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Research Article

Perishable Inventory System with Postponed Demands and Negative Customers

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This article considers a continuous review perishable (s,S) inventory system in which the demands arrive according to a Markovian arrival process (*MAP*). The lifetime of items in the stock and the lead time of reorder are assumed to be independently distributed as exponential. Demands that occur during the stock-out periods either enter a pool which has capacity $N(<\infty)$ or are lost. Any demand that takes place when the pool is full and the inventory level is zero is assumed to be lost. The demands in the pool are selected one by one, if the replenished stock is above s, with time interval between any two successive selections distributed as exponential with parameter depending on the number of customers in the pool. The waiting demands in the pool independently may renege the system after an exponentially distributed amount of time. In addition to the regular demands, a second flow of negative demands following *MAP* is also considered which will remove one of the demands waiting in the pool. The joint probability distribution of the number of customers in the pool and the inventory level is obtained in the steady state case. The measures of system performance in the steady state are calculated and the total expected cost per unit time is also considered. The results are illustrated numerically.

1. Introduction

In most of the inventory models considered in the literature, the demanded items are directly delivered from the stock (if available). The demands occurring during the stock-out period are either lost (lost sales) or satisfied only after the arrival of ordered items (backlogging). In the latter case, it is assumed that either all (full backlogging) or any prefixed number of demands (partial backlogging) that occurred during the stock-out period are satisfied. The often quoted review articles Nahmias [1] and Raafat [2] and the recent review articles N. H. Shah and Y. K. Shah [3] and Goyal and Giri [4] provide excellent summaries of many of these modeling efforts. For some recent references see Chakravarthy and Daniel [5], Yadavalli et al. [6], and Kalpakam and Shanthi [7, 8].

In the case of backlogging, the backlogged demands are satisfied immediately when the ordered items are materialized. But in some real-life situations, the backlogged demand may have to wait even after the replenishment. This type of inventory problems are called inventory with postponed demands. The concept of postponed demand in inventory has been introduced by Berman et al. [9]. They have assumed that both demand and service rates are deterministic. Krishnamoorthy and Islam [10] have considered a Markovian inventory system with exponential lead time and the pooled customers are selected according to an exponentially distributed time lag. The concept of postponed customers in queueing model has considered by Deepak et al. [11].

In this work we have extended the work of Krishnamoorthy and Islam [10] by assuming that the items are perishable in nature, the demands occur according to a *MAP*, and that the lead times are distributed as exponential. The demands that occur during the stock-out periods either enter a pool which has capacity $N(<\infty)$ or considered to be lost. Any demand, that takes place when the pool is full and inventory level is zero, is also assumed to be lost. The demands in the pool are selected one-by-one, if the replenished stock is above s, and the interval time between any two successive selections is distributed as exponential with parameter depending on the number of customers in the pool. The waiting demands in the pool may independently renege the system after an exponentially distributed amount of time. In addition to the regular demands, we consider a second flow of negative customers following *MAP* who will remove one of the waiting customers in the pool. In practice, these negative customers may be viewed as the touts of competing organizations, who take away prospective customers. The concept of negative customer was introduced by Gelenbe [12] and the research on queueing systems with negative arrivals has been greatly motivated by some practical applications in computers, neural networks, and communication networks, and so forth. For comprehensive analysis of queueing networks with negative arrivals, one may refer to Chao et al. [13], Gelenbe and Pujolle [14] and Sivakumar and Arivarignan [15]. A recent review can be found in Artalejo [16].

The rest of the paper is organized as follows. In Section 2, we describe the mathematical model and the notations. The formulation and the steady state analysis of the model are presented in Section 3. In Section 4, we derive various system performance measures in the steady state. In Section 5, we calculate the total expected cost rate in the steady state. Some numerical examples are presented in Section 6.

2. The mathematical model

We consider a continuous review (s,S) perishable inventory system in which the demands occur for single units according to a Markovian arrival process (*MAP*) with representation (D_0,D_1) where D's are of order $m_1 \times m_1$. The underlying Markov chain $J_1(t)$ of the *MAP* has the generator $D(=D_0+D_1)$ and a stationary row vector τ_1 of length m_1. The stationary arrival rate is given by $\lambda_1 = \tau_1 D_1 \mathbf{e}$, where $\mathbf{e}$ is a column vector of appropriate dimension containing all ones. The life time of a unit in the inventory is exponentially distributed with a constant failure rate γ. The operating policy is as follows: as soon as the stock level drops to s, a replenishment order for $Q(=S-s>s)$ items is placed. The lead time is exponentially distributed with parameter $\beta(>0)$. Any arriving demands, that occur during the inventory level is zero, are offered the choice of either leaving the system immediately or of being postponed until the ordered items are received. We assume that the demanding customer accept the offer of postponement according to independent Bernoulli trials with probability p, $0 \leq p < 1$. With probability $q = 1 - p$, the customer declines and is considered to be lost. The postponed customers are retained in a pool, which has a finite capacity $N(<\infty)$. After any replenishment and as long as the inventory level is greater than s, the pooled customers are selected according to an exponentially distributed time lag with rate μ_n where n is the number of customers in the pool.

In addition to the regular demands, we consider a second flow of negative customers following a MAP with representation (F_0, F_1) where F's are of order $m_2 \times m_2$. The underlying Markov chain $J_2(t)$ of this MAP has the generator $F(=F_0+F_1)$ and a stationary row vector τ_{-1} of length m_2. The stationary arrival rate of negative customer is given by $\lambda_{-1} = \tau_{-1}F_1\mathbf{e}$. A negative customer has the effect of removing a waiting demand from the pool. The removal policy adopted in the paper is RCE (removal of a customer from the end of the queue). Further, we have assumed that an impatient customer in the pool leaves the system independently after a random time which is distributed as negative exponential with parameter $\alpha(>0)$.

3. Analysis

Let $L(t)$ denote the inventory level, let $X(t)$ denote the number of customers in the pool, let $J_1(t)$ denote the phase of the regular demand process, and let $J_2(t)$ denote the phase of the negative customer process at time t, respectively. From the assumptions made on the input and output processes, it can be shown that the quadruple $\{(L(t), X(t), J_1(t), J_2(t)),\ t \geq 0\}$ is a Markov process whose state space is

$$E = \{(i,k,j_1,j_2) : i = 0,1,\dots,S,\ k = 0,1,\dots,N,\ j_1 = 1,2,\dots,m_1,\ j_2 = 1,2,\dots,m_2\}. \tag{3.1}$$

We order the elements of E lexicographically. Then the infinitesimal generator P of the Markov process $\{(L(t), X(t), J_1(t), J_2(t)),\ t \geq 0\}$ has the following block partitioned form:

$$[P]_{ij} = \begin{cases} B_i, & j = i-1,\ i = 1,2,\dots,S, \\ C, & j = i+Q,\ i = 0,1,\dots,s, \\ A_i, & j = i,\ i = 0,1,\dots,S, \\ \mathbf{0}, & \text{otherwise}, \end{cases} \tag{3.2}$$

where

$$C = \beta I_{m_1 m_2 (N+1)}. \tag{3.3}$$

For $i = 1,2,\dots,s$,

$$B_i = I_{(N+1)} \otimes [D_1 \otimes I_{m_2} + i\gamma I_{m_1 m_2}]. \tag{3.4}$$

For $i = s+1, s+2, \ldots, S$,

$$[B_i]_{kl} = \begin{cases} D_1 \otimes I_{m_2} + i\gamma I_{m_1} \otimes I_{m_2}, & l = k,\ k = 0,1,\ldots,N, \\ \mu_k I_{m_1} \otimes I_{m_2}, & l = k-1,\ k = 1,2,\ldots,N, \\ \mathbf{0}, & \text{otherwise}. \end{cases} \tag{3.5}$$

For $i = 0$,

$$[A_i]_{kl} = \begin{cases} pD_1 \otimes I_{m_2}, & l = k+1,\ k = 0,1,\ldots,N-1, \\ I_{m_1} \otimes F_1 + k\alpha I_{m_1} \otimes I_{m_2}, & l = k-1,\ k = 1,2,\ldots,N, \\ (D_0 + qD_1) \oplus F - \beta I_{m_1} \otimes I_{m_2}, & l = k,\ k = 0, \\ (D_0 + qD_1) \oplus F_0 - (\beta + k\alpha) I_{m_1} \otimes I_{m_2}, & l = k,\ k = 1,2,\ldots,N-1, \\ D \oplus F_0 - (\beta + k\alpha) I_{m_1} \otimes I_{m_2}, & l = k,\ k = N, \\ \mathbf{0}, & \text{otherwise}. \end{cases} \tag{3.6}$$

For $i = 1,2,\ldots,s$,

$$[A_i]_{kl} = \begin{cases} I_{m_1} \otimes F_1 + k\alpha I_{m_1} \otimes I_{m_2}, & l = k-1,\ k = 1,2,\ldots,N, \\ D_0 \oplus F - (\beta + i\gamma) I_{m_1} \otimes I_{m_2}, & l = k,\ k = 0, \\ D_0 \oplus F_0 - (\beta + k\alpha + i\gamma) I_{m_1} \otimes I_{m_2}, & l = k,\ k = 1,2,\ldots,N, \\ \mathbf{0}, & \text{otherwise}. \end{cases} \tag{3.7}$$

For $i = s+1, s+2, \ldots, S$,

$$[A_i]_{kl} = \begin{cases} I_{m_1} \otimes F_1 + k\alpha I_{m_1} \otimes I_{m_2}, & l = k-1,\ k = 1,2,\ldots,N, \\ D_0 \oplus F - i\gamma I_{m_1} \otimes I_{m_2}, & l = k,\ k = 0, \\ D_0 \oplus F_0 - (\mu_k + k\alpha + i\gamma) I_{m_1} \otimes I_{m_2}, & l = k,\ k = 1,2,\ldots,N, \\ \mathbf{0}, & \text{otherwise}. \end{cases} \tag{3.8}$$

It may be noted that the matrices C, B's, and A's are all square matrices of order $m_1 m_2(N+1)$.

3.1. Steady state analysis. It can be seen from the structure of P that the homogeneous Markov process $\{(L(t), X(t), J_1(t), J_2(t)),\ t \geq 0\}$ on the finite state space E is irreducible. Hence, the limiting distribution

$$\phi_{(i,k,j_1,j_2)} = \lim_{t \to \infty} \Pr\left[L(t) = i,\ X(t) = k,\ J_1(t) = j_1,\ J_2(t) = j_2 \mid L(0), X(0), J_1(0), J_2(0)\right] \tag{3.9}$$

exists. Let

$$\begin{aligned} \phi_{(i,k,j_1)} &= (\phi_{(i,k,j_1,1)},\phi_{(i,k,j_1,2)},\dots,\phi_{(i,k,j_1,m_2)}), \quad j_1 = 1,2,\dots,m_1,\\ \phi_{(i,k)} &= (\phi_{(i,k,1)},\phi_{(i,k,2)},\dots,\phi_{(i,k,m_1)}), \quad k = 0,1,\dots,N,\\ \boldsymbol{\phi}^{(i)} &= (\phi_{(i,0)},\phi_{(i,1)},\dots,\phi_{(i,N)}), \quad i = 0,1,\dots,S,\\ \boldsymbol{\Phi} &= (\boldsymbol{\phi}^{(0)},\boldsymbol{\phi}^{(1)},\boldsymbol{\phi}^{(2)},\dots,\boldsymbol{\phi}^{(S-1)},\boldsymbol{\phi}^{(S)}). \end{aligned} \tag{3.10}$$

Then the vector of limiting probabilities $\boldsymbol{\Phi}$ satisfies

$$\boldsymbol{\Phi}P = \mathbf{0}, \qquad \boldsymbol{\Phi}\mathbf{e} = 1. \tag{3.11}$$

The first equation of the above yields the following set of equations:

$$\boldsymbol{\phi}^{(i+1)}B_{i+1} + \boldsymbol{\phi}^{(i)}A_i = \mathbf{0}, \quad i = 0,1,\dots,Q-1, \tag{3.12}$$

$$\boldsymbol{\phi}^{(i+1)}B_{i+1} + \boldsymbol{\phi}^{(i)}A_i + \boldsymbol{\phi}^{(i-Q)}C = \mathbf{0}, \quad i = Q, \tag{3.13}$$

$$\boldsymbol{\phi}^{(i+1)}B_{i+1} + \boldsymbol{\phi}^{(i)}A_i + \boldsymbol{\phi}^{(i-Q)}C = \mathbf{0}, \quad i = Q+1,Q+2,\dots,S-1, \tag{3.14}$$

$$\boldsymbol{\phi}^{(i)}A_i + \boldsymbol{\phi}^{(i-Q)}C = \mathbf{0}, \quad i = S. \tag{3.15}$$

The equations (except (3.13)) can be recursively solved to get

$$\boldsymbol{\phi}^{(i)} = \boldsymbol{\phi}^{(Q)}\theta_i, \quad i = 0,1,\dots,S, \tag{3.16}$$

where

$$\theta_i = \begin{cases} (-1)^{Q-i}B_QA_{Q-1}^{-1}B_{Q-1}\cdots B_{i+1}A_i^{-1}, & i = 0,1,\dots,Q-1,\\ I, & i = Q,\\ (-1)^{2Q-i+1}\sum_{j=0}^{S-i}\big[(B_QA_{Q-1}^{-1}B_{Q-1}\cdots B_{s+1-j}A_{s-j}^{-1})CA_{S-j}^{-1} & \\ \qquad \times(B_{S-j}A_{S-j-1}^{-1}B_{S-j-1}\cdots B_{i+1}A_i^{-1})\big], & i = Q+1,\dots,S. \end{cases} \tag{3.17}$$

Substituting the values of θ_i in (3.13) and in the normalizing condition, we get

$$\begin{aligned} &\boldsymbol{\phi}^{(Q)}\Bigg[(-1)^Q\sum_{j=0}^{s-1}\big[(B_QA_{Q-1}^{-1}B_{Q-1}\cdots B_{s+1-j}A_{s-j}^{-1})CA_{S-j}^{-1}\\ &\qquad\qquad \times(B_{S-j}A_{S-j-1}^{-1}B_{S-j-1}\cdots B_{Q+2}A_{Q+1}^{-1})\big]B_{Q+1} + A_Q\\ &\qquad + (-1)^QB_QA_{Q-1}^{-1}B_{Q-1}\cdots B_1A_0^{-1}C\Bigg] = \mathbf{0},\\ &\boldsymbol{\phi}^{(Q)}\Bigg[\sum_{i=0}^{Q-1}\big((-1)^{Q-i}B_QA_{Q-1}^{-1}B_{Q-1}\cdots B_{i+1}A_i^{-1}\big) + I\\ &\qquad + \sum_{i=Q+1}^{S}\Bigg((-1)^{2Q-i+1}\sum_{j=0}^{S-i}\big[(B_QA_{Q-1}^{-1}B_{Q-1}\cdots B_{s+1-j}A_{s-j}^{-1})CA_{S-j}^{-1}\\ &\qquad\qquad \times(B_{S-j}A_{S-j-1}^{-1}B_{S-j-1}\cdots B_{i+1}A_i^{-1})\big]\Bigg)\Bigg]\mathbf{e} = 1. \end{aligned} \tag{3.18}$$

Solving the above two equations, we get $\boldsymbol{\phi}^{(\boldsymbol{Q})}$.

4. System performance measures

In this section, we derive some stationary performance measures of the system. Using these measures, we can construct the total expected cost per unit time.

4.1. Mean inventory level. Let ζ_I denote the mean inventory level in the steady state. Since $\boldsymbol{\phi}^{(\boldsymbol{i})}$ is the steady state probability vector for ith inventory level with each component specifying a particular combination of number of customers in the pool, the phase of the regular demand process and phase of the negative arrival process, the quantity $\boldsymbol{\pi}^{(\boldsymbol{i})}\mathbf{e}$ gives the probability that the inventory level is i in the steady state. Hence, the mean inventory level is given by

$$\zeta_I = \sum_{i=1}^{S} i\boldsymbol{\phi}^{(\boldsymbol{i})}\mathbf{e}. \tag{4.1}$$

4.2. Mean reorder rate. Let ζ_R denote the expected reorder rate in the steady state. A reorder is triggered when the inventory level drops to s from the level $s+1$, due to anyone of the following events:

(1) a regular demand occurs,

(2) anyone of the $(s+1)$ items fails,

(3) anyone of the customers in the pool is selected.

This leads to

$$\zeta_R = \frac{1}{\lambda_1}\sum_{k=0}^{N}\left(\phi_{(s+1,k)}\left(D_1 \otimes I_{m_2}\right)\mathbf{e}\right) + \sum_{k=1}^{N}\mu_k\phi_{(s+1,j)}\mathbf{e} + \sum_{k=0}^{N}(s+1)\gamma\phi_{(s+1,k)}\mathbf{e}. \tag{4.2}$$

4.3. Mean perishable rate. The mean perishable rate ζ_{PR} in the steady state is given by

$$\zeta_{\text{PR}} = \sum_{i=1}^{S}\sum_{k=0}^{N} i\gamma\phi_{(i,k)}\mathbf{e}. \tag{4.3}$$

4.4. Expected number of pool customers. Let ζ_{PC} denote the expected number of pool customers in the steady state. Since $\phi_{(i,k)}$ is a vector of probabilities with the inventory level is i and the number of customer in the pool is k, the mean number of pool customers ζ_{PC} in the steady state is given by

$$\zeta_{\text{PC}} = \sum_{i=0}^{S}\sum_{k=1}^{N} k\phi_{(i,k)}\mathbf{e}. \tag{4.4}$$

4.5. Expected reneging rate. The expected reneging rate ζ_{RR} is given by

$$\zeta_{RR} = \sum_{i=0}^{S} \sum_{k=1}^{N} k\alpha\phi_{(i,k)}\mathbf{e}. \tag{4.5}$$

4.6. Mean rate of arrivals of negative demands. Let ζ_N denote the mean arrival rate of negative demands in the steady state. This is given by

$$\zeta_N = \frac{1}{\lambda_{-1}} \sum_{i=0}^{S} \sum_{k=1}^{N} \phi_{(i,k)}(I_{m_1} \otimes F_1)\mathbf{e}. \tag{4.6}$$

4.7. Average customers lost to the system. Let ζ_L be the average number of customers lost to the system. Then ζ_L is given by

$$\zeta_L = \frac{1}{\lambda_1}\left(\sum_{k=0}^{N-1} \phi_{(0,k)}(qD_1 \otimes I_{m_2})\mathbf{e} + \phi_{(0,N)}(D_1 \otimes I_{m_2})\mathbf{e}\right). \tag{4.7}$$

4.8. Mean waiting time. Let ζ_W denote the mean waiting time of the demands in the pool. Then by Little's formula

$$\zeta_W = \frac{\zeta_{PC}}{\lambda_e}, \tag{4.8}$$

where ζ_{PC} is the mean number of demands in the pool and the effective arrival rate (Ross [17]), λ_e is given by

$$\lambda_e = \frac{1}{\lambda_1} \sum_{k=0}^{N-1} \phi_{(0,k)}(pD_1 \otimes I_{m_2})\mathbf{e}. \tag{4.9}$$

5. Cost analysis

The expected total cost per unit time (expected total cost rate) in the steady state for this model is defined to be

$$\mathrm{TC}(S,s,N) = c_h\zeta_I + c_p\zeta_{PR} + c_r\zeta_{RR} + c_w\zeta_{PC} + c_s\zeta_R + c_n\zeta_N + c_{cl}\zeta_L, \tag{5.1}$$

where

(i) c_s: setup cost per order,
(ii) c_h: the inventory carrying cost per unit item per unit time,
(iii) c_r: reneging cost per customer per unit time,
(iv) c_w: waiting cost of a customer in the pool per unit time,
(v) c_n: loss per unit time due to arrival of a negative customer,

(vi) c_p: perishable cost per unit item per unit time,

(vii) c_{cl}: cost of a customer lost per unit time.

Substituting ζ's, we get

$$\begin{aligned}
\text{TC}(S,s,N) = {} & c_h\left(\sum_{i=1}^{S} i\phi^{i}\mathbf{e}\right) + c_p\left(\sum_{i=1}^{S}\sum_{k=0}^{N} i\gamma\phi_{(i,k)}\mathbf{e}\right) + c_r\left(\sum_{i=0}^{S}\sum_{k=1}^{N} k\alpha\phi_{(i,k)}\mathbf{e}\right) \\
& + c_s\left(\frac{1}{\lambda_1}\sum_{k=0}^{N}\left(\phi_{(s+1,k)}\left(D_1\otimes I_{m_2}\right)\mathbf{e}\right) + \sum_{k=1}^{N}\mu_k\phi_{(s+1,j)}\mathbf{e}\right. \\
& \qquad \left. + \sum_{k=0}^{N}(s+1)\gamma\phi_{(s+1,k)}\mathbf{e}\right) + c_w\left(\sum_{i=0}^{S}\sum_{k=1}^{N} k\phi_{(i,k)}\mathbf{e}\right) \\
& + c_n\left(\frac{1}{\lambda_{-1}}\sum_{i=0}^{S}\sum_{k=1}^{N}\phi_{(i,k)}\left(I_{m_1}\otimes F_1\right)\mathbf{e}\right) \\
& + c_{\text{cl}}\left(\frac{1}{\lambda_1}\left[\sum_{k=0}^{N-1}\phi_{(0,k)}\left(qD_1\otimes I_{m_2}\right)\mathbf{e} + \phi_{(0,N)}\left(D_1\otimes I_{m_2}\right)\mathbf{e}\right]\right).
\end{aligned} \tag{5.2}$$

Since the expected total cost function per unit time is obtained only implicitly, the analytical properties such as convexity of the cost function cannot be studied in general. However, we present some numerical examples in the next section to demonstrate the computability of the results derived in our work, and to illustrate the existence of local optimum when the expected total cost function is treated as a function of only two variables.

6. Numerical illustrations

We first consider the following case: the regular demand process is represented by the *MAP* with

$$D_0 = \begin{pmatrix} -50 & 0 \\ 0 & -5 \end{pmatrix}, \qquad D_1 = \begin{pmatrix} 39 & 11 \\ 3.9 & 1.1 \end{pmatrix} \tag{6.1}$$

and the arrival process of negative customer is represented by a *MAP* with

$$F_0 = \begin{pmatrix} -20 & 0 \\ 0 & -2 \end{pmatrix}, \qquad F_1 = \begin{pmatrix} 19 & 1 \\ 1.9 & 0.1 \end{pmatrix}. \tag{6.2}$$

The parameter and the costs are assumed to have the following values: $N = 5$, $p = .7$, $\beta = 25$, $\alpha = 1.3$, $\gamma = 0.8$, $\mu_i = 4i$, $i = 1,2,\dots,5$, $c_h = 0.1$, $c_w = 10$, $c_r = 6$, $c_p = 0.2$, $c_s = 10$, $c_n = 25$, $c_{\text{cl}} = 5$.

By taking $N = 5$, the total expected cost function per unit time, namely $\text{TC}(S,s,5)$, is considered as a function of two arguments, $\overline{\text{TC}}(S,s)$. The values of $\overline{\text{TC}}(S,s)$ are given in Table 6.1 for $s = 1,2,\dots,5$ and $S = 24,25,\dots,31$.

The optimal total expected cost rate for each s is shown in bold case and for each S is underlined. These values show that $\overline{\text{TC}}(S,s)$ is a convex function in (S,s) for the selected

Table 6.1. Total expected cost rate.

S	s				
	1	2	3	4	5
24	6.466367	6.320651	6.555001	7.027032	7.658907
25	**6.460348**	**6.317161**	6.542545	6.996300	7.601485
26	6.461928	6.321608	**6.539325**	6.977032	7.558750
27	6.470193	6.332993	6.544132	6.967672	7.528633
28	6.484365	6.350471	6.555952	**6.966925**	7.509426
29	6.503780	6.373324	6.573928	6.973706	7.499711
30	6.527864	6.400938	6.597334	6.987100	**7.498298**
31	6.556120	6.432785	6.625548	7.006328	7.504181

values of S and s and for the fixed values of other parameters and costs. The local optimum occurs at $(S,s) = (25,2)$.

Next, we will consider the following five *MAP*s for arrival of regular customers as well as for arrival negative customers so that these processes are normalized to have a specific (given) demand rate $\lambda_1(\lambda_{-1})$ when considered for arrival of regular (negative) customers.

(1) *Exponential (Exp)*

$$H_0 = (-1) \qquad H_1 = (1). \tag{6.3}$$

(2) *Erlang (Erl)*

$$H_0 = \begin{pmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \qquad H_1 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}. \tag{6.4}$$

(3) *Hyper-exponential (HExp)*

$$H_0 = \begin{pmatrix} -10 & 0 \\ 0 & -1 \end{pmatrix}, \qquad H_1 = \begin{pmatrix} 9 & 1 \\ 0.9 & 0.1 \end{pmatrix}. \tag{6.5}$$

(4) *MAP with Negative correlation (MNC)*

$$H_0 = \begin{pmatrix} -2 & 2 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -450.50 \end{pmatrix}, \qquad H_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0.02 & 0 & 1.98 \\ 445.995 & 0 & 4.505 \end{pmatrix}. \tag{6.6}$$

(5) *MAP with positive correlation (MPC)*

$$H_0 = \begin{pmatrix} -2 & 2 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -450.50 \end{pmatrix}, \qquad H_1 = \begin{pmatrix} 0 & 0 & 0 \\ 1.98 & 0 & 0.02 \\ 4.505 & 0 & 445.995 \end{pmatrix}. \tag{6.7}$$

Table 6.2. Values of S^* and s^* (in the upper row) and the optimum cost rate (in the lower row) for arrival processes for regular and negative customer.

		*MAP*s of negative arrivals				
		Exp	Erl	HExp	MNC	MPC
*MAP*s of regular arrival	Exp	24 3 1.572453	22 2 1.499994	27 4 1.796481	25 3 1.640885	25 3 1.667935
	Erl	22 2 1.479162	21 2 1.479162	26 4 1.690255	24 3 1.548586	24 3 1.556385
	HExp	27 3 1.815940	26 3 1.726598	32 5 2.087582	29 4 1.894005	30 4 1.946499
	MNC	24 3 1.595167	23 2 1.531924	28 4 1.815005	25 3 1.652637	26 3 1.698333
	MPC	19 0 3.757670	18 0 3.680964	23 1 4.031168	21 1 3.801462	22 1 4.047543

All the above *MAP*s are qualitatively different in that they have different variance and correlation structures. The first three processes are special cases of renewal processes and the correlation between arrival times is 0. The demand process labeled as MNC has correlated arrival with correlation coefficient -0.488909 and the arrivals corresponding to the process labeled MPC has positive correlation coefficient 0.488909.

For the next example, we take $\lambda_1 = 15$, $\lambda_{-1} = 60$, $N = 5$, $p = 0.3$, $\beta = 5$, $\alpha = 2$, $\gamma = 0.6$, $\mu_i = 4i$, $i = 1,2,\dots,5$, $c_h = 0.01$, $c_w = 15$, $c_r = 6$, $c_p = 0.1$, $c_s = 2$, $c_n = 3$, $c_{\text{cl}} = 5$.

Table 6.2 gives the optimum values, S^* and s^*, that minimize the expected total cost for each of the five *MAP*s for arrivals of regular demands considered with each of the five *MAP*s of negative customers. The associated expected total cost values are also given.

For the third example, the various costs and parameters are assumed to be as follows: $\lambda_1 = 8$, $\lambda_{-1} = 60$, $S = 23$, $s = 4$, $p = 0.3$, $\beta = 5$, $\alpha = 1.5$, $\gamma = 0.2$, $\mu_i = 4i$, $i = 1,2,\dots,N$, $c_h = 0.1$, $c_w = 3$, $c_r = 4$, $c_p = 0.2$, $c_s = 2$, $c_n = 3$, $c_{\text{cl}} = 8$.

Table 6.3 summarizes the optimum N^* values along with the optimum total cost rate. The upper entries in each cell gives N^* value and the lower entry corresponding to optimal cost rate.

7. Conclusion

In this work, we modeled an inventory system of perishable commodities in which the arrival of regular and negative customers have independent *MAP*s. The customers whose requirement cannot be met immediately join a pool of finite capacity; their demands are satisfied after a randomly distributed time when the inventory level is above s after the replenishment. The customers in the pool may renege according to exponentially distributed times. We have derived the steady state solutions of the joint processes and illustrated the results by numerical examples. Since we have assumed *MAP*s for arrivals, the proposed model covers a large collection of renewal and nonrenewal processes and can be applied to wide range of inventory systems.

Table 6.3. Value of N^* (in the upper row) and the optimum cost rate (in the lower row) for arrival processes for regular and negative customer.

		*MAP*s of negative arrivals				
		Exp	Erl	HExp	MNC	MPC
*MAP*s of regular arrival	Exp	8 1.9835	8 1.9813	4 1.9929	9 1.9861	4 1.9867
	Erl	6 1.9442	5 1.9429	4 1.94977	7 1.9458	4 1.9459
	HExp	7 2.0913	8 2.0864	4 2.1123	8 2.09686	4 2.0989
	MNC	8 1.9910	7 1.9888	4 2.0009	8 1.9934	4 1.9947
	MPC	4 5.1857	4 5.1788	3 5.2106	4 5.1884	2 5.2512

Notations

$[A]_{ij}$: The element/submatrix at (i, j)th position of A

$\mathbf{0}$: Zero matrix

I: An identity matrix

I_k: An identity matrix of order k

$A \otimes B$: Kronecker product of matrices A and B

$A \oplus B$: Kronecker sum of matrices A and B

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