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AN M^{X}/GI/1/N QUEUE WITH CLOSE-DOWN AND VACATION TIMES

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An $\mathrm{M}^{\mathrm{X}}/\mathrm{GI}/1/N$ finite capacity queue with close-down time, vacation time and exhaustive service discipline is considered under the partial batch acceptance strategy as well as under the whole batch acceptance strategy. Applying the supplementary variable technique the queue length distribution at an arbitrary instant and at a departure epoch is obtained under both strategies, where no assumption on the batch size distribution is made. The loss probabilities and the Laplace-Stieltjes transforms of the waiting time distribution of the first customer and of an arbitrary customer of a batch are also given. Numerical examples give some insight into the behavior of the system.

Key words: $\mathrm{M}^{\mathrm{X}}/\mathrm{GI}/1/N$ Type Queue, Finite Capacity, Close-Down Time, Server Vacation, Exhaustive, Queue Length, Acceptance Strategy, Loss Probability, Waiting Time.

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1. Introduction

There has been much interest in batch arriving queueing systems during the last three and a half decades, both from theoretical and practical points of view. Those systems are frequently encountered in the real world as can be seen in Chaudhry and

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Templeton [5]. In telecommunications, a batch can correspond to a message while a customer can correspond to a packet; see Manfield and Tran-Gia [11].

Many techniques have been developed or extended to deal with the additional analytical complexities that result from the introduction of batch arrivals. For instance, both the embedded Markov chain (EMC) technique and the supplementary variable (SV) technique can be applied to the Poisson (non-batch) arrival M/GI/1/N finite capacity queue. As for the batch-Poisson arrival M^{X}/GI/1/N finite capacity queue, however, the EMC technique cannot be straightforwardly applied under the whole batch acceptance strategy, WBAS (see Section 2 for WBAS), while the SV technique can be applied with an artificial condition by Baba [1] (see also Baba [3], and Takagi [15]). The main purpose of the paper is to present an SV-technique based analysis for a batch-Poisson arrival M^{X}/GI/1/N finite capacity queue with close-down time and server vacation, asserting that Baba's [1] assumption can be omitted.

The batch-Poisson arrival M^{X}/GI/1/N finite capacity queue with server vacation is now common in telecommunications. For example, a processor (server) has secondary jobs (customers) to be performed aside from primary jobs. The processor is scheduled to perform secondary jobs only when it finds no primary jobs. The processing time for a secondary job corresponds to a vacation time in queueing terminology. Another example is a buffer (queue) under the time division multiple access (TDMA) environment (see Stuck and Arthurs [14]). An arriving packet (customer) who finds the system idle cannot be transmitted (served) immediately, and it has to wait until the slotted boundary comes. A constant slotted time period corresponds to a vacation time. Performances issues in these examples then necessitate our M^{X}/GI/1/N queue with vacation time.

A queueing situation with vacation time and close-down time can be recently seen in the switched virtual channel connection (SVCC) for internet protocol (IP) over asynchronous transfer mode (ATM) networks, where the close-down time corresponds to an inactivity timer in the SVCC operation. See Hassan and Atiquzzaman [9], and Sakai et al. [13] for SVCC.

Assuming infinite queueing capacity, Baba [2] analyzed the M^{X}/GI/1 queue with vacation time via the SV technique. Because the queueing capacity is infinite, the well-known stochastic decomposition formula is known to be valid (see Doshi [6], Furhmann and Cooper [8], and Miyazawa [12]). Indeed, the results in Baba [2] show this formula straightforwardly.

Assuming (non-batch) Poisson input and a finite capacity queue M/GI/1/N, Lee [10] provided a numerical algorithm for this system via the EMC technique; see also Frey and Takahashi [7]. However, as for the ordinary queue (without vacation), it seems hard to generalize Lee's EMC technique to our queue with vacation under the WBAS. Thus, here, we take the SV approach to obtain the queue length distribution, based on which practical performance measures of interest can be derived.

To the best of our knowledge, there is no literature on finite-capacity server-vacation models with close-down times, except for one [13]. In [13] Sakai et al. treated a single-arrival M/GI/1/N queue via the SV approach.

Note that our formulated equations are reduced to Baba's equations if we make the close-down time be constantly zero.

This paper is organized as follows. In Section 2 we describe our model together with the considered strategies, the so-called partial batch acceptance strategy (PBAS) and the WBAS. In Section 3 we apply the SV technique to obtain the queue length distribution at an arbitrary instant as well as at a departure epoch under both strategies. Here, we show that Baba's [1] assumption can be omitted. The loss probability of the first customer of a batch and the loss probability of an arbitrary customer are considered in Section 4. In Section 5 we derive the Laplace-Stieltjes transforms of the waiting time distribution of the first customer and of an arbitrary customer of an actual arrived batch. Extensive numerical calculations are performed in Section 6 which give a feeling for the influence of the service time, close-down time and vacation time distributions. Final conclusions and remarks about possible future research are made in Section 7.

2. The Model

We consider an $\mathrm{M}^{\mathrm{X}}/\mathrm{GI}/1/N$ queue, where N equals the number of waiting places in the queue, including the space for the customer that may be in service. We assume that the arrival epochs of the batches form a homogeneous Poisson process with intensity λ and that consecutive batch sizes are independent and have the common probability function $\{g_i\}_{i=1}^{\infty}$. Furthermore, we assume that the service times form a sequence of i.i.d. random variables with distribution function $S(x)$ and Laplace-Stieltjes transform $S^*(\theta) = \int_0^\infty e^{-\theta x} S(dx)$. Customers accepted by the system are served by a single server exhaustively, i.e. the server serves the queue continuously until the queue is empty. Whenever the queue becomes empty a close-down period starts with distribution function $C(x)$ and Laplace-Stieltjes transform $C^*(\theta) = \int_0^\infty e^{-\theta x} C(dx)$. During the close-down period, if a customer arrives, the server immediately begins service for that customer. On the other hand, if no customer arrives until the end of a close-down period, the server starts a vacation with distribution function $V(x)$ and Laplace-Stieltjes transform $V^*(\theta) = \int_0^\infty e^{-\theta x} V(dx)$. If the queue is still empty upon his return, he takes another independent vacation with the same distribution function. Otherwise, he starts service. We assume further that the service discipline is FIFO. By $\widehat{S}$ we denote the steady-state remaining service time for the customer in service and by $\widehat{C}(\widehat{V})$ the steady-state remaining close-down time (vacation time).

We will analyze this queueing system under two batch acceptance strategies which are common in the field of telecommunication. One is the partial batch acceptance strategy (PBAS) and the other one is the whole batch acceptance strategy (WBAS). Under the PBAS we understand that when an arrival batch is larger in size than the number of available free waiting places, the free positions will be filled up and the remaining customers of the batch will be lost. On the other hand, under the WBAS, an arriving batch will be lost, when the batch is larger in size than the number of available free waiting places. For the WBAS we also assume that $\sum_{i=1}^{N} g_i > 0$, which means that there is a positive probability that customers are accepted by the system.

3. The Queue Length Distribution

By g_j^{Σ} we will denote the probability that the batch size is at most j, i.e.

$$g_j^{\Sigma} = \sum_{i=1}^{j} g_i, \qquad j = 1, 2, \ldots$$

and by g_j^c we will denote the probability that the batch size is greater or equal to j, i.e.,

$$g_j^c = \sum_{i=j}^{\infty} g_i, \quad j = 1, 2, \ldots.$$

At an arbitrary time in steady state, if we define by L the number of customers present in the system and if we set $\xi = 0$ if the server is on vacation, $\xi = 1$ if the system is closing down and $\xi = 2$ if the server is busy, we can formulate the quantities of main interest, i.e. the joint distribution of the queue length and the remaining service (close-down, vacation) time

$$\pi_n(x)dx = \mathbf{P}(L = n, x < \widehat{S} \le x + dx, \xi = 2), \quad n = 1, \ldots, N$$

$$c_n(x)dx = \mathbf{P}(L = n, x < \widehat{C} \le x + dx, \xi = 1), \quad n = 0, \ldots, N$$

$$\omega_n(x)dx = \mathbf{P}(L = n, x < \widehat{V} \le x + dx, \xi = 0), \quad n = 0, \ldots, N$$

and

$$\pi_n^*(\theta) = \int_0^{\infty} e^{-\theta x} \pi_n(x)dx,$$

$$c_n^*(\theta) = \int_0^{\infty} e^{-\theta x} c_n(x)dx,$$

$$\omega_n^*(\theta) = \int_0^{\infty} e^{-\theta x} \omega_n(x)dx.$$

Our main goal will be to calculate $\pi_n(0)$, $\pi_n^*(0)$, $n = 1, \ldots, N$ and $c_0^*(0)$, $\omega_n(0)$, $\omega_n^*(0)$, $n = 0, \ldots, N$ for the PBAS as well as for the WBAS.

3.1 Partial Batch Acceptance Strategy (PBAS)

Relating the probabilities at time $t + dt$ to those at time t, we obtain the following equations:

$$-\frac{d\pi_1(x)}{dx} = -\lambda \pi_1(x) + (\pi_2(0) + c_1(0) + \omega_1(0))\frac{dS(x)}{dx}, \tag{1}$$

$$-\frac{d\pi_n(x)}{dx} = 0\lambda \pi_n(x) + \lambda \sum_{i=1}^{n-1} g_{n-i}\pi_i(x) \tag{2}$$

$$+ (\pi_{n+1}(0) + c_n(0) + \omega_n(0))\frac{dS(x)}{dx}, \quad n = 2, \ldots, N-1,$$

$$-\frac{d\pi_N(x)}{dx} = \lambda \sum_{i=1}^{N-1} g_{N-i}^c \pi_i(x) + (c_N(0) + \omega_N(0))\frac{dS(x)}{dx} \tag{3}$$

$$-\frac{dc_0(x)}{dx} = -\lambda c_0(x) + \pi_1(0)\frac{dC(x)}{dx}, \tag{4}$$

$$-\frac{d\omega_0(x)}{dx} = -\lambda \omega_0(x) + (c_0(0) + \omega_0(0))\frac{dV(x)}{dx}, \tag{5}$$

$$-\frac{d\omega_n(x)}{dx} = -\lambda\omega_n(x) + \lambda\sum_{i=0}^{n-1} g_{n-i}\omega_i(x), n = 1,\ldots,N-1, \tag{6}$$

$$-\frac{d\omega_N(x)}{dx} = \lambda\sum_{i=0}^{N-1} g^c_{N-i}\omega_i(x), \tag{7}$$

from which it follows that

$$(\lambda-\theta)\pi_1^*(\theta) = -\pi_1(0) + (\pi_2(0) + c_1(0) + \omega_1(0))S^*(\theta), \tag{8}$$

$$(\lambda-\theta)\pi_n^*(\theta) = -\pi_n(0) + \lambda\sum_{i=1}^{n-1} g_{n-i}\pi_i^*(\theta) \tag{9}$$

$$+ (\pi_{n+1}(0) + c_n(0) + \omega_n(0))S^*(\theta), \quad n = 2,\ldots,N-1,$$

$$-\theta\pi_N^*(\theta) = -\pi_N(0) + \lambda\sum_{i=1}^{N-1} g^c_{N-i}\pi_i^*(\theta) + (c_N(0) + \omega_N(0))S^*(\theta), \tag{10}$$

$$(\lambda-\theta)c_0^*(\theta) = -c_0(0) + \pi_1(0)C^*(\theta), \tag{11}$$

$$(\lambda-\theta)\omega_0^*(\theta) = -\omega_0(0) + (c_0(0) + \omega_0(0))V^*(\theta), \tag{12}$$

$$(\lambda-\theta)\omega_n^*(\theta) = -\omega_n(0) + \lambda\sum_{i=0}^{n-1} g_{n-i}\omega_i^*(\theta), \quad n = 1,\ldots,N-1, \tag{13}$$

$$-\theta\omega_N^*(\theta) = -\omega_N(0) + \lambda\sum_{i=0}^{N-1} g^c_{N-i}\omega_i^*(\theta). \tag{14}$$

During the close-down time, an arriving customer may end the period, thus

$$c_n(0) = \lambda g_n c_0^*(0), \qquad n = 1,\ldots,N-1,$$
$$c_N(0) = \lambda g^c_N c_0^*(0).$$

Hence, it suffices to calculate $\pi_n(0), n = 1,\ldots,N, c_n(0)$ and $\omega_n(0)$, $n = 0,\ldots,N$. From (8)-(14) we can obtain the following lemma.

Lemma 3.1: *It holds that*

$$c_0(0) = \lambda\omega_0^*(0), \tag{15}$$

$$\pi_1(0) = \frac{\lambda\omega_0^*(0)}{C^*(\lambda)}, \tag{16}$$

$$\pi_2(0) = \frac{\pi_1(0)}{S^*(\lambda)} - \omega_1(0) - c_1(0), \tag{17}$$

$$\pi_n(0) = \frac{\pi_{n-1}(0)}{S^*(\lambda)} - \lambda\sum_{i=1}^{n-2}\frac{g_{n-1-i}}{S^*(\lambda)}\pi_i^*(\lambda) - \omega_{n-1}(0) - c_{n-1}(0), \tag{18}$$
$$n = 3,\ldots,N,$$

$$\omega_0(0) = \frac{\lambda V^*(\lambda)}{1 - V^*(\lambda)} \omega_0^*(0), \tag{19}$$

$$\omega_n(0) = \lambda \sum_{i=0}^{n-1} g_{n-i} \omega_i^*(\lambda), \quad n = 1, \ldots, N-1, \tag{20}$$

$$\begin{aligned} \omega_N(0) = &- \sum_{i=1}^{N-1} \sum_{j=1}^{N-i} \sum_{\delta \in A_j^{=N}} g_{\delta_1}^c g_{\delta_2} \cdots g_{\delta_j} \omega_i(0), \\ &+ \sum_{j=1}^{N} \sum_{\delta \in A_j^{=N}} g_{\delta_1}^c g_{\delta_2} \cdots g_{\delta_j} \omega_0^*(0), \end{aligned} \tag{21}$$

where $A_j^{=n}$ *is the set of all j-tuples* $\delta = (\delta_1, \ldots, \delta_j)$ *with* $\delta_k \in \mathbb{N} = \{1, 2, \ldots\}$ $(k = 1, \ldots, j)$ *and* $\sum_{k=1}^{j} \delta_k = n$.

Proof: By inserting $\theta = 0$ into (12) we obtain (15) and by inserting $\theta = \lambda$ into (8), (9), (11)-(13) and by using (15) we obtain (16)-(20). Substituting $\theta = 0$ into (13) and (14), we obtain

$$\lambda \omega_n^*(0) = -\omega_n(0) + \lambda \sum_{i=0}^{n-1} g_{n-i} \omega_i^*(0), \quad n = 1, \ldots, N-1, \tag{22}$$

$$\omega_N(0) = \lambda \sum_{i=0}^{N-1} g_{N-i}^c \omega_i^*(0), \tag{23}$$

from which (21) follows. □

Thus, we need $c_0^*(0)$, $\omega_0^*(0)$, $\omega_i^*(\lambda)$, $i = 0, \ldots, N-2$ and $\pi_i^*(\lambda)$, $i = 1, \ldots, N-2$ to obtain the desired quantities $\pi_k(0)$, $k = 1, \ldots, N$ and $c_k(0)$, $\omega_k(0)$, $k = 0, \ldots, N$. Differentiating (8), (9), (12), (13) and inserting $\theta = \lambda$ yield

$$\pi_1^{*(n)}(\lambda) = -\frac{1}{n+1}(\pi_2(0) + c_1(0) + \omega_1(0)) S^{*(n+1)}(\lambda), \quad n = 0, \ldots, N-3, \tag{24}$$

$$\begin{aligned} \pi_k^{*(n)}(\lambda) = -\frac{1}{n+1} &\left(\lambda \sum_{i=1}^{k-1} g_{k-i} \pi_i^{*(n+1)}(\lambda) \right. \\ &\left. + (\pi_{k+1}(0) + c_k(0) + \omega_k(0)) S^{*(n+1)}(\lambda) \right), \quad k = 2, \ldots, N-2, \quad n = 0, \ldots, N-2-k, \end{aligned} \tag{25}$$

$$\omega_0^{*(n)}(\lambda) = -\frac{1}{n+1}(c_0(0) + \omega_0(0)) V^{*(n+1)}(\lambda), n = 0, \ldots, N-1, \tag{26}$$

$$\omega_k^{*(n)}(\lambda) = -\frac{1}{n+1}\left(\lambda \sum_{i=0}^{k-1} g_{k-i} \omega_i^{*(n+1)}(\lambda) \right), \tag{27}$$

$$k = 1, \ldots, N-1, \quad n = 0, \ldots, N-1-k.$$

Hence we can express $\omega_i^*(\lambda)$, $i = 0, \ldots, N-2$ and $\pi_i^*(\lambda)$, $i = 1, \ldots, N-2$ by $\omega_0^*(0)$ and $c_0^*(0)$. Inserting $\theta = 0$ into (11) and using (16) and (15), we obtain

$$c_0^*(0) = \frac{1}{\lambda}(-c_0(0) + \pi_1(0))$$

$$= \frac{1 - C^*(\lambda)}{C^*(\lambda)}\omega_0^*(0)$$

and therefore we can obtain $\pi_k(0)$, $k = 1,\ldots,N$ and $c_k(0)$, $\omega_k(0)$, $k = 0,\ldots,N$ by using $\omega_0^*(0)$.

Remark 3.1: In the sequel, we are expressing $\pi_i^*(0)$ and $\omega_i^*(0)$ by $\pi_k(0)$, $k = 1,\ldots,N$ and $c_k(0)$, $\omega_k(0)$, $k = 0,\ldots,N$ and hence by $\omega_0^*(0)$. Finally, using the normalization condition

$$c_0^*(0) + \omega_0^*(0) + \sum_{i=1}^{N}(\omega_i^*(0) + \pi_i^*(0)) = 1 \tag{28}$$

we obtain $\omega_0^*(0)$.

Inserting $\theta = 0$ into (8), (9), (11)-(13), and the derivatives of (10), (14), we get

$$\lambda\pi_1^*(0) = -\pi_1(0) + \pi_2(0) + \omega_1(0) + c_1(0), \tag{29}$$

$$\lambda\pi_n^*(0) = -\pi_n(0) + \lambda\sum_{i=1}^{n-1} g_{n-1}\pi_i^*(0) + \pi_{n+1}(0) + \omega_n(0) + c_n(0), \tag{30}$$

$$n = 2,\ldots,N-1,$$

$$\lambda\omega_0^*(0) = c_0(0), \tag{31}$$

$$\lambda\omega_n^*(0) = -\omega_n(0) + \lambda\sum_{i=0}^{n-1} g_{n-i}\omega_i^*(0), \quad n = 1,\ldots,N-1, \tag{32}$$

$$\pi_N^*(0) = -\left(\lambda\sum_{i=1}^{N-1} g_{N-i}^c\pi_i^{*(1)}(0) + (c_N(0) + \omega_N(0))S^{*(1)}(0)\right), \tag{33}$$

$$\omega_N^*(0) = -\lambda\sum_{i=0}^{N-1} g_{N-i}^c\omega_i^{*(1)}(0). \tag{34}$$

Further differentiating (8), (9), (12), (13) and inserting $\theta = 0$ yield

$$\lambda\pi_1^*(0) = (\pi_2(0) + c_1(0) + \omega_1(0))S^{*(1)}(0) + \pi_1^*(0), \tag{35}$$

$$\lambda\pi_n^{*(1)}(0) = \lambda\sum_{i=1}^{n-1} g_{n-i}\pi_i^{*(1)}(0) + \Big(\pi_{n+1}(0) + c_n(0) \tag{36}$$

$$+ \omega_n(0)\Big)S^{*(1)}(0) + \pi_n^*(0), \quad n = 2,\ldots,N-1,$$

$$\lambda\omega_0^{*(1)}(0) = (c_0(0) + \omega_0(0))V^{*(1)}(0) + \omega_0^*(0), \tag{37}$$

$$\lambda\omega_n^{*(1)}(0) = \lambda\sum_{i=0}^{n-1} g_{n-i}\omega_i^{*(1)}(0) + \omega_n^*(0), \quad n = 1,\ldots,N-1, \tag{38}$$

from which we can evaluate the right-hand sides of (33) and (34), and hence, obtain $\omega_0^*(0)$. Using (28) we can now obtain $\omega_0^*(0)$ and $\pi_n(0)$, $n=1,\ldots,N$ and $c_n(0)$, $\omega_n(0)$, $n=0,\ldots,N$.

3.2 Whole Batch Acceptance Strategy (WBAS)

We are now considering the whole batch acceptance strategy (WBAS), i.e., a batch is either accepted as a whole or lost as a whole. Similarly as to the PBAS, we obtain the following equations by relating the probabilities at time $t+dt$ to those at time t:

$$-\frac{d\pi_1(x)}{dx} = -\lambda g_{N-1}^{\Sigma}\pi_1(x) + (\pi_2(0)+c_1(0)+\omega_1(0))\frac{dS(x)}{dx}, \tag{39}$$

$$-\frac{d\pi_n(x)}{dx} = -\lambda g_{N-n}^{\Sigma}\pi_n(x) + \lambda\sum_{i=1}^{n-1} g_{n-i}\pi_i(x) \tag{40}$$

$$+(\pi_{n+1}(0)+c_n(0)+\omega_n(0))\frac{dS(x)}{dx}, \quad n=2,\ldots,N-1,$$

$$-\frac{d\pi_N(x)}{dx} = \lambda\sum_{i=1}^{N-1} g_{N-i}\pi_i(x) + (c_N(0)+\omega_N(0))\frac{dS(x)}{dx}, \tag{41}$$

$$-\frac{dc_0(x)}{dx} = -\lambda g_N^{\Sigma}c_0(x) + \pi_1(0)\frac{dC(x)}{dx}, \tag{42}$$

$$-\frac{d\omega_0(x)}{dx} = -\lambda g_N^{\Sigma}\omega_0(x) + (c_0(0)+\omega_0(0))\frac{dV(x)}{dx}, \tag{43}$$

$$-\frac{d\omega_n(x)}{dx} = -\lambda g_{N-n}^{\Sigma}\omega_n(x) + \lambda\sum_{i=0}^{n-1} g_{n-i}\omega_i(x), \quad n=1,\ldots,N-1, \tag{44}$$

$$-\frac{d\omega_N(x)}{dx} = \lambda\sum_{i=0}^{N-1} g_{N-i}\omega_i(x), \tag{45}$$

from which it follows that

$$(\lambda g_{N-1}^{\Sigma}-\theta)\pi_1^*(\theta) = -\pi_1(0) + (\pi_2(0)+c_1(0)+\omega_1(0))S^*(\theta), \tag{46}$$

$$(\lambda g_{N-n}^{\Sigma}-\theta)\pi_n^*(\theta) = -\pi_n(0) + \lambda\sum_{i=1}^{n-1} g_{n-i}\pi_i^*(\theta) \tag{47}$$

$$+(\pi_{n+1}(0)+c_n(0)+\omega_n(0))S^*(\theta), \quad n=2,\ldots,N-1,$$

$$-\theta\pi_N^*(\theta) = -\pi_N(0) + \lambda\sum_{i=1}^{N-1} g_{N-i}\pi_i^*(\theta) + (c_N(0)+\omega_N(0))S^*(\theta), \tag{48}$$

$$(\lambda g_N^{\Sigma}-\theta)c_0^*(\theta) = -c_0(0) + \pi_1(0)C^*(\theta), \tag{49}$$

$$(\lambda g_N^{\Sigma}-\theta)\omega_0^*(\theta) = -\omega_0(0) + (c_0(0)+\omega_0(0))V^*(\theta), \tag{50}$$

$$(\lambda g_{N-n}^{\Sigma}-\theta)\omega_n^*(\theta) = -\omega_n(0)+\lambda\sum_{i=0}^{n-1} g_{n-i}\omega_i^*(\theta), \quad n=1,\ldots,N-1, \tag{51}$$

$$-\theta\omega_N^*(\theta) = -\omega_N(0)+\lambda\sum_{i=0}^{N-1} g_{N-i}\omega_i^*(\theta). \tag{52}$$

During the close-down time, an arriving customer may end the period, thus

$$c_n(0)=\lambda g_n c_0^*(0), \quad n=1,\ldots,N.$$

Hence it suffices to calculate the quantities $\pi_n(0)$, $n=1,\ldots,N$ and $c_n(0)$, $\omega_n(0)$, $n=0,\ldots,N$. Equivalent to Lemma 3.1 we obtain the following lemma.

Lemma 3.2: *It holds that*

$$c_0(0)=\lambda g_N^{\Sigma}\omega_0^*(0), \tag{53}$$

$$\pi_1(0)=\frac{\lambda g_N^{\Sigma}\omega_0^*(0)}{C^*(\lambda g_N^{\Sigma})}, \tag{54}$$

$$\pi_2(0)=\frac{\pi_1(0)}{S^*(\lambda g_{N-1}^{\Sigma})}-c_1(0)-\omega_1(0), \tag{55}$$

$$\pi_n(0)=\frac{\pi_{n-1}(0)-\lambda\sum_{i=1}^{n-2} g_{n-1-i}\pi_i^*(\lambda g_{N-n+1}^{\Sigma})}{S^*(\lambda g_{N-n+1}^{\Sigma})}-c_{n-1}(0)-\omega_{n-1}(0), \tag{56}$$

$$n=3,\ldots,N,$$

$$\omega_0(0)=\frac{\lambda g_N^{\Sigma}V^*(\lambda g_N^{\Sigma})}{1-V^*(\lambda g_N^{\Sigma})}\omega_0^*(0), \tag{57}$$

$$\omega_n(0)=\lambda\sum_{i=0}^{n-1} g_{n-i}\omega_i^*(\lambda g_{N-n}^{\Sigma}), \quad n=1,\ldots,N-1, \tag{58}$$

$$\omega_N(0)=\lambda g_N^{\Sigma}\omega_0^*(0)-\sum_{i=1}^{N-1}\omega_i(0). \tag{59}$$

Proof: This can be easily seen by inserting $\theta=0$ into (50) and $\theta=\lambda g_{N-1}^{\Sigma}$ into (46), $\theta=\lambda g_{N-n}^{\Sigma}$ into (47), $\theta=\lambda g_N^{\Sigma}$ into (49) and (50), $\theta=\lambda g_{N-n}^{\Sigma}$ into (51) and combining $\theta=0$ into (52) with $\theta=0$ into (51). □

Thus we need $c_0^*(0)$, $\omega_0^*(0)$, $\omega_i^*(\lambda g_{N-n}^{\Sigma})$, $i=0,\ldots,n-1$, $n=1,\ldots,N-1$ and $\pi_i^*(\lambda g_{N-n+1}^{\Sigma})$, $i=1,\ldots,n-2, n=3,\ldots,N$ to obtain the desired quantities $\pi_k(0)$, $k=1,\ldots,N$ and $c_k(0)$, $\omega_k(0)$, $k=0,\ldots,N$.

This yields two different cases:

Case 1: $g_i>0$, $\forall i\in\{1,\ldots,N-1\}$.

In this case, from (47), (50) and (51) we can obtain

$$\pi_i^*(\lambda g_{N-n+1}^{\Sigma})=\frac{1}{\lambda(g_{N-i}^{\Sigma}-g_{N-n+1}^{\Sigma})}\left\{-\pi_i(0)+\lambda\sum_{j=1}^{i-1} g_{i-j}\pi_j^*(\lambda g_{N-n+1}^{\Sigma})\right.$$

$$+(\pi_{i+1}(0)+c_i(0)+\omega_i(0))S^*(\lambda g^{\Sigma}_{N-n+1})\Big\}, \quad n=3,\ldots,N, \tag{60}$$

$$\omega_0^*(\lambda g^{\Sigma}_{N-n})=\frac{1}{\lambda(g^{\Sigma}_N-g^{\Sigma}_{N-n})}\Big\{-\omega_0(0)+(c_0(0)+\omega_0(0))V^*(\lambda g^{\Sigma}_{N-n})\Big\}, \tag{61}$$

$$\omega_i^*(\lambda g^{\Sigma}_{N-n})=\frac{1}{\lambda(g^{\Sigma}_{N-i}-g^{\Sigma}_{N-n})}\left\{-\omega_i(0)+\lambda\sum_{j=0}^{i-1}g_{i-j}\omega_j^*(\lambda g^{\Sigma}_{N-n})\right\},$$

$$n=1,\ldots,N-1. \tag{62}$$

Inserting $\theta=0$ into (49) and using (53) and (54), we obtain

$$\begin{aligned}c_0^*(0)&=\frac{-c_0(0)+\pi_1(0)}{\lambda g^{\Sigma}_N},\\ &=\frac{1-C^*(\lambda g^{\Sigma}_N)}{C^*(\lambda g^{\Sigma}_N)}\omega_0^*(0).\end{aligned}$$

Hence, $\pi_i(0), i=1,\ldots,N$ and $c_i(0)$, $\omega_i(0)$, $i=0,\ldots,N$ can be expressed in terms of $\omega_0^*(0)$.

Remark 3.2: In order to obtain $\omega_0^*(0)$ we will express $\pi_i^*(0)$ and $\omega_i^*(0)$, $i=1,\ldots,N$, by $\omega_0^*(0)$ (via $\pi_k(0)$, $c_k(0)$ and $\omega_k(0)$) and then make use of the normalizing condition

$$c_0^*(0)+\omega_0^*(0)+\sum_{i=1}^{N}(\omega_i^*(0)+\pi_i^*(0))=1. \tag{63}$$

Inserting $\theta=0$ into (46), (47) and (51) yields

$$\lambda g^{\Sigma}_{N-1}\pi_1^*(0)=-\pi_1(0)+\pi_2(0)+c_1(0)+\omega_1(0), \tag{64}$$

$$\lambda g^{\Sigma}_{N-n}\pi_n^*(0)=-\pi_n(0)+\lambda\sum_{i=1}^{n-1}g_{n-i}\pi_i^*(0)+\pi_{n+1}(0)+c_n(0)+\omega_n(0), \tag{65}$$

$$n=2,\ldots,N-1,$$

$$\lambda g^{\Sigma}_{N-n}\omega_n^*(0)=-\omega_n(0)+\lambda\sum_{i=0}^{n-1}g_{n-i}\omega_i^*(0),\quad n=1,\ldots,N-1. \tag{66}$$

Inserting $\theta=0$ into the derivatives of (48) and (52) yields

$$\pi_N^*(0)=-\left(\lambda\sum_{i=1}^{N-1}g_{N-i}\pi_i^{*(1)}(0)+(c_N(0)+\omega_N(0))S^{*(1)}(0)\right)$$

$$\omega_N^*(0)=-\lambda\sum_{i=0}^{N-1}g_{N-i}\omega_i^{*(1)}(0).$$

The right-hand sides can be expressed as follows by inserting $\theta=0$ into the derivatives of (46), (47), (50) and (51),

$$\lambda g_{N-1}^{\Sigma} \pi_1^{*(1)}(0) = (\pi_2(0) + c_1(0) + \omega_1(0)) S^{*(1)}(0) + \pi_1^*(0), \tag{67}$$

$$\lambda g_{N-n}^{\Sigma} \pi_n^{*(1)}(0) = \lambda \sum_{i=1}^{n-1} g_{n-i} \pi_i^{*(1)}(0) + (\pi_{n+1}(0) + c_n(0) + \omega_n(0)) S^{*(1)}(0) + \pi_n^*(0), \quad n = 2, \ldots, N-1, \tag{68}$$

$$\lambda g_N^{\Sigma} \omega_0^{*(1)}(0) = (c_0(0) + \omega_0(0)) V^{*(1)}(0) + \omega_0^*(0), \tag{69}$$

$$\lambda g_{N-n}^{\Sigma} \omega_n^{*(1)}(0) = \lambda \sum_{i=0}^{n-1} g_{n-i} \omega_i^{*(1)}(0) + \omega_n^*(0), \quad n = 1, \ldots, N-1, \tag{70}$$

from which we obtain $\omega_0^*(0)$ and hence we are done by Remark 3.2.

Case 2: $\exists k \in \{1, \ldots, N-2\}$: $g_{N-k} = 0$ and $g_{N-j} \neq 0 \forall j \in \{1, \ldots, N-1\}$, $j \neq k$.

Remark 3.3: This assumption implies that $g_{N-k}^{\Sigma} - g_{N-(k+1)}^{\Sigma} = 0$ and therefore, $\pi_k^*(\lambda g_{N-(k+1)}^{\Sigma})$ and $\omega_k^*(\lambda g_{N-(k+1)}^{\Sigma})$ cannot be obtained in the above-mentioned manner.

Differentiating (46) and (47) and inserting $\theta = \lambda_{N-(k+1)}^{\Sigma}$ yield

$$\pi_k^*(\lambda g_{N-(k+1)}^{\Sigma}) = -\left(\lambda \sum_{i=1}^{k-1} g_{k-i} \pi_i^{*(1)}(\lambda g_{N-(k+1)}^{\Sigma}) + (\pi_{k+1}(0) + c_k(0) + \omega_k(0)) S^{*(1)}(\lambda g_{N-(k+1)}^{\Sigma}) \right), \tag{71}$$

$$\pi_1^{*(1)}(\lambda g_{N-(k+1)}^{\Sigma}) = \frac{1}{\lambda(g_{N-1}^{\Sigma} - g_{N-(k+1)}^{\Sigma})} \Big\{ \lambda g_{N-(k+1)}^{\Sigma} \pi_1^*(\lambda g_{N-(k+1)}^{\Sigma}) + (\pi_2(0) + c_1(0) + \omega_1(0)) S^{*(1)}(\lambda g_{N-(k+1)}^{\Sigma}) \Big\}, \tag{72}$$

$$\pi_i^{*(1)}(\lambda g_{N-(k+1)}^{\Sigma}) = \frac{1}{\lambda(g_{N-1}^{\Sigma} - g_{N-(k+1)}^{\Sigma})} \Big\{ \lambda g_{N-(k+1)}^{\Sigma} \pi_i^*(\lambda g_{N-(k+1)}^{\Sigma}) + \lambda \sum_{j=1}^{i-1} g_{i-j} \pi_j^{*(1)}(\lambda g_{N-(k+1)}^{\Sigma}) + (\pi_{i+1}(0) + c_i(0) + \omega_i(0)) S^*(\lambda g_{N-(k+1)}^{\Sigma} \Big\}, \quad 2 \leq i \leq k-1. \tag{73}$$

Analogously, $\omega_k^*(\lambda g_{N-(k+1)}^{\Sigma})$ can be obtained by differentiating (50) and (51) and inserting $\theta = \lambda g_{N-(k+1)}^{\Sigma}$.

Remark 3.4: This calculation can be easily extended to the case $g_k = g_{k-1} = 0$ by considering the second derivatives of (46) and (47), (50) and (51) respectively. The case $g_1 = 0$ can be handled by observing a different set of equations (46)-(47). We do not need any assumptions on the values g_i, $i = 1, \ldots$, in contrast to Baba [1, 3], where he assumed that $g_i \neq 0$ $(i = 1, \ldots, N)$.

4. The Loss Probability

In this section we will consider two different probabilities: First, the probability that the first customer of a batch is being lost and second, the probability that an arbitrary customer is being lost.

Theorem 4.1: *The probability that the first customer of a batch is being lost is given for the PBAS by*

$$\mathbf{P}_{\text{loss}}^{\text{PBAS}} = \pi_N^*(0) + \omega_N^*(0), \tag{74}$$

and is given for the WBAS by

$$\mathbf{P}_{\text{loss}}^{\text{WBAS}} = (c_0^*(0) + \omega_0^*(0))g_{N+1}^c + \sum_{i=1}^{N}(\pi_i^*(0) + \omega_i^*(0))g_{N-i+1}^c. \tag{75}$$

Proof: Because of the Poisson Arrivals See Time Averages (PASTA) property (see Wolff [16]), note that the first customer sees time averages (e.g. $\pi_i^*(0)$, $\omega_i^*(0)$). Under the PBAS, the first customer is lost if there is no waiting place, i.e. if there are already N customers in the system. Hence (74) follows. Under the WBAS, the first customer is lost if the whole batch is lost, i.e., if the batch size is greater than the number of empty waiting places. Hence (75) follows. □

The loss probability of an arbitrary customer is given by the following theorem.

Theorem 4.2: *The probability that an arbitrary customer is lost is given by*

$$\mathbf{P}_{\text{loss}} = 1 \frac{\sum_{n=1}^{N} \pi_n^*(0)}{\lambda \mathbf{E}(X)\mathbf{E}(S)}, \tag{76}$$

where $\mathbf{E}(X)$ *denotes the expectation of the batch size and* $\mathbf{E}(S)$ *the expectation of the service time.*

Proof: We restrict ourselves to only the service facility (excluding the waiting room). The rate $\lambda\mathbf{E}(X)(1-\mathbf{P}_{\text{loss}})$ is the arrival rate of customers accepted by the system, and it is also the throughput of the service facility. On the other hand, the mean number of customers in the service facility is given by $\sum_{n=1}^{N}\pi_n^*(0)$. Applying Little's law, we then obtain (76). □

Remark 4.1: Because of the generality of Little's law, Theorem 4.2 is independent of the acceptance strategy. Nevertheless, the loss probability will depend on the strategy by virtue of the different sets of state probabilities.

Remark 4.2: If we restrict Theorem 4.2 to the non-vacation and non-close-down case, we obtain a formula for the loss probability of an arbitrary customer which is much simpler than the analysis given in Baba [1].

5. Waiting Time Analysis

In this section we will derive the Laplace-Stieltjes transform of the waiting time distribution of the first customer as well as of an arbitrary customer of an actual arrived batch. Furthermore, by using Little's law, we will obtain the mean waiting time for all customers that enter the system.

Following Burke [4] (see also Manfield and Tran-Gia [11]) we can derive the following lemma.

Lemma 5.1: *We will focus on an arbitrary arriving customer. By BS we will denote the size of the batch in which he arrives and by PC his position in the batch. Then the following holds*

$$\mathbf{P}(BS = k) = \frac{kg_k}{\mathbf{E}(X)}, \tag{77}$$

$$\mathbf{P}(PC = i \mid BS = k) = \frac{1}{k}, \quad 1 \le i \le k, \tag{78}$$

and hence

$$\mathbf{P}(PC = i) = \frac{g_i^c}{\mathbf{E}(X)}, \tag{79}$$

$$\mathbf{P}(PC = i \text{ and } BS = k) = \frac{g_k}{\mathbf{E}(X)}, \quad 1 \le i \le k. \tag{80}$$

Remark 5.1: From this lemma we can easily obtain the loss probabilities, i.e., for the partial batch acceptance strategy, the loss probability is given by

$$\mathbf{P}_{\text{loss}} = \frac{1}{\mathbf{E}(X)}\left(\sum_{i=1}^{N}\pi_i^*(0)\sum_{j=N-i+1}^{\infty}(j-N+i)g_j + c_0^*(0)\sum_{j=N+1}^{\infty}(j-N)g_j\right. \tag{81}$$
$$\left. + \sum_{i=0}^{N}\omega_i^*(0)\sum_{j=N-i+1}^{\infty}(j-N+i)g_j\right),$$

and for the whole batch acceptance strategy, the loss probability is given by

$$\mathbf{P}_{\text{loss}} = \frac{1}{\mathbf{E}(X)}\left(\sum_{i=1}^{N}\pi_i^*(0)\sum_{j=N-i+1} jg_j + c_0^*(0)\sum_{j=N+1}^{\infty} jg_j\right. \tag{82}$$
$$\left. + \sum_{i=0}^{N}\omega_i^*(0)\sum_{j=N-i+1}^{\infty} jg_j\right).$$

Under PBAS, when the batch is too large for the available space, the test customer occupies a position in the rejected portion of the batch with probability $(j-N+i)/j$ and hence (81) follows. Equation (82) follows from the fact that when the batch containing the test customer is larger than the available waiting space, all the customers of this batch are lost.

We are now in a position to derive the following theorem.

Theorem 5.1: *The Laplace-Stieltjes transform of the waiting time distribution of the first customer in a batch under the PBAS is given by*

$$W_F^*(\theta) = \frac{\sum_{i=1}^{N-1}\pi_i^*(\theta)[S^*(\theta)]^{i-1} + \sum_{i=0}^{N-1}\omega_i^*(\theta)[S^*(\theta)]^i + c_0^*(0)}{1-\pi_N^*(0)-\omega_N^*(0)} \tag{83}$$

and is given by

$$W_F^*(\theta) = \frac{\sum_{i=1}^{N-1}\pi_i^*(\theta)g_{N-i}^{\Sigma}[S^*(\theta)]^{i-1} + \sum_{i=0}^{N-1}\omega_i^*(\theta)g_{N-i}^{\Sigma}[S^*(\theta)]^i + c_0^*(0)g_N^{\Sigma}}{\sum_{i=1}^{N-1}\pi_i^*(0)g_{N-i}^{\Sigma} + c_0^*(0)g_N^{\Sigma} + \sum_{i=0}^{N-1}\omega_i^*(0)g_{N-i}^{\Sigma}} \tag{84}$$

under the WBAS.

Proof: The denominator of (83) is the probability that a batch is not totally rejected under the PBAS and hence the first customer is accepted. The numerator just adds the remaining service (vacation) time and the service times of the customers already in the system. Under the WBAS the first customer is accepted if and only if the whole batch is accepted, hence (83) follows. □

For the waiting time of an arbitrary customer the following theorem holds.

Theorem 5.2: *The Laplace-Stieltjes transform of the waiting time distribution of an arbitrary customer in a batch under the PBAS is given by*

$$W_A^*(\theta) = \left\{\sum_{i=1}^{N-1} \pi_i^*(0) \sum_{j=1}^{N-i} g_j^c + c_0^*(0) \sum_{j=1}^{N} g_j^c + \sum_{i=0}^{N-1} \omega_i^*(0) \sum_{j=1}^{N-i} g_j^c\right\}^{-1}$$
$$\times \left\{\sum_{i=1}^{N-1} \pi_i^*(\theta) \sum_{j=1}^{N-i} g_j^c [S^*(\theta)]^{i+j-2} + c_0^*(0) \sum_{j=1}^{N} g_j^c [S^*(\theta)]^{j-1} + \sum_{i=0}^{N-1} \omega_i^*(\theta) \sum_{j=1}^{N-i} g_j^c [S^*(\theta)]^{i+j-1}\right\} \tag{85}$$

and is given by

$$W_A^*(\theta) = \left\{\sum_{i=1}^{N-1} \pi_i^*(0) \sum_{j=1}^{N-i} j g_j + c_0^*(0) \sum_{j=1}^{N} j g_j + \sum_{i=0}^{N-1} \omega_i^*(0) \sum_{j=1}^{N-i} j g_j\right\}^{-1}$$
$$\times \left\{\sum_{i=1}^{N-1} \pi_i^*(\theta) \sum_{j=1}^{N-i} g_j \sum_{k=0}^{j-1} [S^*(\theta)]^{i+k-1} + c_0^*(0) \sum_{j=1}^{N} g_j \sum_{k=0}^{j-1} [S^*(\theta)]^{k} + \sum_{i=0}^{N-1} \omega_i^*(\theta) \sum_{j=1}^{N-i} g_j \sum_{k=0}^{j-1} [S^*(\theta)]^{i+k}\right\} \tag{86}$$

under the WBAS.

Proof: These results can be obtained by using Lemma 5.1. □

From these equations the mean waiting time can be obtained, but it is easier to obtain it directly from Little's law using effective arrival rates.

Theorem 5.3: *The mean waiting time for all customers that enter the system is given by*

$$\mathbf{E}(W) = \frac{\mathbf{E}(S)\left(\sum_{k=2}^{N} (k-1)\pi_k^*(0) + \sum_{k=1}^{N} k\omega_k^*(0)\right)}{\sum_{n=1}^{N} \pi_n^*(0)} \tag{87}$$

independent of the acceptance strategy.

Proof: We will restrict our attention to the waiting places (excluding the service facility) only. Following Theorem 4.2 the effective arrival rate of customers λ' is given by

$$\lambda' = \frac{\sum_{n=1}^{N} \pi_n^*(0)}{\mathbf{E}(S)}.$$

Noting that the waiting room consists of $N-1$ places if the server is serving and N places if the server is closing down or on vacation, we obtain the mean queue length

$\mathbf{E}(L)$ as

$$\mathbf{E}(L) = \sum_{k=1}^{N}(k-1)\pi_k^*(0) + \sum_{k=0}^{N}k(c_k^*(0)+\omega_k^*(0))$$
$$= \sum_{k=2}^{N}(k-1)\pi_k^*(0) + \sum_{k=1}^{N}k\omega_k^*(0).$$

Using Little's law yields the assertion. □

6. Numerical Results

In this section we will apply the above-derived results to the following setting:

- the batch size is deterministic and equals 5, i.e., $g_5 = 1$;
- the number of waiting places, including the customer that may be in service, equals 11, i.e., $N = 11$;
- the service time, close-down time and vacation time distribution is either deterministics (Det), Erlang of order 2 (Erl), exponential (Exp) or hyperexponential of order 2 (Hyp);
- by Det(x), Erl(x), Exp($1/x$), Hyp(x), we denote the corresponding distribution with mean x.

We will calculate the loss of probabilities and the expected waiting time of an arbitrary customer for this setting under the partial batch acceptance strategy (PBAS) and the whole batch acceptance strategy (WBAS).

Remark 6.1: Note that under the whole batch acceptance strategy this setting cannot be analyzed by Baba's [1, 3] result.

Example 6.1: The expected waiting time of an arbitrary customer under the partial batch acceptance strategy w.r.t. the close-down time distribution for different values of the traffic load ρ is given in Figure 1. In this example the service time and vacation time are exponentially distributed with mean 1. It can be seen that the waiting time is decreasing if the close-down time is increasing.

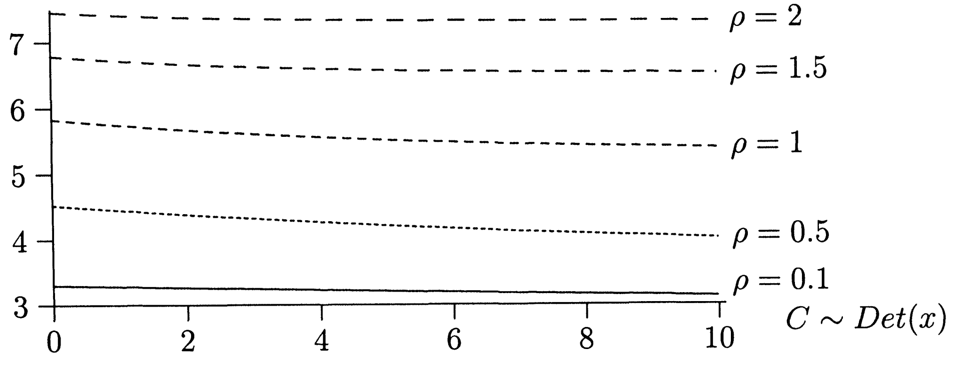

Figure 1. Expected waiting time of an arbitrary customer under the PBAS w.r.t. the close-down time distribution ($S \sim$ Exp(1), $V \sim$ Exp(1)).

Example 6.2: The loss probability of the first customer and of an arbitrary customer under the partial batch acceptance strategy w.r.t. the close-down time distribution for different values of the traffic load ρ are given in Figure 2 and in Figure 3. In this example, the service time and vacation time are exponentially distributed with mean 1. It can be seen that the close-down distribution has almost no influence on the loss probabilities. This comes from the fact that the main part of the loss probability comes from a loss during a service period. The joint probability of being on vacation and losing a customer is very small, so either the traffic load is small and hence the probability of being lost is high or the the traffic load is high and hence the probability of being on vacation is small.

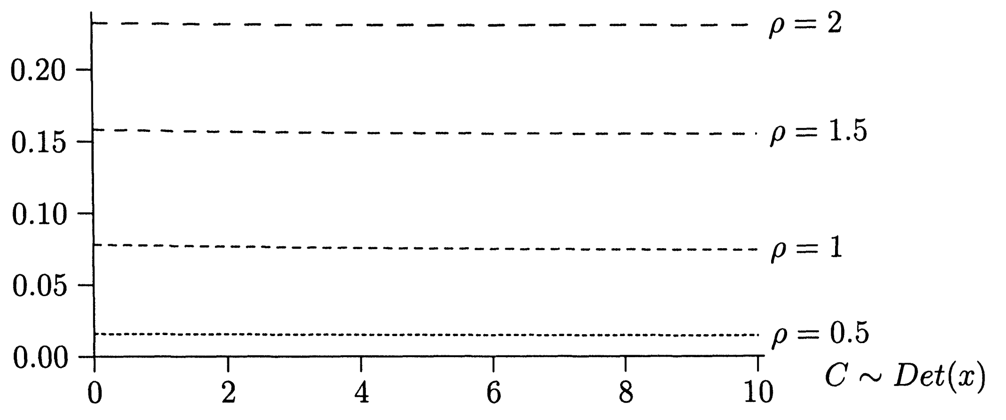

Figure 2. Loss probability of the first customer under the PBAS w.r.t. the close-down time distribution ($S \sim \text{Exp}(1)$, $V \sim \text{Exp}(1)$).

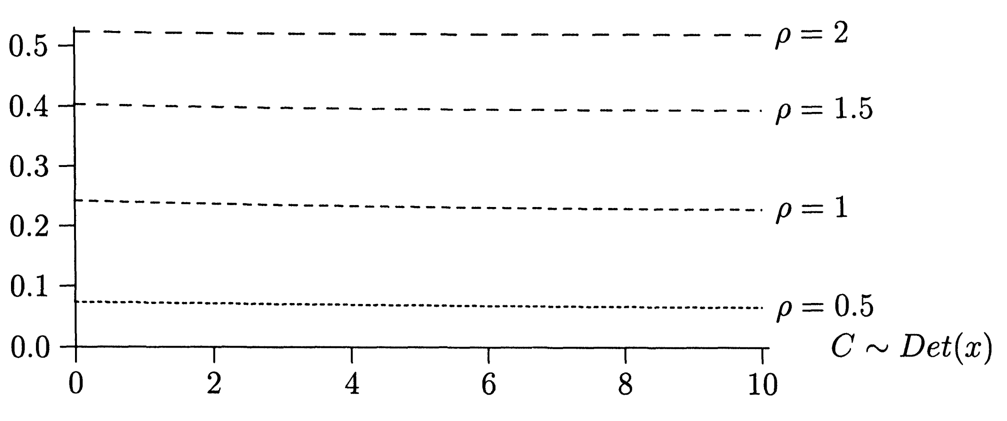

Figure 3. Loss probability of an arbitrary customer under the PBAS w.r.t. the close-down time distribution ($S \sim \text{Exp}(1)$, $V \sim \text{Exp}(1)$).

Example 6.3: The expected waiting time of an arbitrary customer under the PBAS w.r.t. the vacation distribution for different values of the close-down time distribution is given in Figure 4. In this example the service time is exponentially distributed with mean 1 and the traffic load $\rho = 0.5$. It can be seen that the influence of the close-down time distribution is high if the mean vacation time is also high.

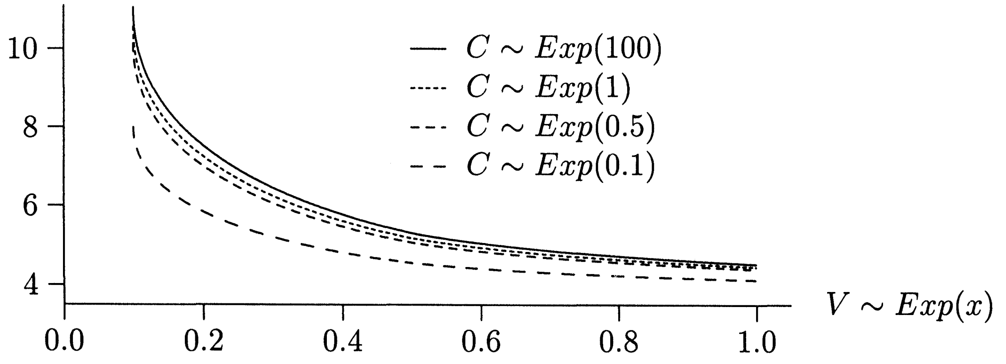

Figure 4. Expected waiting time of an arbitrary customer under the PBAS ($\rho = 0.5$, $S \sim \text{Exp}(1)$).

Example 6.4: The loss probability of the first customer under the partial batch acceptance strategy w.r.t. the traffic load ρ for different values of the vacation time distribution is given in Figure 5. In this example, the service time and close-down time are exponentially distributed with mean 1. It can be seen that the vacation time distribution is not as important as the service time distribution illustrated in the next example.

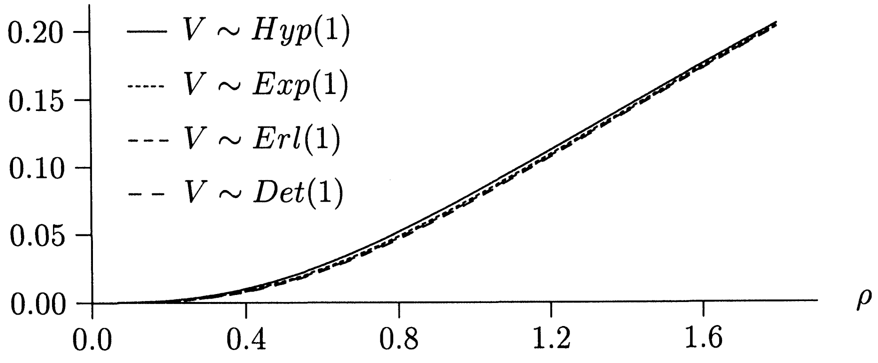

Figure 5. Loss probability of the first customer under the PBAS ($S \sim \text{Exp}(1)$, $C \sim \text{Exp}(1)$).

Example 6.5: The loss probability of the first customer under the partial batch acceptance strategy w.r.t. the traffic load ρ for different values of the service time distribution is given in Figure 6. In this example, the vacation time and close-down time are exponentially distributed with mean 1. It can be seen that the service time distribution plays an important role and that the different curves are strictly ordered by their service time distributions.

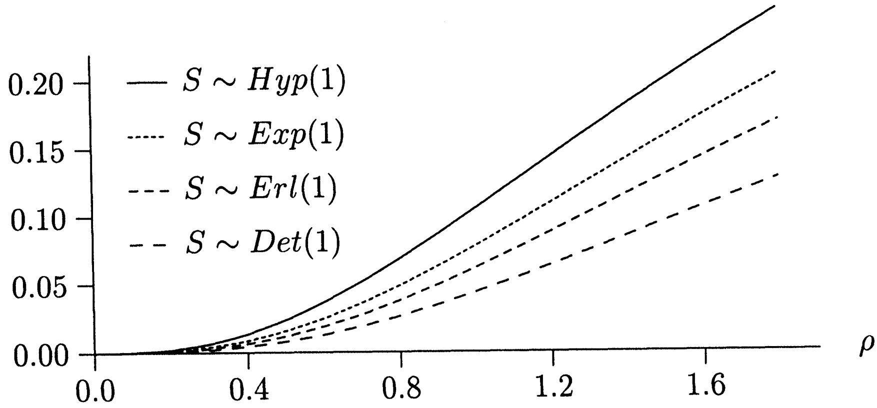

Figure 6. Loss probability of the first customer under the PBAS ($V \sim$ Exp(1), $C \sim$ Exp(1)).

Example 6.6: The loss probability under the whole batch acceptance strategy w.r.t. the traffic load ρ is given in Figure 7. In this example, the service time, vacation time, and close-down time are exponentially distribution with mean 1.

Example 6.7: The expected waiting time of an arbitrary customer under the whole batch acceptance strategy w.r.t. the traffic load ρ for different values of the service time (close-down time, vacation time) distribution is given in Figure 8 (Figure 9, Figure 10, respectively). It can be seen from these figures, that the service time distribution is dominant for heavy traffic, the vacation time distribution is important under light traffic and the close-down time distribution has almost no influence (under the same mean).

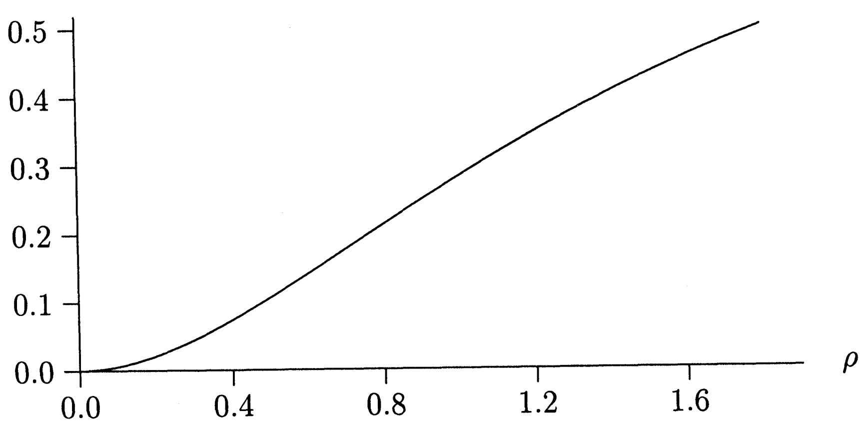

Figure 7. Loss probability under the WBAS ($S \sim$ Exp(1), $C \sim$ Exp(1), $V \sim$ Exp(1)).

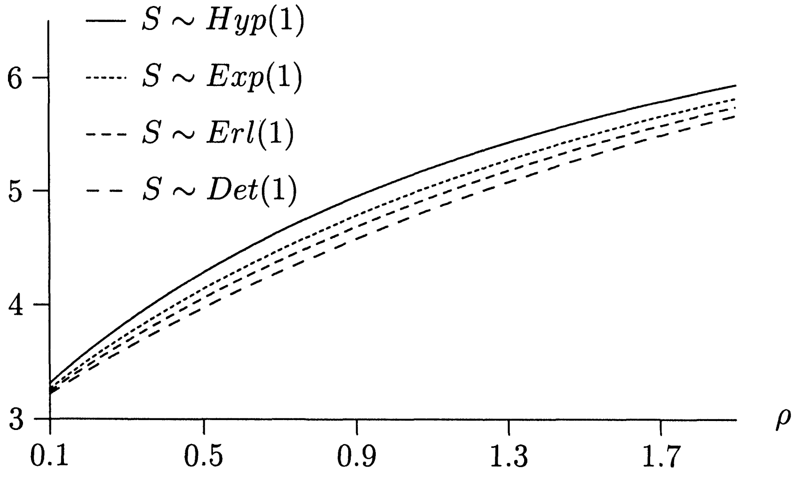

Figure 8. Expected waiting time of an arbitrary customer under the WBAS ($V \sim$ Exp(1), $C \sim$ Exp(1)).

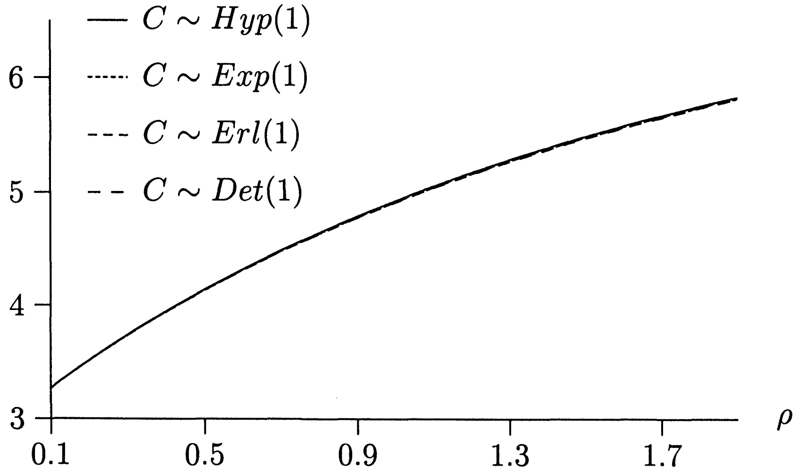

Figure 9. Expected waiting time of an arbitrary customer under the WBAS ($S \sim$ Exp(1), $V \sim$ Exp(1)).

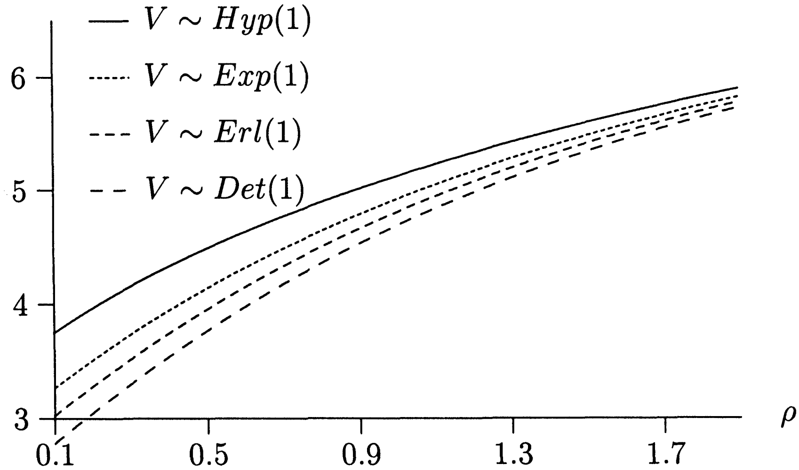

Figure 10. Expected waiting time of an arbitrary customer under the WBAS ($S \sim$ Exp(1), $C \sim$ Exp(1)).

7. Conclusions

We consider an $M^X/GI/1/N$ finite capacity queue with close-down, server vacation and exhaustive service discipline. Introducing the remaining service time (close-down time, vacation time resp.) as a supplementary variable enabled us to obtain a set of differential equations under both the partial batch acceptance strategy (PBAS) and the whole batch acceptance strategy (WBAS). It should be noted that the embedded Markov chain technique could not be applied to the WBAS system. We showed how to solve these sets of equations to obtain the queue length at an arbitrary time as well as at a departure epoch. Furthermore, we obtained the loss probabilities and the Laplace-Stieltjes transforms of the waiting time distributions for the first customer and for an arbitrary customer of a batch. Numerical calculations showed the influence of the close-down time, vacation time and the service time distribution to the loss probability and the mean waiting time. Hereby it turned out that the vacation time distribution is dominant over the service time distribution in light traffic whereas the opposite holds under heavy traffic. For moderate traffic the service time distribution has more influence than the vacation time distribution.

For further research we are thinking about a single-server finite capacity queue with vacation time and batch-Markov arrival input, where we have to consider the arrival phases.

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