3.24}$$

Substituting (3.24) into (3.20) and boundary conditions (3.21), and equating the coefficients of the harmonic and nonharmonic term after neglecting the coefficients of ϵ^2, we get

$$\frac{d^2\theta_0}{d\eta^2}+P_r\frac{d\theta_0}{d\eta}=-E_c\left[\left(\frac{du_1}{d\eta}\right)^2+\left(\frac{dv_1}{d\eta}\right)^2\right]+P\left[\frac{du_1}{d\eta}\frac{d^2u_1}{d\eta^2}+\frac{dv_1}{d\eta}\frac{d^2v_1}{d\eta^2}\right],\tag{3.25}$$

$$\begin{aligned}\frac{d^2\theta_1}{d\eta^2}+P_r\frac{d\theta_1}{d\eta}-\frac{P_r}{4}i\omega\theta_1=-P_rA\frac{d\theta_0}{d\eta}-2E_c\left[\frac{du_1}{d\eta}\frac{du_2}{d\eta}+\frac{dv_1}{d\eta}\frac{dv_2}{d\eta}\right]\\ -P\begin{bmatrix} i\omega\left(\dfrac{du_1}{d\eta}\dfrac{du_2}{d\eta}+\dfrac{dv_1}{d\eta}\dfrac{dv_2}{d\eta}\right)\\ -\left(\dfrac{du_1}{d\eta}\dfrac{d^2u_2}{d\eta^2}+\dfrac{dv_1}{d\eta}\dfrac{d^2v_2}{d\eta^2}\right)\\ -A\left(\dfrac{du_1}{d\eta}\dfrac{d^2u_1}{d\eta^2}+\dfrac{dv_1}{d\eta}\dfrac{d^2v_1}{d\eta^2}\right)\\ -\left(\dfrac{du_2}{d\eta}\dfrac{d^2u_1}{d\eta^2}+\dfrac{dv_2}{d\eta}\dfrac{d^2v_1}{d\eta^2}\right)\end{bmatrix},\end{aligned}\tag{3.26}$$

$$\begin{aligned}\theta_0&=0\quad\text{at }\eta=0,\\ \theta_0&\longrightarrow 1\quad\text{at }\eta\longrightarrow\infty,\\ \theta_1&=0\quad\text{at }\eta=0,\\ \theta_1&\longrightarrow 0\quad\text{at }\eta\longrightarrow\infty.\end{aligned}\tag{3.27}$$

Solving (3.25) and (3.26) along with the boundary conditions (3.27), we obtain

$$\theta_0 = 1 - (1 + d_7)e^{-P_r\eta} + (d_7 + d_8\eta)e^{-2h_r\eta}, \tag{3.28}$$

$$\begin{aligned}\theta_1 = &-m_{16}e^{-f\eta} + (m_7 + m_9 + m_8\eta)e^{-2h_r\eta}\\ &+ (m_{10} + m_{14} + m_{12}\eta)e^{-(h_r+g_r)\eta}\cos(h_i - g_i)\eta\\ &+ (m_{11} + m_{15} + m_{13}\eta)e^{-(h_r+g_r)\eta}\sin(h_i - g_i)\eta,\end{aligned} \tag{3.29}$$

where

$$d_1 = -E_c(h_r^2 + h_i^2 - 2\alpha L_i h_i), \qquad d_2 = -2E_c\alpha L_r(h_r^2 + h_i^2),$$

$$d_3 = P(-h_r^3 + 3\alpha L_r h_r^2 - h_r h_i^2 + 2\alpha L_i h_i h_r + \alpha L_r h_i^2),$$

$$d_4 = -2P\alpha L_r h_r(h_r^2 + h_i^2), \qquad d_5 = d_1 + d_3, \qquad d_6 = d_2 + d_4,$$

$$d_7 = \frac{d_5}{4h_r^2 - 2P_r h_r} - \frac{d_6(4h_r - P_r)}{(4h_r^2 - 2P_r h_r)^2}, \qquad d_8 = \frac{d_6}{(4h_r^2 - 2P_r h_r)},$$

$$d_9 = \begin{bmatrix} -\frac{4Ah_r}{\omega}(h_r g_r + h_i^2) + \left(1 - \frac{4Ah_i}{\omega}\right)(h_i g_r - h_r g_i)\\ +\alpha(h_r(g_r c_{5i} + g_i c_{5r}) - h_i(g_r c_{5r} - g_i c_{5i}))\\ +\alpha(h_r M_i - h_i M_r) + \frac{4\alpha A h_r}{\omega}(g_r L_r - g_i L_i)\\ +\alpha\left(1 - \frac{4Ah_i}{\omega}\right)(g_i L_r - g_r L_i) \end{bmatrix},$$

$$d_{10} = \begin{bmatrix} -\frac{4\alpha A h_r^2}{\omega}(L_r g_r + L_i g_i) - \alpha h_r\left(1 - \frac{4Ah_i}{\omega}\right)(L_r g_i - L_i g_r)\\ -\alpha(h_r(g_r M_i + g_i M_r) - h_i(g_r M_r - g_i M_i))\\ -\frac{4\alpha A h_r h_i}{\omega}(g_i L_r - g_r L_i) + \alpha h_i\left(1 - \frac{4Ah_i}{\omega}\right)(g_r L_r - g_i L_i) \end{bmatrix},$$

$$d_{11} = \begin{bmatrix} -\frac{4Ah_r h_i}{\omega}(h_r - g_r) + \alpha\left(1 - \frac{4Ah_i}{\omega}\right)(L_r g_r - L_i g_i)\\ -\alpha(h_r(g_r c_{5r} - g_i c_{5i}) - h_i(g_r c_{5i} + g_i c_{5r}))\\ +\frac{4\alpha A h_r}{\omega}(g_i L_r - g_r L_i) + \left(1 - \frac{4Ah_i}{\omega}\right)(g_r h_r + g_i h_i)\\ -\alpha(M_r h_r + M_i h_i) \end{bmatrix},$$

$$d_{12} = \begin{bmatrix} -\frac{4\alpha A h_r^2}{\omega}(L_r g_i - L_i g_r) + \alpha h_r\left(1 - \frac{4Ah_i}{\omega}\right)(L_r g_r + L_i g_i)\\ -\alpha(h_r(g_r M_r + g_i M_i) - h_i(g_r M_i + g_i M_r))\\ -\frac{4\alpha A h_r h_i}{\omega}(g_r L_r - g_i L_i) + \alpha h_i\left(1 - \frac{4Ah_i}{\omega}\right)(g_i L_r - g_r L_i) \end{bmatrix},$$

$$d_{13}=\begin{bmatrix}\dfrac{-8Ah_rh_i}{\omega}(h_r-\alpha L_r)+\alpha c_{5r}(h_r^2+h_i^2)\\ +\dfrac{4Ah_i}{\omega}(h_r^2-h_i^2)-\dfrac{4\alpha AL_i}{\omega}(h_r^2-h_i^2)-\alpha(h_rc_{3r}+h_ic_{3i})\end{bmatrix},$$

$$d_{14}=\begin{bmatrix}\dfrac{-8Ah_rh_i}{\omega}(h_rL_r-h_iL_i)+\dfrac{4\alpha A}{\omega}(h_r^2-h_i^2)(h_rL_i-h_iL_r)\\ +\alpha c_{3r}(h_r^2+h_i^2)\end{bmatrix},$$

$$d_{15}=\begin{bmatrix}\dfrac{4Ah_r}{\omega}(g_r^2-g_i^2)(h_r-\alpha L_r)+2g_rg_i\left(1-\dfrac{4Ah_i}{\omega}\right)(h_r-\alpha L_r)\\ -2\alpha(h_r(g_rM_i-g_iM_r)-h_i(g_rM_r-g_iM_i))\\ -\alpha(g_r^2-g_i^2)(h_rc_{5i}-h_ic_{5r})\\ -\left(1-\dfrac{4Ah_i}{\omega}\right)(g_r^2-g_i^2)(h_i-\alpha L_i)\\ -2\alpha g_rg_i(h_rc_{5r}+h_ic_{5i})+\dfrac{8Ah_rg_rg_i}{\omega}(h_i-\alpha L_i)\end{bmatrix},$$

$$d_{16}=\begin{bmatrix}\dfrac{4\alpha Ah_r}{\omega}(g_r^2-g_i^2)(h_rL_r-h_iL_i)+2\alpha g_rg_i(h_rM_r+g_iM_i)\\ +\alpha(g_r^2-g_i^2)(h_rM_i-h_iM_r)+\dfrac{8\alpha Ah_rg_rg_i}{\omega}(h_rL_i+h_iL_r)\\ -\alpha\left(1-\dfrac{4Ah_i}{\omega}\right)(g_r^2-g_i^2)(h_rL_i+h_iL_r)\\ +2\alpha g_rg_i\left(1-\dfrac{4Ah_i}{\omega}\right)(h_rL_r-h_iL_i)\end{bmatrix},$$

$$d_{17}=\begin{bmatrix}-\dfrac{4Ah_r}{\omega}(g_r^2-g_i^2)(h_i-\alpha L_i)+2g_rg_i\left(1-\dfrac{4Ah_i}{\omega}\right)(h_i-\alpha L_i)\\ +2\alpha(h_r(g_rM_r-g_iM_i)+h_i(g_rM_i-g_iM_r))\\ +\alpha(g_r^2-g_i^2)(h_rc_{5r}+h_ic_{5i})+\dfrac{8Ah_rg_rg_i}{\omega}(h_r+\alpha L_r)\\ -\left(1-\dfrac{4Ah_i}{\omega}\right)(g_r^2-g_i^2)(h_r-\alpha L_r)-2\alpha g_rg_i(h_rc_{5i}-h_ic_{5r})\end{bmatrix},$$

$$d_{18}=\begin{bmatrix}-\dfrac{4\alpha Ah_r}{\omega}(g_r^2-g_i^2)(h_rL_i+h_iL_r)+\dfrac{8\alpha Ah_rg_rg_i}{\omega}(h_rL_r-h_iL_i)\\ -2\alpha g_rg_i\left(1-\dfrac{4Ah_i}{\omega}\right)(h_rL_i+h_iL_r)\\ +2\alpha g_rg_i(h_rM_i-h_iM_r)-\alpha(g_r^2-g_i^2)(h_rM_r+h_iM_i)\\ -\alpha\left(1-\dfrac{4Ah_i}{\omega}\right)(g_r^2-g_i^2)(h_rL_r-h_iL_i)\end{bmatrix},$$

$$d_{19}=\begin{bmatrix}\dfrac{8Ah_rh_i}{\omega}(h_r^2+h_i^2)-\alpha(h_r^3c_{5r}+h_i^3c_{5i})+\alpha h_r^2h_ic_{5i}\\ -\dfrac{12\alpha Ah_rh_i}{\omega}(h_rL_r-h_iL_i)+\dfrac{4\alpha A}{\omega}(L_rh_i^3-L_ih_r^3)\\ +2\alpha c_{3r}(h_r^2+h_i^2)+\alpha h_rh_i(h_rc_{5i}-h_ic_{5r})\end{bmatrix},$$

$$d_{20} = \begin{bmatrix} \dfrac{8\alpha A h_r h_i L_r}{\omega}(h_r^2 + h_i^2) - \dfrac{4\alpha A L_i}{\omega}(h_r^4 - h_i^4) \\ -\alpha(h_r^3 c_{3r} + h_i^3 c_{3i}) + \alpha h_r h_i (h_r c_{3i} - h_i c_{3r}) \end{bmatrix},$$

$$d_{21} = \begin{bmatrix} g_i\left(1 - \dfrac{4Ah_i}{\omega}\right)(h_r^2 - h_i^2) - \alpha h_r^2 (g_r c_{5i} + g_i c_{5r}) + 2\alpha h_r h_i M_r \\ +2\alpha h_r h_i (g_r c_{5r} - g_i c_{5i}) - 2h_r g_r \left(1 - \dfrac{4Ah_i}{\omega}\right)(h_i - \alpha L_i) \\ +\dfrac{4Ah_r g_r}{\omega}(h_r^2 - h_i^2) + \alpha h_i^2 (g_r c_{5i} + g_i c_{5r}) - \dfrac{8\alpha A h_r^2 g_r L_r}{\omega} \\ +\dfrac{8Ah_r^2 g_i}{\omega}(h_i - \alpha L_i) - 2\alpha L_r \left(1 - \dfrac{4Ah_i}{\omega}\right)(h_r g_i - h_i g_r) \\ -\dfrac{8\alpha A h_r h_i}{\omega}(L_r g_i - L_i g_r) + 2\alpha L_i h_i g_i \left(1 - \dfrac{4Ah_i}{\omega}\right) \end{bmatrix},$$

$$d_{22} = \begin{bmatrix} \alpha(g_r M_i + g_i M_r)(h_r^2 - h_i^2) \\ -\dfrac{8\alpha A h_r}{\omega}(h_r^2 - h_i^2)(L_r g_r + L_i g_i) \\ +2\alpha h_r h_i (g_r M_r - g_i M_i) + \dfrac{8\alpha A h_r h_i}{\omega}(L_r g_i - L_i g_r) \\ +\alpha\left(1 - \dfrac{4Ah_i}{\omega}\right)(L_r g_i - L_i g_r)(h_r^2 g_i - h_i^2) \\ -2\alpha h_r h_i \left(1 - \dfrac{4Ah_i}{\omega}\right)(L_r g_r + L_i g_i) \end{bmatrix},$$

$$d_{23} = \begin{bmatrix} -g_r\left(1 - \dfrac{4Ah_i}{\omega}\right)(h_r^2 - h_i^2) + \alpha g_r c_{5r}(h_r^2 - h_i^2) \\ +\alpha g_i c_{5i}(h_r^2 + h_i^2) - \dfrac{8Ah_r^2 g_r}{\omega}(h_i - \alpha L_i) \\ +\dfrac{4Ah_r g_i}{\omega}(h_r^2 - h_i^2) + 2\alpha h_r h_i M_i \\ +2\alpha h_r h_i (g_r c_{5i} + g_i c_{5r}) \\ -2h_r g_i \left(1 - \dfrac{4Ah_i}{\omega}\right)(h_i + \alpha L_i) \\ -\dfrac{8\alpha A h_r^2 g_i L_r}{\omega} + \dfrac{8\alpha A h_r h_i}{\omega}(L_r g_r + L_i h_i) \\ +2\alpha h_i \left(1 - \dfrac{4Ah_i}{\omega}\right)(L_r g_i - L_i g_r) \end{bmatrix},$$

$$d_{24} = \begin{bmatrix} -\alpha\left(1 - \dfrac{4Ah_i}{\omega}\right)(h_r^2 - h_i^2)(L_r g_r + L_i g_i) \\ -\alpha(h_r^2 - h_i^2)(M_r g_r - M_i g_i) - \dfrac{8\alpha A h_r^2 h_i}{\omega}(L_r g_r + L_i g_i) \\ +\dfrac{4\alpha A h_r}{\omega}(L_r g_i - L_i g_r)(h_r^2 - h_i^2) \\ -2\alpha h_r h_i \left(1 - \dfrac{4Ah_i}{\omega}\right)(L_r g_i - L_i g_r) \\ -2\alpha h_r h_i (M_i g_r + M_r g_i) \end{bmatrix},$$

$$d_{25} = \begin{bmatrix} -\alpha(h_r^3 c_{5r} + h_i^3 c_{5i}) + \alpha c_{3r}(h_r^2 - h_i^2) + \dfrac{8\alpha A h_r L_i}{\omega}(h_r^2 - h_i^2) \\ +2\alpha h_r h_i c_{3i} - \alpha h_r h_i (h_r c_{5i} + h_i c_{5r}) - \dfrac{8\alpha A h_i^3 L_r}{\omega} \end{bmatrix},$$

$$d_{26} = \begin{bmatrix} -\dfrac{4\alpha A L_i}{\omega}(h_r^4 + h_i^4) - \dfrac{8\alpha A h_i^2 L_i h_r^2}{\omega} \\ -\alpha(h_r^3 c_{3r} + h_i^3 c_{3i}) - \alpha h_r h_i (h_r c_{3i} + h_i c_{3r}) \end{bmatrix},$$

$$d_{27} = \left[-h_r^3 + 3\alpha h_r^2 L_r - h_r h_i^2 + 2\alpha h_r h_i L_i + \alpha L_r h_i^2 \right],$$

$$d_{28} = -2\alpha L_r h_r (h_r^2 + h_i^2),$$

$$m_1 = m_{1r} + i m_{1i} = \begin{bmatrix} -P_r A(P_r(1+d_7) - 2h_r d_7 + d_8) + PAd_{27} \\ -(2E_c + i\omega P) d_{13} + P(d_{19} + d_{25}) \end{bmatrix},$$

$$m_2 = m_{2r} + i m_{2i} = \begin{bmatrix} 2P_r A h_r d_8 - (2E_c + i\omega P) d_{14} \\ +PAd_{28} + P(d_{20} + d_{26}) \end{bmatrix},$$

$$m_3 = m_{3r} + i m_{3i} = \left[-(2E_c + i\omega P) d_9 + P(d_{15} + d_{21}) \right],$$

$$m_4 = m_{4r} + i m_{4i} = \left[-(2E_c + i\omega P) d_{10} + P(d_{16} + d_{22}) \right],$$

$$m_5 = m_{5r} + i m_{5i} = \left[-(2E_c + i\omega P) d_{11} + P(d_{17} + d_{23}) \right],$$

$$m_6 = m_{6r} + i m_{6i} = \left[-(2E_c + i\omega P) d_{12} + P(d_{18} + d_{24}) \right],$$

$$m_7 = m_{7r} + i m_{7i},$$

$$m_{7r} = \frac{m_{1r}(4h_r^2 - 2h_r P_r) - m_{1i}(\omega P_r/4)}{(4h_r^2 - 2h_r P_r)^2 + (\omega P_r/4)^2}, \qquad m_{7i} = \frac{m_{1i}(4h_r^2 - 2h_r P_r) + m_{1r}(\omega P_r/4)}{(4h_r^2 - 2h_r P_r)^2 + (\omega P_r/4)^2},$$

$$m_8 = m_{8r} + i m_{8i},$$

$$m_{8r} = \frac{m_{2r}(4h_r^2 - 2h_r P_r) - m_{2i}(\omega P_r/4)}{(4h_r^2 - 2h_r P_r)^2 + (\omega P_r/4)^2}, \qquad m_{8i} = \frac{m_{2i}(4h_r^2 - 2h_r P_r) + m_{2r}(\omega P_r/4)}{(4h_r^2 - 2h_r P_r)^2 + (\omega P_r/4)^2},$$

$$n_1 = n_{1r} + i n_{1i} = \begin{bmatrix} (4h_r^2 - 2h_r P_r)^2 + \left(\dfrac{\omega P_r}{4}\right)^2 \\ -2i(4h_r^2 - 2h_r P_r)\left(\dfrac{\omega P_r}{4}\right) \end{bmatrix},$$

$$n_{1r} = (4h_r^2 - 2h_r P_r)^2 + \left(\frac{\omega P_r}{4}\right)^2, \qquad n_{1i} = -2(4h_r^2 - 2h_r P_r)\left(\frac{\omega P_r}{4}\right),$$

$$m_9 = m_{9r} + i m_{9i},$$

$$m_{9r} = \frac{(4h_r - P_r)(n_{1r} m_{2r} - m_{2i} n_{1i})}{n_{1r}^2 + n_{1i}^2}, \qquad m_{9i} = \frac{(4h_r - P_r)(n_{1i} m_{2r} + m_{2i} n_{1r})}{n_{1r}^2 + n_{1i}^2},$$

$$n_2 = n_{2r} + i n_{2i} = \begin{bmatrix} (h_r + g_r)^2 - (h_i - g_i)^2 - P_r(h_r + g_r) \\ +i\left(-2(h_r + g_r)(h_i - g_i) + P_r(h_i - g_i) - \dfrac{\omega P_r}{4} \right) \end{bmatrix},$$

$$n_{2r} = \left[(h_r + g_r)^2 - (h_i - g_i)^2 - P_r(h_r + g_r) \right],$$

$$n_{2i} = -2(h_r + g_r)(h_i - g_i) + P_r(h_i - g_i) - \frac{\omega P_r}{4},$$

$$m_{10} = m_{10r} + im_{10i}, \qquad m_{10r} = \frac{(n_{2r}m_{5r} - m_{3r}n_{2i})}{n_{2r}^2 + n_{2i}^2},$$

$$m_{10i} = \frac{(n_{2r}m_{5i} - m_{3i}n_{2i})}{n_{2r}^2 + n_{2i}^2}, \qquad m_{11} = m_{11r} + im_{11i},$$

$$m_{11r} = \frac{(n_{2i}m_{5r} + m_{3r}n_{2r})}{n_{2r}^2 + n_{2i}^2}, \qquad m_{11i} = \frac{(n_{2i}m_{5i} + m_{3i}n_{2r})}{n_{2r}^2 + n_{2i}^2},$$

$$m_{12} = m_{12r} + im_{12i}, \qquad m_{12r} = \frac{(n_{2r}m_{6r} - m_{4r}n_{2i})}{n_{2r}^2 + n_{2i}^2},$$

$$m_{12i} = \frac{(n_{2r}m_{6i} - m_{4i}n_{2i})}{n_{2r}^2 + n_{2i}^2}, \qquad m_{13} = m_{13r} + im_{13i},$$

$$m_{13r} = \frac{(n_{2i}m_{6r} + m_{6r}n_{2r})}{n_{2r}^2 + n_{2i}^2}, \qquad m_{13i} = \frac{(n_{2i}m_{6i} + m_{6i}n_{2r})}{n_{2r}^2 + n_{2i}^2},$$

$$n_3 = n_{3r} + in_{3i} = \left[(2(h_r + g_r) - P_r) + i(-2(h_i - g_i))\right],$$

$$n_{3r} = 2(h_r + g_r) - P_r, \qquad n_{3i} = -2(h_i - g_i), \qquad n_4 = n_{4r} + in_{4i},$$

$$n_{4r} = \begin{bmatrix} (h_r + g_r)^4 + (h_i - g_i)^4 + \dfrac{\omega P_r^2}{2}(h_i - g_i) \\ -6(h_r + g_r)^2(h_i - g_i)^2 + \dfrac{\omega^2 P_r^2}{16} \\ +P_r^2\left((h_r + g_r)^2 - (h_i - g_i)^2\right) - \omega P_r(h_r + g_r)(h_i - g_i) \\ +2P_r\left(-(h_r + g_r)^3 + 3(h_r + g_r)(h_i - g_i)^2\right) \end{bmatrix},$$

$$n_{4i} = \begin{bmatrix} -4(h_r + g_r)^3(h_i - g_i) + 4(h_r + g_r)(h_i - g_i)^3 \\ +2P_r(3(h_r + g_r)^2(h_i - g_i) - (h_i - g_i)^3) - \dfrac{\omega P_r^2}{2}(h_r + g_r) \\ -2P_r^2(h_r + g_r)(h_i - g_i) - \dfrac{\omega P_r}{2}\left((h_r + g_r)^2 - (h_i - g_i)^2\right) \end{bmatrix},$$

$$m_{14} = m_{14r} + im_{14i},$$

$$m_{14r} = \frac{m_{6r}(n_{4r}n_{3r} + n_{4i}n_{3i}) + m_{4r}(n_{4r}n_{3i} - n_{4i}n_{3r})}{n_{4r}^2 + n_{4i}^2},$$

$$m_{14i} = \frac{m_{6i}(n_{4r}n_{3r} + n_{4i}n_{3i}) + m_{4i}(n_{4r}n_{3i} - n_{4i}n_{3r})}{n_{4r}^2 + n_{4i}^2},$$

$$m_{15} = m_{15r} + im_{15i},$$

$$m_{15r} = \frac{-m_{6r}(n_{4r}n_{3i} - n_{4i}n_{3r}) + m_{4r}(n_{4r}n_{3r} + n_{4i}n_{3i})}{n_{4r}^2 + n_{4i}^2},$$

$$m_{14i} = \frac{-m_{6i}(n_{4r}n_{3i} - n_{4i}n_{3r}) + m_{4i}(n_{4r}n_{3r} + n_{4i}n_{3i})}{n_{4r}^2 + n_{4i}^2},$$

$$m_{16} = m_{16r} + im_{16i} = m_7 + m_9 + m_{10} + m_{14},$$

$$m_{16r} = m_{7r} + m_{9r} + m_{10r} + m_{146r}, \qquad m_{16i} = m_{7i} + m_{9i} + m_{10i} + m_{146i},$$

$$f = f_r + if_i = \frac{\left(P_r + \sqrt{P_r^2 + i\omega P_r}\right)}{2},$$

$$f_r = \frac{P_r}{2} + \frac{a_2}{2} = \frac{P_r}{2} + \frac{1}{2}\left[\frac{P_r^2 + \sqrt{P_r^4 + \omega^2 P_r^2}}{2}\right]^{1/2},$$

$$f_i = \frac{b_2}{2} = \frac{1}{2}\left[\frac{-P_r^2 + \sqrt{P_r^4 + \omega^2 P_r^2}}{2}\right]^{1/2},$$

$$a_2 = \left[\frac{P_r^2 + \sqrt{P_r^4 + \omega^2 P_r^2}}{2}\right]^{1/2}, \qquad b_2 = \left[\frac{-P_r^2 + \sqrt{P_r^4 + \omega^2 P_r^2}}{2}\right]^{1/2},$$

$$r_2 = a_2^2 + b_2^2 = \sqrt{P_r^4 + \omega^2 P_r^2}. \tag{3.30}$$

From (3.24), (3.28), and (3.29), we can write

$$\theta = \theta_0 + \epsilon(\theta_{1r}\cos\omega t - \theta_{1i}\sin\omega t), \tag{3.31}$$

in which

$$\theta_{1r} = -e^{-f_r\eta}(m_{16r}\cos f_i\eta - m_{16i}\sin f_i\eta) + (m_{7r} + m_{9r} + m_{8r}\eta)e^{-2h_r\eta} + \begin{bmatrix} (m_{10r} + m_{14r} + m_{12r}\eta)\cos(h_i - g_i)\eta \\ +(m_{11r} + m_{15r} + m_{13r}\eta)\sin(h_i - g_i)\eta \end{bmatrix} e^{-(h_r+g_r)\eta}, \tag{3.32}$$

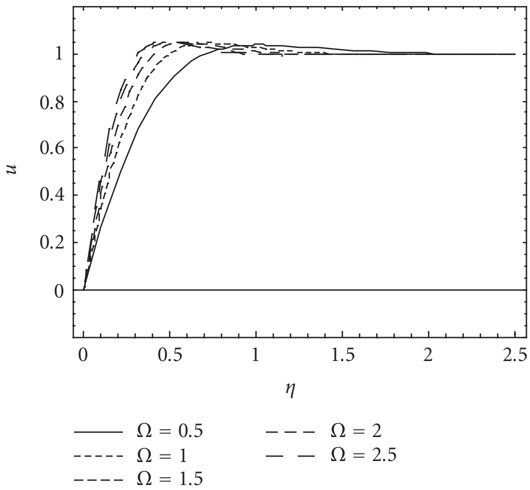
$$\theta_{1i} = -e^{-f_r\eta}(m_{16r}\sin f_i\eta + m_{16i}\cos f_i\eta) + (m_{7i} + m_{9i} + m_{8i}\eta)e^{-2h_r\eta} + \begin{bmatrix} (m_{10i} + m_{14i} + m_{12i}\eta)\cos(h_i - g_i)\eta \\ +(m_{11i} + m_{15i} + m_{13i}\eta)\sin(h_i - g_i)\eta \end{bmatrix} e^{-(h_r+g_r)\eta}. \tag{3.33}$$

4. Discussion of results

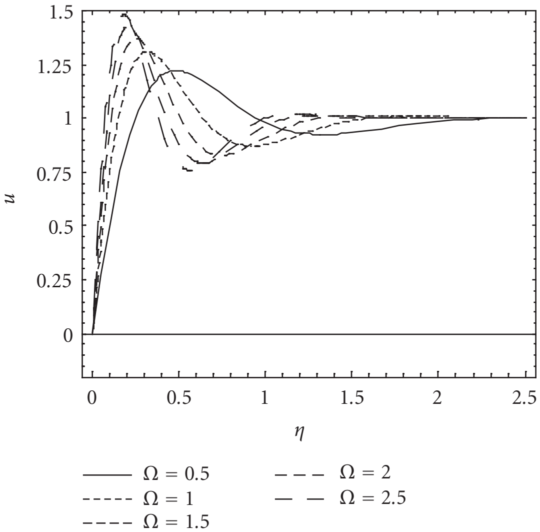
In this paper, we consider the problem of heat transfer in rotating flow of an incompressible fluid of second grade. A perturbation procedure has been used to obtain the analytic solution. The effects of various parameters such as Ω, P_r, and E_c on the real and imaginary parts of velocity (u,v) and temperature (θ_r,θ_i) distributions are studied and the results have been presented by several graphs.

To study the effect of Ω on the velocity components, we have plotted u and v against η in Figures 4.1, and 4.2 for Newtonian and second-grade fluids. From Figure 4.1(a), it is observed that near the plate u increases with the increase of Ω. Figure 4.1(b) indicates that u increases very near to the plate and then fluctuates through an increase in Ω. The comparison of these two figures reveals that u in case of second-grade fluid is greater than that of Newtonian fluid. Also, the velocity boundary layer thickness for second-grade fluid is larger than the Newtonian fluid. It is also seen from Figures 4.2(a) and 4.2(b) that v increases near the plate and then decreases for large value of Ω. The fluctuations in second-grade fluid are more visible than that of Newtonian fluid. Also, the value of v for second-grade fluid is smaller than in the case of Newtonian fluid.

Figures 4.3 and 4.4 show the effect of Ω on the real (θ_r) and imaginary (θ_i) parts of temperature distributions. Figure 4.3(a) shows that with the increase of Ω, θ_r decreases near the wall. As shown in Figure 4.3(b), we can see that as Ω increases, θ_r increases near

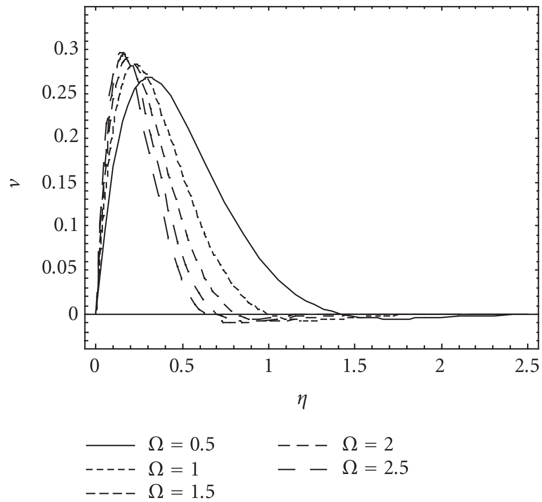


(a)

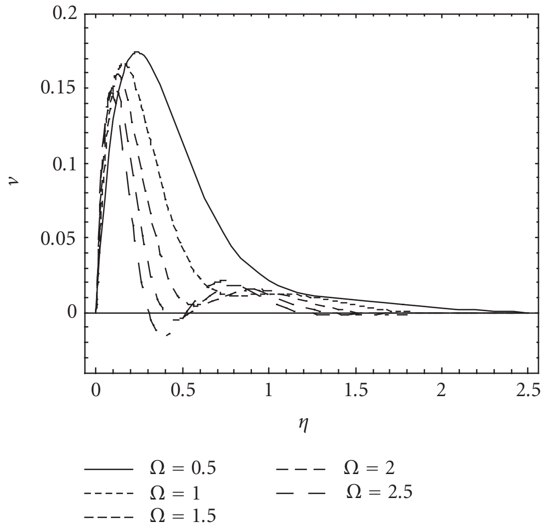


(b)

Figure 4.1. Effect of Ω on real part of velocity profile u versus η. In (a) for Newtonian fluid at $\alpha = 0$, $\omega t = \pi/2$, $A = 0.2$, $\epsilon = \omega = 0.5$, $W_0 = -0.5$, $\nu = 0.1$. In (b) for second-grade fluid at $\alpha = 0.4$.

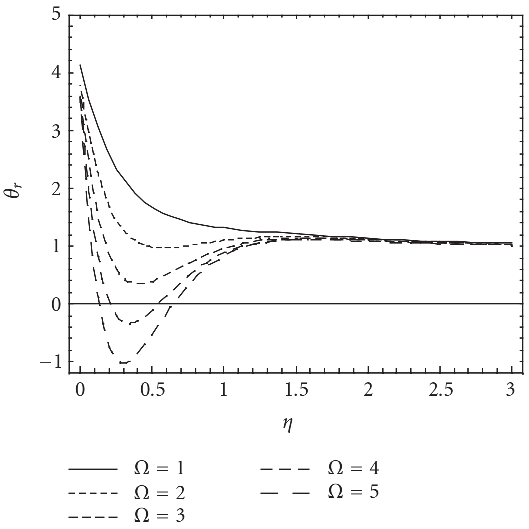


(a)

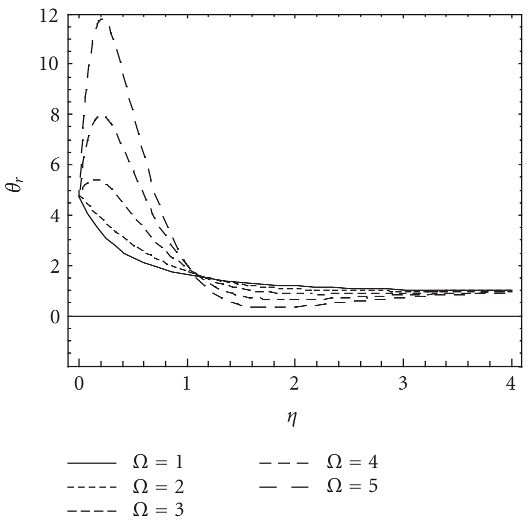


(b)

Figure 4.2. Effect of Ω on imaginary part of velocity profile v versus η. In (a) for Newtonian fluid at $\alpha = 0$, $\omega t = \pi/2$, $A = 0.2$, $\epsilon = \omega = 0.5$, $W_0 = -0.5$, $\nu = 0.1$. In (b) for second-grade fluid at $\alpha = 0.1$.

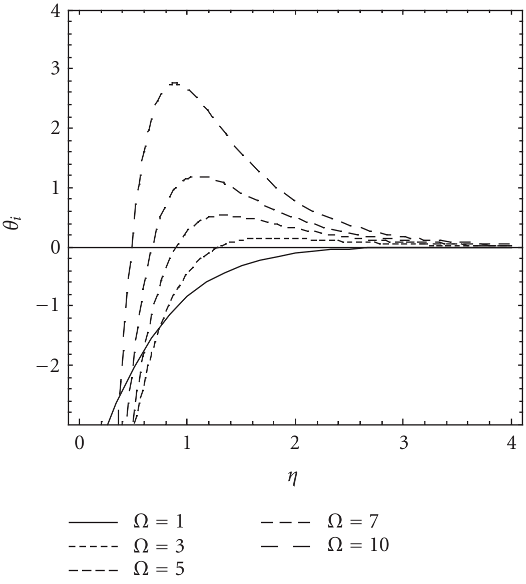


(a)

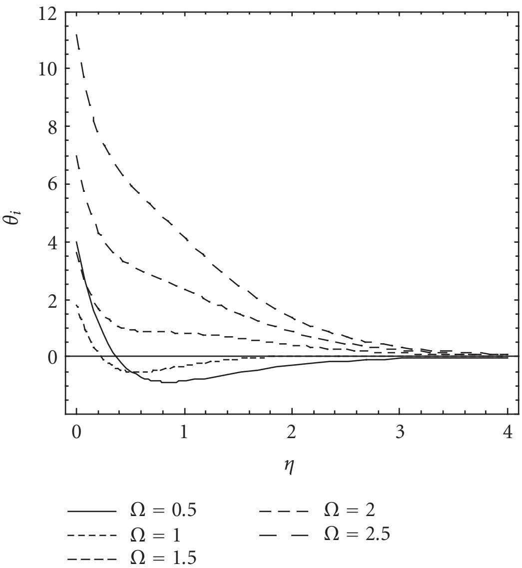


(b)

Figure 4.3. Effect of Ω on real part of temperature profile θ_r versus η. In (a) for Newtonian fluid at $\alpha = 0$, $\omega t = \pi/2$, $A = \epsilon = \omega = 0.5$, $W_0 = -0.5$, $\nu = 0.1$, $P_r = 1.5$, $E_c = 5.0$, $k = 0.2$, $P = 0.3$. In (b) for second-grade fluid at $\alpha = 0.04$.

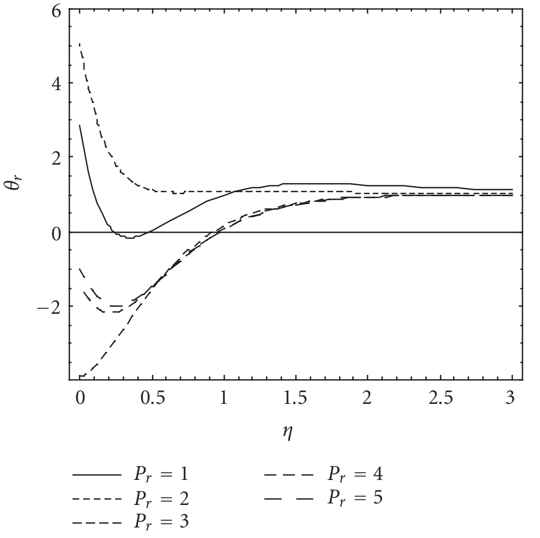


(a)

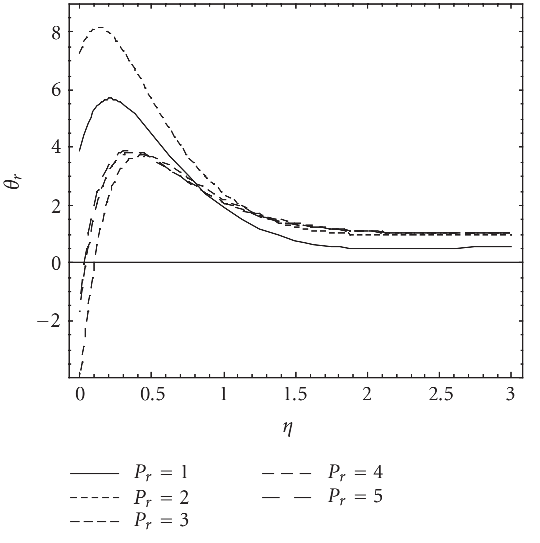


(b)

Figure 4.4. Effect of Ω on imaginary part of temperature profile θ_i versus η. In (a) for Newtonian fluid at $\alpha = 0$, $\omega t = \pi/2$, $A = \epsilon = \omega = 0.5$, $W_0 = -0.5$, $\nu = 0.1$, $P_r = 1.5$, $E_c = 5.0$, $k = 0.2$, $P = 0.3$. In (b) for second-grade fluid at $\alpha = 0.05$.

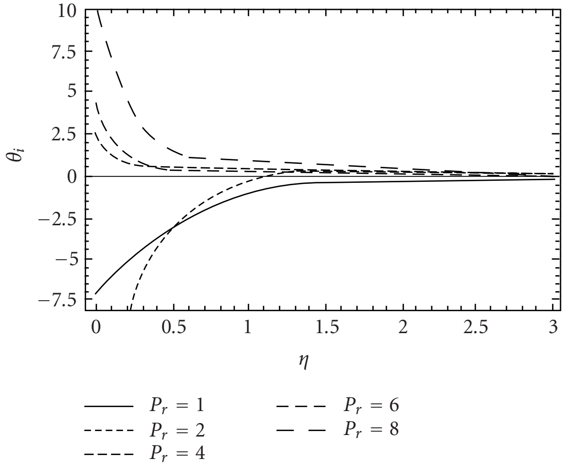


(a)

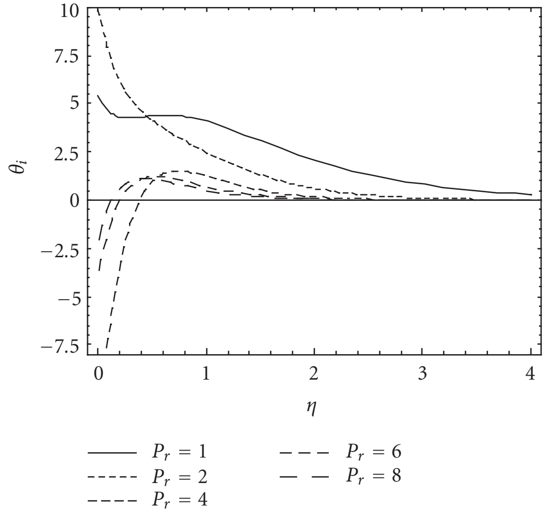


(b)

Figure 4.5. Effect of P_r on real part of temperature profile θ_r versus η. In (a) for Newtonian fluid at $\alpha = 0$, $\omega t = \pi/2$, $A = \epsilon = \omega = 0.5$, $W_0 = -0.5$, $\nu = 0.1$, $\Omega = 3.0$, $E_c = 5.0$, $k = 0.2$, $P = 0.3$. In (b) for second-grade fluid at $\alpha = 0.05$.

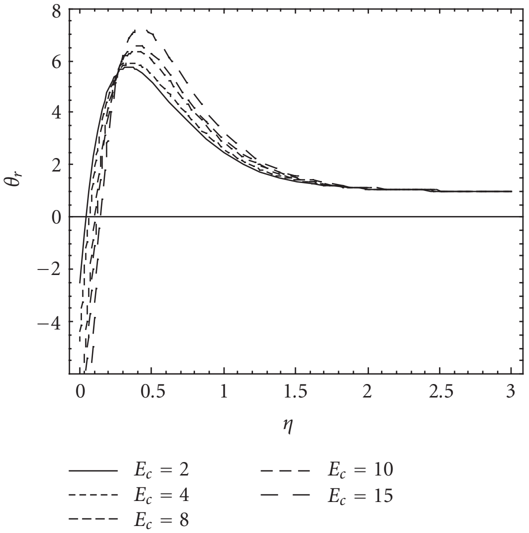


(a)

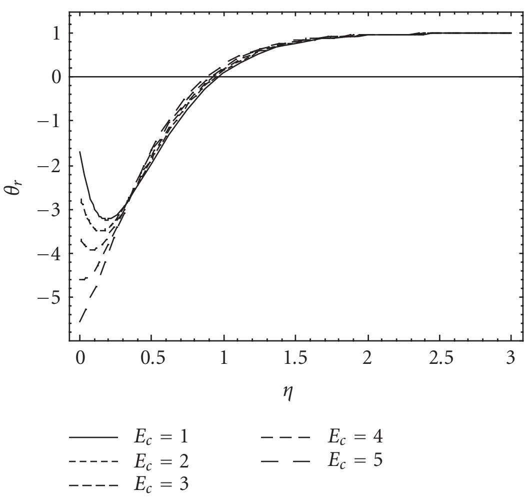


(b)

Figure 4.6. Effect of P_r on imaginary part of temperature profile θ_i versus η. In (a) for Newtonian fluid at $\alpha = 0$, $\omega t = \pi/2$, $A = \epsilon = \omega = 0.5$, $W_0 = -0.5$, $\nu = 0.1$, $\Omega = 2.5$, $E_c = 5.0$, $k = 0.2$, $P = 0.3$. In (b) for second-grade fluid at $\alpha = 0.04$.

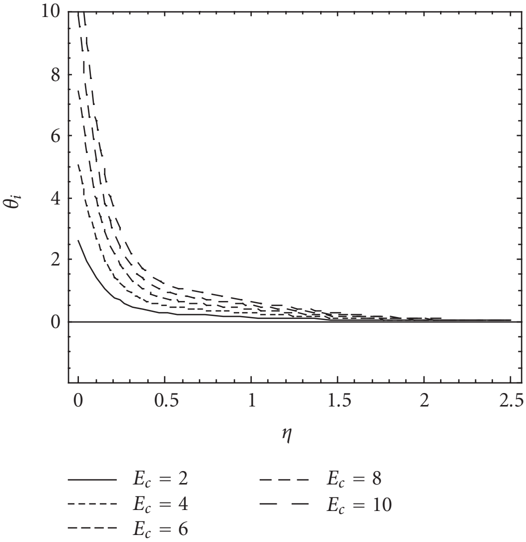


(a)

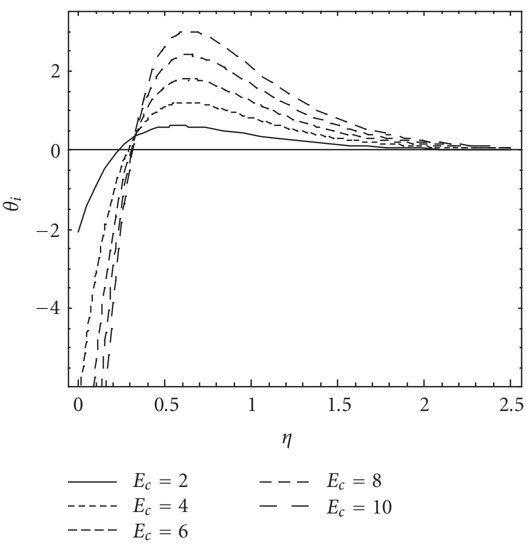


(b)

Figure 4.7. Effect of E_c on real part of temperature profile θ_r versus η. In (a) for Newtonian fluid at $\alpha = 0$, $\omega t = \pi/2$, $A = \epsilon = \omega = 0.5$, $W_0 = -0.5$, $\nu = 0.1$, $\Omega = 4.0$, $P_r = 5.0$, $k = 0.2$, $P = 0.3$. In (b) for second-grade fluid at $\alpha = 0.04$.



(a)



(b)

Figure 4.8. Effect of E_c on imaginary part of temperature profile θ_i versus η. In (a) for Newtonian fluid at $\alpha = 0$, $\omega t = \pi/2$, $A = \epsilon = \omega = 0.5$, $W_0 = -0.5$, $\nu = 0.1$, $\Omega = 2.5$, $P_r = 5.0$, $k = 0.2$, $P = 0.3$. In (b) for second-grade fluid at $\alpha = 0.05$.

the plate, and then at a distance of $\eta = 1$, the θ_r begins to decrease. That is, the behavior of θ_r is quite opposite for Newtonian and second-grade fluids near the plate. Figure 4.4(a) shows the variation of Ω on θ_i. It can be seen that as Ω increases, the value of θ_i decreases at a distance of approximately $\eta = 0.8$ and then increases. Figure 4.4(b) indicates that θ_i increases near the wall for $\Omega > 1$.

In order to illustrate the variation of P_r on θ_r and θ_i, we have prepared Figures 4.5 and 4.6. Figures 4.5(a) and 4.6(a) explain the effect of P_r on θ_r and θ_i, respectively, for Newtonian fluid case. From these figures, it is revealed that near the plate, θ_r decreases and θ_i increases for $P_r > 2$. The thermal boundary layer thickness in θ_r increases where as for θ_i decreases. For second-grade fluid, we note that from Figures 4.5(b) and 4.6(b) that for $P_r > 2$, θ_r decreases near the wall and increases far away. Also θ_i decreases for $P_r > 2$.

Figures 4.7 and 4.8 show the effect of E_c on θ_r and θ_i. From Figures 4.7(a) and 4.7(b), we observe that θ_r decreases near the wall with the increase in E_c and increases far away. The thermal boundary layer thickness increases for large E_c. Moreover, it can be seen from Figure 4.8(a) that θ_i increases for large values of E_c. From Figure 4.8(b), it can be seen that with the increase in the values of E_c, the temperature θ_i decreases near the plate and increases far away. The thermal boundary layer thicknesses in both fluids increases.

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Inspired on the rediscovering of the richness of nonlinear and chaotic phenomena, engineers started using analytical tools from "Qualitative Theory of Differential Equations," allowing more precise analysis and synthesis, in order to produce new vital products and services. Bifurcation theory, dynamical systems and chaos started to be part of the mandatory set of tools for design engineers.

This proposed special edition of the *Mathematical Problems in Engineering* aims to provide a picture of the importance of the bifurcation theory, relating it with nonlinear and chaotic dynamics for natural and engineered systems. Ideas of how this dynamics can be captured through precisely tailored real and numerical experiments and understanding by the combination of specific tools that associate dynamical system theory and geometric tools in a very clever, sophisticated, and at the same time simple and unique analytical environment are the subject of this issue, allowing new methods to design high-precision devices and equipment.

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