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MARK ALEXANDROVICH KRASNOSEL'SKII

(April 27, 1920 - February 13, 1997)

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On February 13, 1997, the world famous mathematician, Mark Alexandrovich Krasnosel'skii, died unexpectedly. This article gives a brief overview of his life and research.

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1. A Biographical Note

Krasnosel'skii was born on April 27, 1920 in the town of Starokonstantinov in the Ukraine. His father, Alexander Jakovlevich, worked as a construction engineer for the Asov Fishing Company (Azovrybtrest). His mother, Fanny Moiseevna, taught Russian in an elementary school. There were two sons in the Krasnosel'skii family. The older son, Iosiph, became a well-known metallurgical engineer and one of the founders and principals of a Moscow factory for special alloys. The younger son, Mark, selected mathematics as vocation.

In 1932, the Krasnosel'skii family moved to Berdyansk. In 1938, Mark Krasnosel'skii finished high school and the same year he enrolled in the department of physics and mathematics of Kiev University. At the beginning of World War II, Kiev University was evacuated to Kazakhstan, where it came to be known as the Joint Ukrainian University. In 1942, Krasnosel'skii graduated from the Joint Ukrainian University and then joined the Soviet Army for the next four years. He taught at the Ryazan Artillery School, which was evacuated to Talgar in the Alma-Ata region. In 1946, Krasnosel'skii was demobilized at the rank of lieutenant and in August of the same year he moved to Kiev. There, initially (for several months), Krasnosel'skii taught descriptive geometry at the Kiev Road-Institute, after which he accepted an entry-level scientific position at the Mathematical Institute of the Ukranian Academy of Sciences.

It was in Kiev, during the first years after the war, where Krasnosel'skii began his active scientific life. He attended lectures and participated in seminars lead by prominent scientists, such as N.N. Bogoliubov, A.N. Kolmogorov, M.G. Krein, B.V. Gnedenko, M.A.Lavrent'ev, A.Yu. Ishlinskii, N.V. Efimov, A.G. Kurosh, V.E. Loshkarev, and others.

In 1948, Krasnosel'skii defended his Ph.D. thesis on the theory of extension for Hermite operators, and in 1950, he completed his higher degree doctoral dissertation (known as "habilitation" in Western Europe) on topological methods in nonlinear analysis.

In 1953, he moved to Voronezh, where during the subsequent 15 years he chaired the functional analysis section (kafedra) of the physico-mathematical, and afterwards, mathematico-mechanical faculty of Voronezh University. His seminar on nonlinear analysis became well known far beyond Voronezh.

His scientific interests were extensive and covered many aspects of modern mathematics. Krasnosel'skii opened a variety of new scientific directions, the development of which laid the fundamentals of modern nonlinear analysis. He presented basic and special courses and supervised seminars.

In 1968, Krasnosel'skii moved from Voronezh to Moscow and began to work at the Institute of Control Sciences of the USSR Academy of Sciences, which in those years was referred to as the Institute of Automatics and Telemechanics. Here, Krasnosel'skii supervised the Laboratory of Mathematical Methods in Complex Systems Analysis. The profile of the Institute of Control Sciences was reflected in the more applied direction of the Moscow period of Krasnosel'skii's scientific activities (control theory, mathematical models of a hysteresis, etc.).

In 1990, Krasnosel'skii moved to the Institute for Information Transmissions Problems of the USSR Academy of Sciences. Here, as well as at the Institute of Control Sciences, high-class results in abstract mathematics alternating with those in applied branches (dynamics of hysteresis systems, impulse desynchronized systems, system with incomplete corrections, etc.) were obtained by Krasnosel'skii.

The scientific life of Krasnosel'skii was always tightly interlaced with his pedagogical activity. From the first days of scientific work, there was exhibited love and the skill of Krasnosel'skii to attract talented young people to science. A large reserve of scientific enthusiasm and optimism, present during the years of joint work with Krasnosel'skii, has inspired his students for many years. Dozens of his former pupils have scientific degrees; more than 30 of them are doctors of sciences and professors with their own scientific directions and scientific schools.

Up to his last days, Krasnosel'skii actively worked and was full of energy and creative plans. On February 13, 1997, he died at his desk.

In more than half a century of his scientific activity, Krasnosel'skii wrote 14 monographs and more than three hundred scientific articles.

Below, we outline the main directions of his research.

2. A Survey of Krasnosel'skii's Research Activities

In connection with classical Von Neumann problems, Krasnosel'skii constructed the general theory of extensions of Hermite operators with nondense domains. He proved theorems for invariance of deficiency indices of non-Hermitian operators and dis-

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covered an unexpected link between invariance problems and Lyusternik-Shnirel'man category theory. He and Mark G. Krein found new invariance conditions for a Noether index.

Krasnosel'skii was the first to analyze the functional properties of fractional degrees of self-adjoint operators; the established theorems were extended later (together with P.E. Sobolevskii) for non-self-adjoint operators in non-Hilbert spaces. A fractional degrees method is widely applied now in the investigation of various boundary value problems in mathematical physics, problems of hydrodynamics, etc.

A large part of Krasnosel'skii's work, which was done together with P.P. Zabreiko, E.A. Lifshits, Yu.V. Pokorny, A.V. Sobolev, V.Ya. Stecenko, and others, included Krein's cone theory and extended the theory of positive operators in new directions. New classes of operators with leading prime eigenvalues were described, spectral gaps were estimated, and various geometric problems were solved. As became apparent afterwards, these new classes of operators covered many operators arising in mathematical physics (for instance, the problem of neutron propagation). The method of fractional degrees of operators and Krasnosel'skii's theorem on interpolation property of complete continuity were keynote elements for a number of works on interpolation theory.

During several years, Krasnosel'skii, together with Ya.B. Rutickii and P.P. Zabreiko, studied properties of integral operators in various functional spaces. He offered powerful conditions for continuity and complete continuity, differentiability on the whole space and at individual points, and concavity and convexity for integral operators and superposition operators. Together with A.V. Pokrovskii, he discovered some unexpected properties of discontinuous integral operators. These results laid a background material to the theory as well as applications in the area of integral equations.

Krasnosel'skii realized that a convenient tool for analyzing integral equations with strong (exponential) nonlinearities is Orlicz spaces. That is why the theory of Orlicz spaces was subjected by Krasnosel'skii and Rutickii to radical reorganization. New classes of convex functions were introduced and investigated and special properties of operators in these spaces were discovered.

The topological methods of nonlinear analysis, originating from contributions of Birkhoff, Kellog, Leray, Schauder, Tikhonov, Nemytskii, and others, were applied when proving existence theorems. In Krasnosel'skii's work, topological methods became a universal tool for deriving the answers to many qualitative problems, such as: evaluating the number of solutions, description of the structure of a solution set and conditions of connectedness of this set, convergence of approximate methods of Galerkin type, and bifurcations of solutions in nonlinear problems. Excellent results of Krasnosel'skii in all of these areas are widely known and applied.

Jointly with P.P. Zabreiko, I.A. Bakhtin, V.V. Strygin, E.M. Mukhamadiev and E.A. Lifshits, Krasnosel'skii offered new general principles of resolvability of nonlinear equations, including the principle of one-sided estimates, the principle of cone expansion and contraction, the drop principle, the fixed-point theorems for monotone operators, a combination of the Schauder principle and the contractive mapping principle (which became the foundation of the intensively developing theory of condensing operators), and the partial inversion principle. Starting from some unknown work of P.S. Uryson on integral equations, Krasnosel'skii with coauthors developed the theory of equations with concave operators. This theory has found applications in various boundary value problems, oscillations theory, stability, market models, and the analysis of processes in nuclear reactors.

Krasnosel'skii was especially concerned with effective evaluation of topological properties of mappings in infinite dimensional spaces. Among other things, he, together with P.P. Zabreiko, E.A. Lifshits, V.V. Strygin, N.A. Bobylev, and others, established new theorems on periodic mappings and special coverings of spheres, introduced an algorithm for calculating the topological index of a singular point in degenerate cases, and established principles of affinity and invariance of rotations (which link various characteristics of different equations, generated by the same problem). The possibilities of exploitation of topological properties have transformed general methods of nonlinear analysis into an effective tool for investigation of a wide range of specific problems.

For a variety of situations, Krasnosel'skii posed and solved stability problems for wide classes of perturbations arising in variational analysis. For this purpose, he introduced, studied, and calculated a new topological characteristic: the genus of a set.

In his final years, Krasnosel'skii, together with N.A. Bobylev, A.M. Dementieva, A.M. Krasnosel'skii and E.M. Mukhamadiev, suggested a new general method to study degenerate extremals. For example, to analyze the degenerate extremals in the classical Euler problem requires finding the first nonzero number in a certain diagram. The method has found many applications. Krasnosel'skii showed how to use variational schemes in problems of general nonlinear analysis.

Krasnosel'skii developed qualitative methods to study critical and bifurcation values of parameters. These well-known methods are useful in many applications (hydrodynamics, forms of stability loss for elastic systems, problems of self-oscillations, etc.), since they are based on rather restricted information of the corresponding nonlinear equations. For example, to analyze bifurcation values of parameters it is sufficient to know the properties of the equations linearized at zero or at infinity. These qualitative methods made it possible to find sets of nontrivial solutions and to study spectra of nonlinear problems.

To investigate bifurcations of solutions for general nonlinear functional equations, Krasnosel'skii (together with P.P. Zabreiko) developed the method of simple solutions, found asymptotic representations of these solutions, and discovered natural relations between the methods of Lyapunov, Schmidt, and Nekrasov.
He suggested a method of parameter functionalization, which enables one to exempt parameters in miscellaneous nonlinear problems. In particular, this method leads to powerful generalizations of the well-known Hopf bifurcation theorem on self-oscillations.

Krasnosel'skii obtained a range of sufficient conditions for uniqueness, nonlocal continuability, stability, correctness, dissipativity, and solvability of Cauchy and other boundary value problems. He, S.G. Krein, and P.E. Sobolevskii proved the first general theorems concerning solvability of nonlinear differential equations with unbounded operators in functional spaces. Krasnosel'skii, together with A.I. Perov, V.V. Strygin, N.A. Bobylev, and E.M. Mukhamadiev suggested the method of leading potentials to study periodic oscillations and bounded solutions of nonlinear systems.

Krasnosel'skii, jointly with V.Sh. Burd and Yu.S. Kolesov, developed essentially new methods for the analysis of nonlinear almost-periodic oscillations with applications to the theory of pendulums and automatic control problems. He and S.G. Krein suggested a new approach to justify basic theorems of the Bogoliubov-Krylov averaging method and applied the averaging method to bifurcation problems. Krasnosel'skii and A.V. Pokrovskii proposed a new method to study absolute stability. This method requires one only to analyze the uniqueness of solutions for some explicitly written equations; it is used to analyze absolute stability for systems with many nonlinear components. Krasnosel'skii and N.A. Bobylev suggested a new approach to study Lyapunov systems.

In a series of papers, in part co-authored with Ya.B. Rutickii, V.A. Chechik, and others, Krasnosel'skii studied the general theory of approximate methods, proved applicability of the Galerkin, Galerkin-Petrov, and Ritz methods to nonlinear problems, and obtained a posteriori estimates of errors for approximations of solutions. He also pioneered new techniques for estimation of spectral radii of linear operators (these estimates are important for analyzing the convergence of iterative procedures) and suggested new modifications of the Seidel-Nekrasov method. Krasnosel'skii and S.G. Krein developed a widely used method of minimum discrepancies for solving linear problems; together with I.V. Yemelin and N.P. Panskikh, he developed a spurt-method based on ideas of control of variable structure systems; jointly with I.V. Yemelin he has studied a halt-method for iterative procedures. This method produces a regularization for a wide class of the incorrect problems. Krasnosel'skii and A.V. Pokrovskii developed and studied a shuttle iterations approach, which is useful for approximate solutions of various boundary value problems and in analysis of oscillations in variable structure systems (when corresponding operators are discontinuous).

Krasnosel'skii, together with N.A. Bobylev and N.A. Kuznetsov, investigated a harmonic balance method, which finds important applications in engineering. He the applicability range of this method, proved convergence theorems proved and evaluated the convergence rate of the harmonic balance method.

Krasnosel'skii developed new techniques in studying dynamics of control systems with nonlinear terms with discontinuous characteristics. He established a number of remarkable theorems to state existence of solutions (well-defined in respect to perturbations) for equations with discontinuous operators, estimate the number of solutions, compute them with a high precision, and to take into account the influence of inevitable small noise. The proposed methods are applicable to systems with variable structures and to the analysis of problems such as the M.A. Lavrent'ev problem on separation flow.

In the middle of the seventies, Krasnosel'skii offered an extensive program for studying systems with hysteresis and attracted a large group of his colleagues to this program, including A.V. Pokrovskii, V.S. Kozyakin, P.P. Zabreiko, A.F. Kleptsyn, E.A. Lifshits, N.I. Grachev, A.A. Vladimirov, V.V. Chernorutskii, and D.I. Rachinskii. This program was based on the introduction of special mathematical structures, corresponding to various phenomenological models in plasticity theory and magnetism. A realization of this program lead to the solution of several non-conventional problems. For example, vibrostable differential equations were described and studied; for stochastic differential equations the possibility of selection of individual trajectories corresponding to individual Wiener processes was investigated, and the role of Frobenius complete integrability condition for stochastic equations was emphasized. Almost all classical models of hysteresis were covered by the constructed mathematical theory. It allowed the reduction of phenomenological models of hysteresis (constructive, magnetic, plastic etc.) to more convenient mathematical models.

In the last years, Krasnosel'skii, together with E.A. Asarin, V.S. Kozyakin, A.F. Kleptsyn and N.A. Kuznetsov was actively involved in the construction of the desynchronized systems theory. He offered methods for the qualitative analysis of desynchronized systems, stability of such systems, and applications to numerous problems in engineering.

Certainly, in the few pages of this paper it was impossible to completely cover the scope of all Krasnosel'skii's achievements. This short review features only some basic directions of his scientific activity.

Monogrphs of M.A. Krasnosel'skii

[1] *Topological Methods in the Theory of Nonlinear Integral Equations*, GTTI, Moscow, 1956, Russian (Eng.: Pergamon Press, Oxford, 1964).

[2] *Convex Functions and Orlicz Space* (with Ya.B. Rutickii), Physmatgiz, Moscow, 1958, Russian (Eng.: Noordhoff, Groningen, 1961).

[3] *Positive Solutions of Operator Equations*, Physmatgiz, Moscow, 1962, Russian (Eng.: Noordhoff, Groningen, 1964).

[4] *Plane Vector Fields* (with P.P. Zabreiko, et al.), Physmatgiz, Moscow, 1963, Russian (Eng.: Acad. Press, New York, 1966).

[5] *Functional Analysis*, SMB, Nauka, Moscow, 1964, 2-nd revised edition, 1972, Russian (Eng.: Noordhoff, Groningen, 1972).

[6] *The Operator of Translation Along the Trajectories of Differential Equations*, Physmatgiz, Moscow, 1966, Russian (Eng.: Trans. Math. Monogr. 19, AMS, Providence, 1968).

[7] *Integral Operators in Spaces of Summable Functions*, (with P.P. Zabreiko, et al.) Nauka, Moscow, 1966, Russian (Eng.: Noordhoff, Leyden, 1976).

[8] *Integral Equations* (with P.P. Zabreiko, et al.), SMB, Nauka, Moscow, 1968, Russian (Eng.: Noordhoff, Leyden, 1975).

[9] *Approximate Solution of Operator Equations* (with G.M. Vainikko, et al.), Nauka, Moscow, 1969, Russian (Eng.: Wolters-Noordhoff Publ., Groningen, 1976).

[10] *Nonlinear Almost-Periodic Oscillations* (with V.Sh. Burd and Yu.S. Kolesov), Nauka, Moscow, 1970, Russian (Eng.: Halsted Press, New York, 1973).

[11] *Geometric Methods of Nonlinear Analysis* (with P.P. Zabreiko), Nauka, Moscow, 1975, Russian (Eng.: Springer; 1989).

[12] *System with Hysteresis* (with A.V. Pokrovskii), Nauka, Moscow, 1983, Russian (Eng.: Springer, 1989).

[13] *Positive Linear Systems* (with E.A. Lifshits and A.V. Sobolev), Nauka, Moscow, 1985, Russian (Eng.: Hildermann, Berlin, 1990).

[14] *Analysis of Stability of Desynchronized Discrete Systems* (with E.A. Asarin et al.), Nauka, Moscow, 1992, Russian.

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