ROBUST DISSIPATIVITY FOR UNCERTAIN IMPULSIVE DYNAMICAL SYSTEMS

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We discuss the robust dissipativity with respect to the quadratic supply rate for uncertain impulsive dynamical systems. By employing the Hamilton-Jacobi inequality approach, some sufficient conditions of robust dissipativity for this kind of system are established. Finally, we specialize the obtained results to the case of uncertain linear impulsive dynamical systems.

1. Introduction

In many engineering problems, stability issues are often linked to the theory of dissipative systems which postulates that the energy dissipated inside a dynamical system is less than the energy supplied from an external source. In the literature of nonlinear control, dissipativity concept was initially introduced by Willems in his seminal two-part papers [14, 15] in terms of an inequality involving the storage function and supply rate. The extension of the work of Willems to the case of affine nonlinear systems was carried out by Hill and Moylan (see [7, 8] and the references therein).

Dissipativity theory along with its connections to Lyapunov stability theory have been mainly applied to dynamical systems possessing continuous motions. However, there are many real-world systems and natural processes which display special dynamic behavior that exhibits both continuous and discrete characteristics. For instance, many evolutionary processes, particularly some biological systems such as biological neural networks and bursting rhythm models in pathology, are characterized by abrupt changes of states at certain time instants. In addition, optimal control models in economics, frequency-modulated signal processing systems, and flying object motions may also exhibit the same feature. This feature is the familiar impulsive phenomenon, and the corresponding systems are called impulsive dynamical systems (see [1, 2, 9, 10, 11, 12, 17]). Recently, researchers have also introduced and studied the stability for other discontinuous dynamical systems such as hybrid systems [18], sampled-data systems [6], and discrete-event systems [13]. For all these systems, discontinuous system motions arise naturally. More recently, Haddad et al. have developed dissipativity and exponential dissipativity concepts

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for nonlinear impulsive dynamical systems and left-continuous dynamical systems (see [3, 4, 5] and the references therein). They have extended the notions of classical dissipativity theory using generalized storage functions and supply rates for impulsive dynamical systems and left-continuous dynamical systems.

In practice, the mathematical model used in design usually has some uncertainties. Therefore, robust control strategy becomes important. Hence, in the dissipative synthesis problems, we should consider the robustness of dissipativity in the presence of the uncertainties.

In this paper, we consider the robust dissipativity with respect to the quadratic supply rate for uncertain impulsive dynamical systems. The uncertainty is assumed to be a nonlinear function of the state that happened on the continuous part of the system, and it is bounded by a known function. Under this condition, we derive a sufficient condition under which an uncertain impulsive dynamical system is a robust dissipative one. Finally, we specialize the results to the case of uncertain linear impulsive dynamical systems.

2. Main results

2.1. Dissipativity for impulsive dynamical systems. We consider the impulsive dynamical system

$$\begin{aligned} \dot{x}(t) &= f_c(x(t)) + g_c(x(t))u_c(t), \quad t_k < t \le t_k, \\ \Delta x(t) &= f_d(x(t)) + g_d(x(t))u_d(t), \quad t = t_k, \\ y_c(t) &= h_c(x(t)), \quad t_k < t \le t_k, \\ y_d(t) &= h_d(x(t)), \quad t = t_k, \end{aligned} \tag{2.1}$$

where $x(t_0) = x_0$, $t \ge 0, x(t) \in \mathbb{R}^n$, $\Delta x(t_k) = x(t_k^+) - x(t_k)$, $u_c(t) \in \mathbb{R}^{m_c}$, $u_d \in \mathbb{R}^{m_d}$, $y_c(t) \in \mathbb{R}^{l_c}$, $y_d \in \mathbb{R}^{l_d}$, $f_c : \mathbb{R}^n \to \mathbb{R}^n$ is Lipschitz continuous and satisfies $f_c(0) = 0$, $g_c : \mathbb{R}^n \to \mathbb{R}^{n \times m_c}$, $f_d : \mathbb{R}^n \to \mathbb{R}^n$ is continuous and satisfies $f_d(0) = 0$, $g_d : \mathbb{R}^n \to \mathbb{R}^{n \times m_d}$, $h_c : \mathbb{R}^n \to \mathbb{R}^{l_c}$ and satisfies $h_c(0) = 0$, and $h_d : \mathbb{R}^n \to \mathbb{R}^{l_d}$ and satisfies $h_d(0) = 0$. We will assume that $u_c(\cdot)$ and $u_d(\cdot)$ are restricted to the class of admissible inputs consisting of measurable functions $(u_c(t), u_d(t)) \in U$ for all $t \ge 0$, where $(0,0) \in U$.

Definition 2.1. A function $(\gamma_c(u_c, y_c), \gamma_d(u_d, y_d))$, where $\gamma_c : \mathbb{R}^{m_c} \times \mathbb{R}^{l_c} \to \mathbb{R}$ and $\gamma_d : \mathbb{R}^{m_d} \times \mathbb{R}^{l_d} \to \mathbb{R}$ are such that $\gamma_c(0,0) = 0$ and $\gamma_d(0,0) = 0$, is called a supply rate of system (2.1) if $\gamma_c(u_c, y_c)$ is locally integrable; that is, for all input-output pairs $u_c(t), y_c(t)$, $\gamma_c(u_c, y_c)$ satisfies $\int_t^{\hat{t}} |\gamma_c(u_c(s), y_c(s))| ds < \infty$ for any $\hat{t} \ge t \ge 0$, and $\gamma_d(u_d, y_d)$ is locally summable. In other words, for all input-output pairs $u_d(t_k)$, $y_d(t_k)$, $\gamma_d(u_d, y_d)$ satisfies

$$\sum_{k \in \mathbb{N}[t,\hat{t})} \left| \gamma_d(u_d(t_k), y_d(t_k)) \right| < \infty, \tag{2.2}$$

where $\mathbb{N}[t, \hat{t}) = \{k : t \le t_k < \hat{t}\}$.

Definition 2.2. An impulsive dynamical system of the form (2.1) is said to be dissipative with respect to supply rate (γ_c, γ_d) if there exists a C^r $(r \ge 0)$ nonnegative function $V : \mathbb{R}^n \to \mathbb{R}$ with $V(0) = 0$, called storage function, such that, for all $(u_c, u_d) \in U$, the following dissipation inequality holds:

$$V(x(t)) \leq V(x(t_0)) + \int_{t_0}^{t} \gamma_c(u_c(s), y_c(s))\,ds + \sum_{k\in\mathbb{N}[t,\hat{t})} \gamma_d(u_d(t_k), y_d(t_k)), \tag{2.3}$$

where $x(t)$ $(t \geq t_0)$ is a solution to (2.1) with $(u_c, u_d) \in U$ and $x(t_0) = x_0$.

In [4], several basic dissipativity results for impulsive dynamical systems have been established, one of which will be introduced in following lemma.

Lemma 2.3 [4]. *An impulsive dynamical system given by (2.1) is dissipative with respect to the supply rate* (γ_c, γ_d) *if and only if there exists a* C^0 *nonnegative definite function* $V : \mathbb{R}^n \to \mathbb{R}$ *such that, for all* $k \in \mathbb{N} = \{0, 1, 2, \ldots\}$,

$$\begin{aligned} V(x(\hat{t})) - V(x(t)) &\leq \int_{t}^{\hat{t}} \gamma_c(u_c(s), y_c(s))\,ds, \quad t_k < t \leq \hat{t} \leq t_{k+1}, \\ V(x(t_k^+)) - V(x(t_k)) &\leq \gamma_d(u_d(t_k), y_d(t_k)). \end{aligned} \tag{2.4}$$

Remark 2.4. If in Lemma 2.3, $V(x(\cdot))$ is C^r $(r \geq 1)$, then, in this case, dissipativity of the impulsive dynamical system with respect to the supply rate (γ_c, γ_d) is given by

$$\begin{aligned} \dot{V}(x(t)) &\leq \gamma_c(u_c(t), y_c(t)), \quad t_k < t \leq t_{k+1}, \\ \Delta V(x(t_k)) &\leq \gamma_d(u_d(t_k), y_d(t_k)), \quad k \in \mathbb{N}, \end{aligned} \tag{2.5}$$

where

$$\begin{aligned} \Delta V(x(t_k)) &= V(x(t_k^+)) - V(x(t_k)) \\ &= V(x(t_k) + f_d(x(t_k)) + g_d(x(t_k))u_d(t_k)) - V(x(t_k)). \end{aligned} \tag{2.6}$$

2.2. Robust dissipativity for uncertain impulsive dynamical systems. The uncertain impulsive dynamical system under our consideration can be described as follows:

$$\begin{aligned} \dot{x}(t) &= f_c(x(t)) + \hat{f}_c(x(t)) + g_c(x(t))u_c(t), \quad t_k < t \leq t_k, \\ \Delta x(t) &= f_d(x(t)) + g_d(x(t))u_d(t), \quad t = t_k, \\ y_c(t) &= h_c(x(t)), \quad t_k < t \leq t_k, \\ y_d(t) &= h_d(x(t)), \quad t = t_k, \end{aligned} \tag{2.7}$$

where $\hat{f}_c : \mathbb{R}^n \to \mathbb{R}^n$ represents uncertainty characterized by $\hat{f}_c(x) = e_c(x)\delta_c(x)$, $\hat{f}_c(0) = 0$, the mapping $e_c : \mathbb{R}^n \to \mathbb{R}^{n\times m}$ is a known functional matrix whose entries are smooth functions of the state and $\delta_c : \mathbb{R}^n \to \mathbb{R}^m$ is an unknown smooth vector function belonging to the set $\Omega_c = \{\delta_c(x) : \|\delta_c(x)\| \leq \|n_c(x)\|\}$, where $n_c : \mathbb{R}^n \to \mathbb{R}^m$ is a given continuous function with $n_c(0) = 0$ and $\|\cdot\|$ stands for the Euclidean norm.

Definition 2.5. System (2.7) is said to be a robust dissipative system with respect to supply rate (γ_c, γ_d) if, for every $\delta_c \in \Omega_c$, the system is dissipative with respect to the supply rate (γ_c, γ_d).

In this paper, we focus our attention on the quadratic supply rate (γ_c, γ_d) as follows:

$$\gamma_c(u_c, y_c) = \frac{1}{2}\{y_c^T R_c y_c + 2y_c^T S_c u_c + u_c^T Q_c u_c\}, \tag{2.8}$$

$$\gamma_d(u_d, y_d) = \frac{1}{2}\{y_d^T R_d y_d + 2y_d^T S_d u_d + u_d^T Q_d u_d\}, \tag{2.9}$$

where R_c, S_c, Q_c, and R_d, S_d, Q_d are symmetric matrices with $R_c \leq 0$, $R_d \leq 0$, and $Q_c > 0$, $Q_d > 0$.

Theorem 2.6. *Suppose that there exist functions $V : \mathbb{R}^n \to \mathbb{R}$, $l_d : \mathbb{R}^n \to \mathbb{R}^{p_d}$, $w_d : \mathbb{R}^n \to \mathbb{R}^{p_d \times m_d}$, $P_{1u_d} : \mathbb{R}^n \to \mathbb{R}^{1\times m_d}$, and $P_{2u_d} : \mathbb{R}^n \to \mathbb{R}^{m_d \times m_d}$ with $P_{2u_d}(x) \geq 0$ for all $x \in \mathbb{R}^n$, such that $V(\cdot)$ is C^1 and positive definite with $V(0) = 0$ and that the following conditions hold:*

(C1) *for all $x \in \mathbb{R}^n$, $u_d \in \mathbb{R}^{m_d}$,*

$$V(x + f_d(x) + g_d(x)u_d) = V(x + f_d(x)) + P_{1u_d}(x)u_d + u_d^T P_{2u_d}(x)u_d; \tag{2.10}$$

(C2) *there exists a positive definite function $\lambda(x) > 0$ satisfying the Hamilton-Jacobi inequality, for all $t_k < t \leq t_{k+1}$, $k \in \mathbb{N}$, given by*

$$\begin{aligned}\frac{\partial V}{\partial x}[f_c - g_c Q_c^{-1} S_c h_c] + \frac{\lambda}{2}\frac{\partial V}{\partial x} e_c e_c^T \frac{\partial V^T}{\partial x} + \frac{1}{2\lambda} n_c^T n_c \\ + \frac{1}{2}\frac{\partial V}{\partial x} g_c Q_c^{-1} g_c^T \frac{\partial V^T}{\partial x} + \frac{1}{2} h_c^T (S_c Q_c^{-1} S_c - R_c) h_c \leq 0;\end{aligned} \tag{2.11}$$

(C3) *for $t = t_k$, $k \in \mathbb{N}$,*

$$\begin{aligned} V(x + f_d(x)) - V(x) - \frac{1}{2} h_d^T(x) R_d h_d(x) + l_d^T(x) l_d(x) = 0, \\ \frac{1}{2} P_{1u_d}(x) - \frac{1}{2} h_d^T(x) S_d + l_d^T(x) w_d(x) = 0, \\ \frac{1}{2} Q_d - P_{2u_d}(x) - w_d^T(x) w_d(x) = 0. \end{aligned} \tag{2.12}$$

Then the uncertain impulsive dynamical system given by (2.7) is a robust dissipative system with respect to the quadratic supply rate (γ_c, γ_d) given by (2.8) and (2.9).

In order to prove Theorem 2.6, we first prove the following lemmas.

Lemma 2.7. *If there exists a C^1 positive definite $V(x)$, with $V(0) = 0$, which satisfies the Hamilton-Jacobi inequality*

$$\frac{\partial V}{\partial x} f_c(x) + \frac{1}{2\gamma^2}\frac{\partial V}{\partial x} g_c(x) g_c^T(x) \frac{\partial V^T}{\partial x} + \frac{1}{2} h_c^T(x) h_c(x) \leq 0, \quad \gamma > 0, \tag{2.13}$$

then $V(x)$ must satisfy the following dissipation inequality

$$\frac{\partial V}{\partial x} f_c(x) + \frac{\partial V}{\partial x} g_c(x) u_c \leq \frac{1}{2}\left\{\gamma^2 ||u_c||^2 - ||y_c||^2\right\}. \tag{2.14}$$

Proof. From (2.13), we get

$$\begin{aligned}
&\frac{\partial V}{\partial x} f_c(x) + \frac{\partial V}{\partial x} g_c(x) u_c \\
&\quad \le -\frac{1}{2} h_c^T(x) h_c(x) - \frac{1}{2\gamma^2} \frac{\partial V}{\partial x} g_c(x) g_c^T(x) \frac{\partial V^T}{\partial x} \\
&\qquad + \frac{\partial V}{\partial x} g_c(x) u_c - \frac{1}{2}\gamma^2 u_c^T u_c + \frac{1}{2}\gamma^2 u_c^T u_c \\
&\quad = \frac{1}{2}\left\{\gamma^2 ||u_c||^2 - ||y_c||^2\right\} - \frac{\gamma^2}{2}\left|\left| u_c - \frac{1}{\gamma^2} g_c^T(x) \frac{\partial V^T}{\partial x}\right|\right|^2 \\
&\quad \le \frac{1}{2}\left\{\gamma^2 ||u_c||^2 - ||y_c||^2\right\}.
\end{aligned} \tag{2.15}$$

Hence, inequality (2.14) follows. □

Lemma 2.8. *For every $\delta_c \in \Omega_c$ and any positive definite function $\lambda(x) > 0$, the following inequality holds*

$$\frac{\partial V}{\partial x} e_c(x)\delta_c(x) \le \frac{\lambda(x)}{2} \frac{\partial V}{\partial x} e_c(x) e_c^T(x) \frac{\partial V^T}{\partial x} + \frac{1}{2\lambda(x)} n_c^T(x) n_c(x). \tag{2.16}$$

Proof. Since $\|\delta_c(x)\| \le \|n_c(x)\|$, so for any positive definite function $\lambda(x) > 0$, we get

$$\begin{aligned}
\frac{\partial V}{\partial x} e_c(x)\delta_c(x) &\le \left|\left|\frac{\partial V}{\partial x} e_c(x)\delta_c(x)\right|\right| \le \left|\left|\frac{\partial V}{\partial x} e_c(x)\right|\right| \cdot ||\delta_c(x)|| \\
&\le \frac{\lambda(x)}{2}\left|\left|\frac{\partial V}{\partial x} e_c(x)\right|\right|^2 + \frac{1}{2\lambda(x)} ||\delta_c(x)||^2 \\
&\le \frac{\lambda(x)}{2}\left|\left|\frac{\partial V}{\partial x} e_c(x)\right|\right|^2 + \frac{1}{2\lambda(x)} ||n_c(x)||^2 \\
&= \frac{\lambda(x)}{2} \frac{\partial V}{\partial x} e_c(x) e_c^T(x) \frac{\partial V^T}{\partial x} + \frac{1}{2\lambda(x)} n_c^T(x) n_c(x).
\end{aligned} \tag{2.17}$$

Thus, inequality (2.16) holds. □

Proof of Theorem 2.6. First, we will show $\dot{V}(x) \le r_c(u_c, y_c)$, for all $t_k < t \le t_{k+1}$, $k \in \mathbb{N}$.

Since $Q_c > 0$, $R_c \le 0$, we can find nonsingular matrices D and E such that $Q_c = E^T E$, $S_c Q_c^{-1} S_c - R_c = D^T D$, and such that the transformation

$$\begin{pmatrix} \tilde{u}_c \\ \tilde{y}_c \end{pmatrix} = \begin{pmatrix} E & E Q_c^{-1} S_c \\ 0 & D \end{pmatrix} \begin{pmatrix} u_c \\ y_c \end{pmatrix} \tag{2.18}$$

is nonsingular.

Let $\tilde{f}_c(x) = f_c(x) - g_c(x) Q_c^{-1} S_c h_c(x)$, $\tilde{g}_c(x) = g_c(x) E^{-1}$, and $\tilde{h}_c(x) = D h_c(x)$, then system (2.7), with respect to the supply rate (r_c, r_d), given by (2.8) and (2.9), changes into the form

$$\begin{aligned} \dot{x}(t) &= \tilde{f}_c(x(t)) + e_c(x)\delta_c(x) + \tilde{g}_c(x(t))\tilde{u}_c(t), \quad t_k < t \le t_k, \\ \Delta x(t) &= f_d(x(t)) + g_d(x(t))u_d(t), \quad t = t_k, \\ \tilde{y}_c(t) &= \tilde{h}_c(x(t)), \quad t_k < t \le t_k, \\ y_d(t) &= h_d(x(t)), \quad t = t_k, \end{aligned} \tag{2.19}$$

with respect to the supply rate (r_c, r_d); here,

$$\tilde{r}_c(\tilde{u}_c, \tilde{y}_c) = \frac{1}{2}\left\{||\tilde{u}_c||^2 - ||\tilde{y}_c||^2\right\}. \tag{2.20}$$

Hence, system (2.7) is robust dissipative with respect to the supply rate (r_c, r_d), given by (2.8) and (2.9), if and only if system (2.19) is robust dissipative with respect to the supply rate $(\tilde{r}_c, r_d)$, given by (2.9) and (2.20).

From (C2) and Lemma 2.8, we get

$$\begin{aligned} &\frac{\partial V}{\partial x}\left(\tilde{f}_c(x) + e_c(x)\delta_c(x)\right) + \frac{1}{2}\frac{\partial V}{\partial x}\tilde{g}_c(x)\tilde{g}_c^T(x)\frac{\partial V^T}{\partial x} + \frac{1}{2}\tilde{h}_c^T(x)\tilde{h}_c(x) \\ &\quad \le \frac{\partial V}{\partial x}\tilde{f}_c(x) + \frac{\lambda(x)}{2}\frac{\partial V}{\partial x}e_c(x)e_c^T(x)\frac{\partial V^T}{\partial x} + \frac{1}{2\lambda(x)}n_c^T(x)n_c(x) \\ &\qquad + \frac{1}{2}\frac{\partial V}{\partial x}\tilde{g}_c(x)\tilde{g}_c^T(x)\frac{\partial V^T}{\partial x} + \frac{1}{2}\tilde{h}_c^T(x)\tilde{h}_c(x) \\ &\quad = \frac{\partial V}{\partial x}\left\{f_c - g_cQ_c^{-1}S_ch_c\right\} + \frac{\lambda}{2}\frac{\partial V}{\partial x}e_ce_c^T\frac{\partial V^T}{\partial x} + \frac{1}{2\lambda}n_c^Tn_c \\ &\qquad + \frac{1}{2}\frac{\partial V}{\partial x}g_cQ_c^{-1}g_c^T\frac{\partial V^T}{\partial x} + \frac{1}{2}h_c^T\left\{S_cQ_c^{-1}S_c - R_c\right\}h_c \le 0. \end{aligned} \tag{2.21}$$

Hence, by Lemma 2.7, $\dot{V}(x) \le \tilde{r}_c(\tilde{u}_c, \tilde{y}_c) = r_c(u_c, y_c)$, for all $t_k < t \le t_{k+1}$, $k \in N$.

We will now show that $\Delta V(x(t_k)) \le r_d(u_d(t_k), y_d(t_k))$, $k \in \mathbb{N}$.

Applying (C1) and (C3), we get

$$\begin{aligned} \Delta V(x) &= V\left(x + f_d(x) + g_d(x)u_d\right) - V(x) \\ &= V\left(x + f_d(x)\right) - V(x) + P_{1u_d}(x)u_d + u_d^TP_{2u_d}(x)u_d \\ &= \frac{1}{2}h_d^T(x)R_dh_d(x) - l_d^T(x)l_d(x) + \left\{h_d^T(x)S_d - 2l_d^T(x)w_d(x)\right\}u_d \\ &\quad + u_d^T\left\{\frac{1}{2}Q_d - w_d^T(x)w_d(x)\right\}u_d \\ &= \frac{1}{2}\left\{h_d^T(x)R_dh_d(x) + 2h_d^T(x)S_du_d + u_d^TQ_du_d\right\} \\ &\quad - \left\{l_d^T(x)l_d(x) + 2l_d^T(x)w_d(x)u_d + u_d^Tw_d^T(x)w_d(x)u_d\right\} \\ &= r_d(u_d, y_d) - ||l_d(x) + w_d(x)u_d||^2 \\ &\le r_d(u_d, y_d). \end{aligned} \tag{2.22}$$

Hence, $\Delta V(x(t_k)) \le r_d(u_d(t_k), y_d(t_k))$ holds for all $k \in \mathbb{N}$.

Thus, by Lemma 2.3, system (2.19) is robust dissipative with respect to the supply rate $(\tilde{r}_c, r_d)$, given by (2.20) and (2.9), that is, system (2.7) is robust dissipative with the supply rate (r_c, r_d), given by (2.8) and (2.9), as required. □

In the following discussion, we will apply Theorem 2.6 to a particular class of nonexpansive impulsive dynamical systems.

Definition 2.9. A system G, given by (2.1) ((2.7)), is (robust) nonexpansive if G is (robust) dissipative with respect to the supply rate

$$(r_c, r_d) = \frac{1}{2}(\gamma_c^2 u_c^T u_c - y_c^T y_c, \gamma_d^2 u_d^T u_d - y_d^T y_d), \tag{2.23}$$

where $\gamma_c > 0$, $\gamma_d > 0$.

Corollary 2.10. *Consider system (2.7) and suppose that there exist the following functions: $V : \mathbb{R}^n \to \mathbb{R}$, $l_d : \mathbb{R}^n \to \mathbb{R}^{p_d}$, $w_d : \mathbb{R}^n \to \mathbb{R}^{p_d \times m_d}$, $P_{1u_d} : \mathbb{R}^n \to \mathbb{R}^{1\times m_d}$, and $P_{2u_d} : \mathbb{R}^n \to \mathbb{R}^{m_d \times m_d}$ with $P_{2u_d}(x) \geq 0$ for all $x \in \mathbb{R}^n$, such that $V(\cdot)$ is C^1 and positive definite with $V(0) = 0$ and that the following conditions hold:*

(C1)′ *for all $x \in \mathbb{R}^n$, $u_d \in \mathbb{R}^{m_d}$,*

$$V(x + f_d(x) + g_d(x)u_d) = V(x + f_d(x)) + P_{1u_d}(x)u_d + u_d^T P_{2u_d}(x)u_d; \tag{2.24}$$

(C2)′ *there exists a positive definite function $\lambda(x) > 0$ satisfying the Hamilton-Jacobi inequality, for all $t_k < t \leq t_{k+1}$, $k \in \mathbb{N}$, given by*

$$\frac{\partial V}{\partial x} f_c + \frac{\lambda}{2}\frac{\partial V}{\partial x} e_c e_c^T \frac{\partial V^T}{\partial x} + \frac{1}{2\lambda} n_c^T n_c + \frac{1}{2\lambda^2}\frac{\partial V}{\partial x} g_c g_c^T \frac{\partial V^T}{\partial x} + \frac{1}{2} h_c^T h_c \leq 0; \tag{2.25}$$

(C3)′ *for $t = t_k$, $k \in \mathbb{N}$,*

$$\begin{gathered} V(x + f_d(x)) - V(x) + \frac{1}{2} h_d^T(x) h_d(x) + l_d^T(x) l_d(x) = 0, \\ \frac{1}{2} P_{1u_d}(x) + l_d^T(x) w_d(x) = 0, \\ \frac{\gamma_d^2}{2} I_{m_d} - P_{2u_d}(x) - w_d^T(x) w_d(x) = 0. \end{gathered} \tag{2.26}$$

Then the uncertain impulsive dynamical system given by (2.7) is robust nonexpansive with respect to the quadratic supply rate (γ_c, γ_d), given by (2.23).

Proof. The result is a direct consequence of Theorem 2.6 with $Q_c = \gamma_c^2 I_{m_c}$, $S_c = 0$, $R_c = -I_{l_c}$, $Q_d = \gamma_d^2 I_{m_d}$, $S_d = 0$, and $R_c = -I_{l_d}$, where I is the identity matrix. □

3. Specialization to uncertain linear impulsive dynamical systems

In this section, we specialize the results of Section 2 to the case of uncertain linear impulsive dynamical systems. This kind of system can be formulated as follows:

$$\begin{aligned} \dot{x}(t) &= A_c x(t) + \hat{A}_c x(t) + B_c u_c(t), \quad t \neq t_k, \\ \Delta x(t) &= (A_d - I_n)x(t) + B_d u_d(t), \quad t = t_k, \\ y_c(t) &= C_c x(t), \quad t \neq t_k, \\ y_d(t) &= C_d x(t), \quad t = t_k, \end{aligned} \tag{3.1}$$

where $x(t_0) = x_0$, $A_c \in \mathbb{R}^{n\times n}$, $B_c \in \mathbb{R}^{n\times m_c}$, $C_c \in \mathbb{R}^{l_c\times n}$, $A_d \in \mathbb{R}^{n\times n}$, $B_d \in \mathbb{R}^{n\times m_d}$, $C_d \in \mathbb{R}^{l_d\times n}$, and $\hat{A}_c \in \mathbb{R}^{n\times n}$ represent uncertainty. By [16, Lemma 2.1], we can conclude that $\hat{A}_c = E_c\Sigma_c F_c$, where $E_c \in \mathbb{R}^{n\times n^2}$ and $F_c \in \mathbb{R}^{n^2\times n}$ are known matrices, and $\Sigma_c \in \mathbb{R}^{n^2\times n^2}$ is the uncertain matrix satisfying

$$\Sigma \in \Omega_c = \{\Sigma_c \in \mathbb{R}^{n^2\times R^2} : \Sigma_c = \operatorname{diag}\{\varepsilon_{11},\dots,\varepsilon_{nn}\},\ |\varepsilon_{ij}| \le 1;\ i,j = 1,2,\dots,n\}. \tag{3.2}$$

Remark 3.1. From (3.2), it is easy to get that Σ_c is characterized by $\Sigma_c^T\Sigma_c \le I_{n^2}$.

THEOREM 3.2. *Suppose there exist matrices $P \in \mathbb{R}^{n\times n}$, $L_d \in \mathbb{R}^{p_d\times n}$, and $W_d \in \mathbb{R}^{p_d\times m_d}$, with P positive definite, such that the following conditions are satisfied.*

(a1) *There exists a positive constant λ satisfying the Riccati inequality*

$$\begin{aligned} &A_c^T P + PA_c - (PB_cQ_c^{-1}S_cC_c + C_c^TS_cQ_c^{-1}B_c^TP) + 2\lambda PE_cE_c^TP \\ &\quad + \frac{1}{2\lambda}F_c^TF_c + 2PB_cQ_c^{-1}B_c^TP + \frac{1}{2}C_c^T(S_cQ_c^{-1}S_c - R_c)C_c \le 0. \end{aligned} \tag{3.3}$$

(a2) *The following equations hold*

$$\begin{aligned} A_d^TPA_d - P - \frac{1}{2}C_d^TR_dC_d + L_d^TL_d &= 0, \\ A_d^TPB_d - \frac{1}{2}C_d^TS_d + L_d^TW_d &= 0, \\ \frac{1}{2}Q_d - B_d^TPB_d - W_d^TW_d &= 0. \end{aligned} \tag{3.4}$$

Then the uncertain linear impulsive dynamical system (3.1) is robustly dissipative with respect to the supply rate (γ_c,γ_d), given by (2.8) and (2.9).

Proof. Let $V(x) = x^TPx$, then V is C^1 and positive definite. Furthermore, let

$$\begin{aligned} &f_c(x) = A_cx, \quad g_c(x) = B_c, \quad e_c(x) = E_c, \\ &\delta_c(x) = \Sigma_cF_cx, \quad f_d(x) = (A_d - I_n)x, \quad g_d(x) = B_d, \\ &h_c(x) = C_cx, \quad h_d(x) = C_dx, \quad n_c(x) = F_cx. \end{aligned} \tag{3.5}$$

This implies that $\|\delta_c(x)\| \le \|n_c(x)\| = \sqrt{x^TF_c^TF_cx}$.

Clearly, $P_{1u_d}(x) = 2x^TA_d^TPB_d$ and $P_{2u_d}(x) = B_d^TPB_d \ge 0$.

Thus, by using Theorem 2.6, the conclusion of this theorem follows. □

Corollary 3.3. *Suppose there exist matrices $P \in \mathbb{R}^{n\times n}$, $L_d \in \mathbb{R}^{p_d\times n}$, and $W_d \in \mathbb{R}^{p_d\times m_d}$, with P positive definite, such that the following conditions are satisfied.*

(a1)′ *There exists a positive constant λ satisfying the Riccati inequality*

$$A_c^T P + PA_c + 2\lambda PE_cE_c^T P + \frac{1}{2\lambda}F_c^T F_c + \frac{2}{\gamma_c^2}PB_cB_c^T P + \frac{1}{2}C_c^T C_c \leq 0. \tag{3.6}$$

(a2)′ *The following equations hold*

$$\begin{aligned} A_d^T PA_d - P + \frac{1}{2}C_d^T C_d + L_d^T L_d &= 0, \\ A_d^T PB_d + L_d^T W_d &= 0, \\ \frac{\gamma_d^2}{2}I_{m_d} - B_d^T PB_d - W_d^T W_d &= 0. \end{aligned} \tag{3.7}$$

Then the uncertain linear impulsive dynamical system (3.1) is robustly nonexpansive with respect to the supply rate (γ_c, γ_d), given by (2.23).

Proof. The result is a direct consequence of Theorem 3.2 with $Q_c = \gamma_c^2 I_{m_c}$, $S_c = 0$, $R_c = -I_{l_c}$, $Q_d = \gamma_d^2 I_{m_d}$, $S_d = 0$, and $R_d = -I_{l_d}$. □

4. Conclusions

We have studied the robust dissipativity with respect to the quadratic supply rate for uncertain impulsive dynamical systems. By employing the Hamilton-Jacobi inequality approach, some sufficient conditions of robust dissipativity for this kind of system are established. As for the robust dissipativity with respect to the generalized supply rate for uncertain impulsive dynamical systems, we will discuss it in future papers.

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